Chapter 7

Electromagnetic Waves

Overview

- This chapter shows that most of the physics principles related to electric and magnetic fields that we have studied so far can be summarized in only four equations, known as Maxwell's equations.

- We examine the science and engineering of magnetic materials.

7.1. Maxwell's Equations:

7.1.1. Gauss's Law for Magnetic Particles:

The simplest magnetic structure that can exist is a magnetic dipole. Magnetic monopoles do not exist (as far as we know).

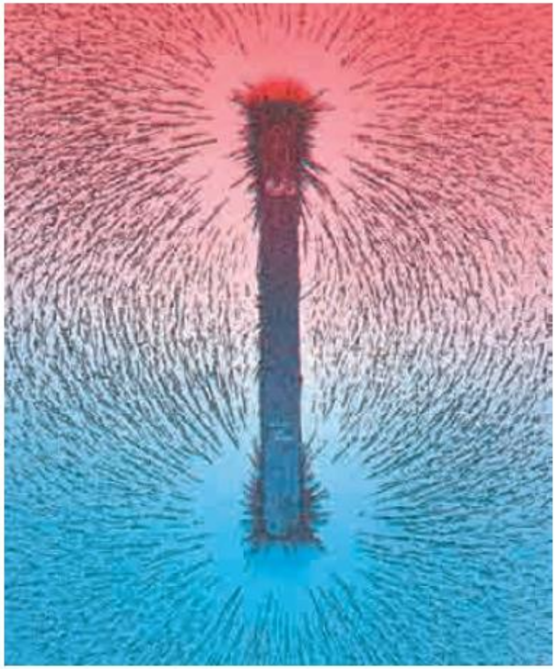


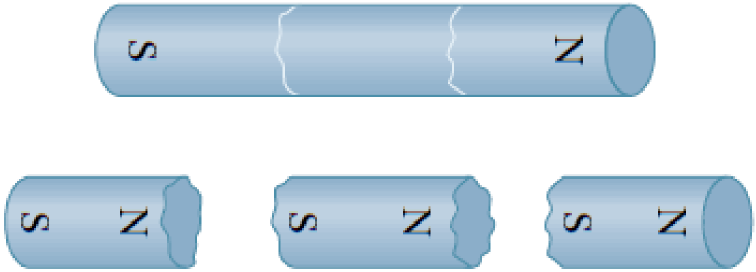
Fig. 32-2 A bar magnet is a magnetic dipole. The iron filings suggest the magnetic field lines. (Colored light fills the background.) *(Runk/Schoenberger/Grant Heilman Photography)*

$$\Phi_B = \oint \vec{B} d\vec{A} = 0 \text{ (Gauss' law for magnetic fields)}$$

The law asserts that the net magnetic flux F_B through any closed Gaussian surface is zero. Here **B** is the magnetic field. Recall Gauss' law for electric fields:

$$\Phi_E = \oint \vec{E} d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

Gauss' law for magnetic fields is a formal way of saying that magnetic monopoles do not exist

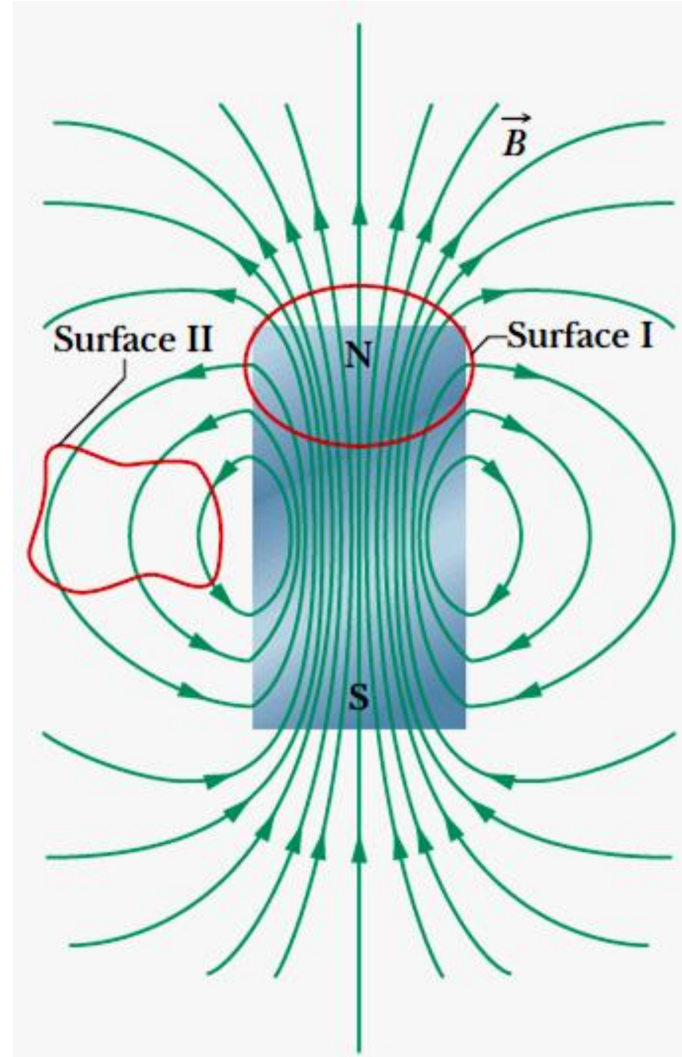


If you break a magnet, each fragment becomes a separate magnet, with its own north and south poles

Gauss' law for magnetic fields holds for structures even if the Gaussian surface does not enclose the entire structure:

1. *Gaussian surface II* near the bar magnet of Fig. 32-4 encloses no poles, and we can easily conclude that the net magnetic flux through it is zero.

2. For *Gaussian surface I*, it may seem to enclose only the north pole of the magnet because it encloses the label N and not the label S. However, a south pole must be associated with the lower boundary of the surface because magnetic field lines enter the surface there. Thus, *Gaussian surface I* encloses a magnetic dipole, and the net flux through the surface is zero.



7.1.2. Induced Magnetic Fields:

We saw that a changing magnetic flux induces an electric field and we ended up with **Faraday's law of induction**:

$$\oint \vec{E}d\vec{s} = -\frac{d\phi_B}{dt}$$

So, we should be tempted to ask: *Can a changing electric flux induce a magnetic field?* The answer: *It can*.

We have **Maxwell's law of induction**:

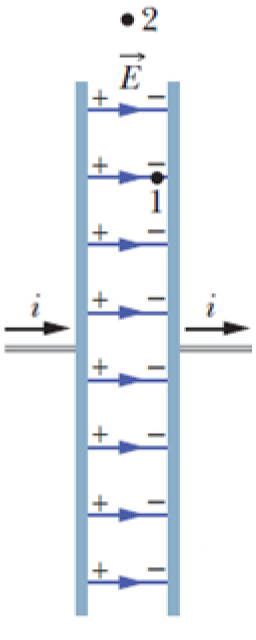
$$\oint \vec{B}d\vec{s} = \mu_0\varepsilon_0 \frac{d\phi_E}{dt}$$

Here B is the magnetic field induced along a closed loop by the changing electric flux ϕ_E in the region encircled by that loop

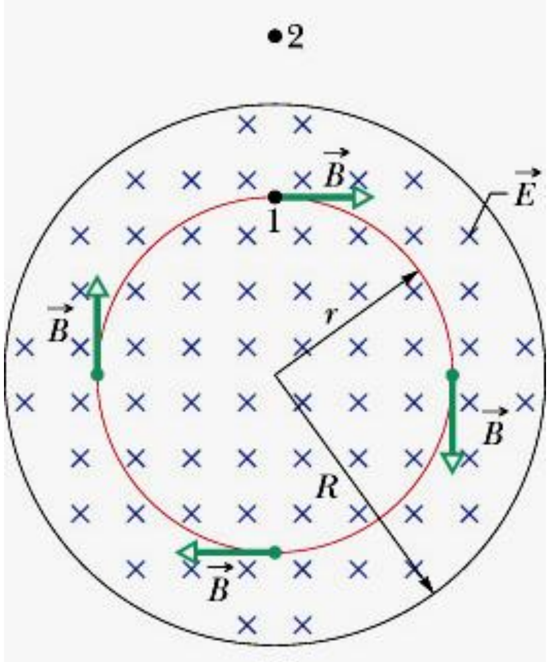
Example: A parallel-plate capacitor with circular plates is charging. We assume that the charge on the capacitor is being **increased** at a steady rate. So, the electric field magnitude between the plates is also increasing at a steady rate.

• Experiment proves that:

- B is induced a loop directed as shown.



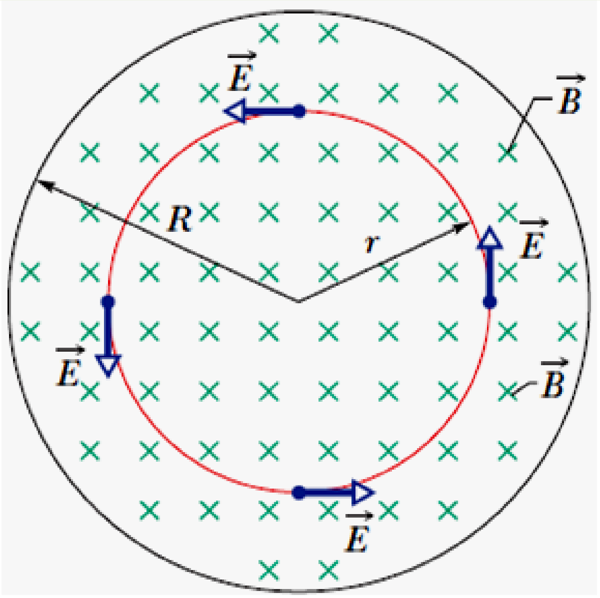
(a)



(b)

- B has the same magnitude at every point around the loop and thus has circular symmetry about the central axis of the plates.
- We also find that a magnetic field is induced around a larger loop as well, e.g. through point 2 outside the plates.
- The change of electric field induces B between the plates, both inside and outside the gap.

The induced $\vec{E}$ direction here is opposite the induced $\vec{B}$ direction in the preceding figure.



Ampere-Maxwell Law:

Recall Ampere's law:

$$\oint \vec{B} d\vec{s} = \mu_0 i_{enc} \text{ (Ampere's law)}$$

Here i_{enc} is the current encircled by the closed loop. In a more complete form:

$$\oint \vec{B} d\vec{s} = \mu_0 \varepsilon_0 \frac{d\phi_E}{dt} + \mu_0 i_{enc} \text{ (Ampere - Maxwell law)}$$

- If there is a current but no change in electric flux (such as with a wire carrying a constant current), the first term on the right side of the second equation is zero, and so it reduces to the first equation, Ampere's law.

7.1.3. Displacement Current:

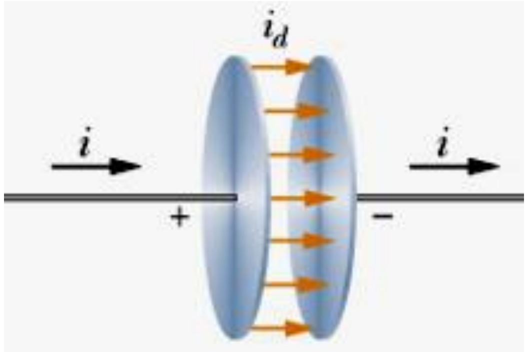
- In the Maxwell-Ampere equation, $\varepsilon(d\phi_E/dt)$ must have the dimension of a current. This product has been treated as being a fictitious current called the **displacement current** i_d:

$$\boxed{i_d = \varepsilon_0 \frac{d\phi_E}{dt} \text{ (displacement current)}}$$

- So:

$$\oint \vec{B}d\vec{s} = \mu_0 i_{d,enc} + \mu_0 i_{enc} \text{ (Ampere - Maxwell law)}$$

$i_{d,enc}$: the displacement current encircled by the integration loop.



- Consider a charging capacitor, the charge q on the plates:

$$q = \varepsilon_0 AE$$

The real current i:

$$\frac{dq}{dt} = i = \varepsilon_0 A \frac{dE}{dt}$$

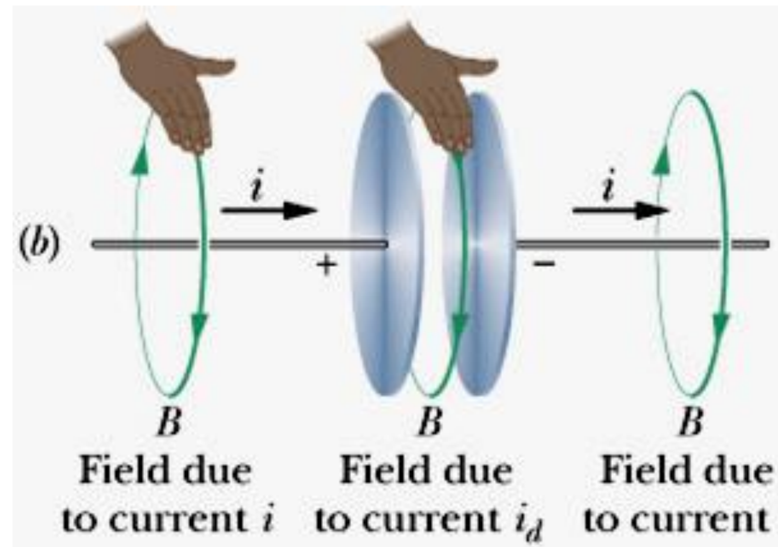
The displacement current i_d:

$$i_d = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 \frac{d(EA)}{dt} = \varepsilon_0 A \frac{dE}{dt} = i$$

$$\boxed{i_d = i \text{ (displacement current in a capacitor)}}$$

• Thus, we can consider the fictitious current i_d to be simply a continuation of the real current i from one plate across the capacitor gap, to the other plate.

• Although **no charge** actually moves across the gap between the plates, the idea of the fictitious current i_d can help us to quickly find the direction and magnitude of an induced magnetic field:



Recall: Inside a long straight wire with current:

$$B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r$$

We consider the space between the plates to be an imaginary circular wire of radius R carrying the imaginary current i_d. So, B at a point inside the capacitor:

$$B = \left(\frac{\mu_0 i_d}{2\pi R^2}\right) r \text{ (inside a circular capacitor)}$$

At a point outside the capacitor: $$B = \frac{\mu_0 i_d}{2\pi r} \text{ (outside a circular capacitor)}$$

7.1.4. Maxwell's Equations (Integral form):

With the assumption that no dielectric or magnetic materials are present:

Name	Equation	
Gauss' law for electricity	$\oint \vec{E} d\vec{A} = q_{enc} / \varepsilon_0$	Relates net electric flux to net enclosed electric charge
Gauss' law for magnetism	$\oint \vec{B} d\vec{A} = 0$	Relates net magnetic flux to net enclosed magnetic charge
Faraday's law	$\oint \vec{E} d\vec{s} = -\dfrac{d\phi_B}{dt}$	Relates induced electric field to changing magnetic flux
Ampere-Maxwell law	$\oint \vec{B} d\vec{s} = \mu_0 \varepsilon_0 \dfrac{d\phi_E}{dt} + \mu_0 i_{enc}$	Relates induced magnetic field to changing electric flux and to current

Maxwell's Equations (Differential form):

$$\boxed{\begin{aligned}&\nabla\cdot E=\rho/\varepsilon_0\\&\nabla\cdot B=0\\&\nabla\times E=-\frac{\partial B}{\partial t}\\&\nabla\times B=\mu_0\varepsilon_0\frac{\partial E}{\partial t}+\mu_0 j_c\\&\text{where }\nabla=\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}\end{aligned}}$$

ρ: charge density; ε_0: permittivity;
μ_0: permeability; j_c: current density

∇: del or nabla operator
$\nabla\bullet$: the divergence operator
$\nabla\times$: the curl operator

For example, a vector function F(x,y,z):

$$\mathrm{div}F=\nabla\cdot F=\frac{\partial F_x}{\partial x}+\frac{\partial F_y}{\partial y}+\frac{\partial F_z}{\partial z}$$

$$\mathrm{curl}F=\nabla\times F=\hat{i}\left(\frac{\partial F_z}{\partial y}-\frac{\partial F_y}{\partial z}\right)+\hat{j}\left(\frac{\partial F_x}{\partial z}-\frac{\partial F_z}{\partial x}\right)+\hat{k}\left(\frac{\partial F_y}{\partial x}-\frac{\partial F_x}{\partial y}\right)$$

$$\text{or}\qquad \nabla\times F=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\ \frac{\partial}{\partial x}&\frac{\partial}{\partial y}&\frac{\partial}{\partial z}\\ F_x&F_y&F_z\end{vmatrix}$$

7.2. Magnetism of Matter:

7.2.1. Magnets: The Magnetism of Earth:

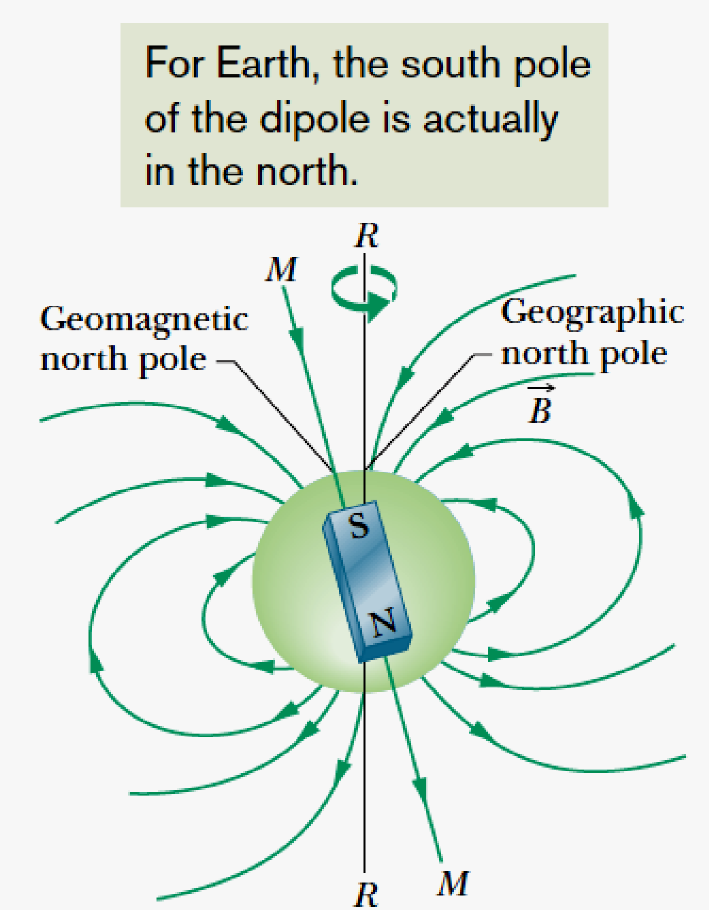


Fig. 32-8 Earth's magnetic field represented as a dipole field. The dipole axis MM makes an angle of 11.5° with Earth's rotational axis RR. The south pole of the dipole is in Earth's Northern Hemisphere.

- *A magnet is a material or object that produces a magnetic field.*

- Earth is a huge magnet, its magnetic field can be approximated as the field of a huge bar magnet – a magnetic dipole – that straddles the center of the planet.

- The magnitude of Earth's magnetic field at the Earth's surface ranges from 25 to 65 μT (0.25-0.65 G).

- At any point on Earth's surface, the measured magnetic field may differ appreciably, in both magnitude and direction, from the idealized dipole field.

7.2.2. Magnetism and Electrons:

- *Magnetic materials, from lodestones to videotapes, are magnetic because of the electrons within them.*
- We have already seen one way in which electrons generate a magnetic field: send them through a wire as an electric current, and their motion produces a magnetic field around the wire.
- There are two more ways, each involving a magnetic dipole moment that produces a magnetic field in the surrounding space.

Spin Magnetic Dipole Moment:

An electron has an intrinsic angular momentum called its spin angular momentum (or just spin), **S**; associated with this spin is an intrinsic spin magnetic dipole moment, μ_s .

$$\vec{\mu}_S = -\frac{e}{m}\vec{S}$$

in which *e* is the elementary charge (1.60×10^{-19} C) and *m* is the mass of an electron (9.11×10^{-31} kg).

Spin $\vec{S}$ is different from the angular momentum $\vec{l} = \vec{r} \times \vec{p}$ in two respects:

• Spin $\vec{S}$ itself cannot be measured. However, its component along any axis can be measured.
• A measured component of $\vec{S}$ is quantized, which is a general term that means it is restricted to certain values. A measured component of $\vec{S}$ can have only two values, which differ only in sign.

Assuming that the component of spin S is measured along the z axis of a coordinate system:

$$S_z = m_S \frac{h}{2\pi}, \text{for } m_S = \pm\frac{1}{2}$$

where m_S is called the *spin magnetic quantum number* and h=6.63 $\times$ 10^{-34} Js is the Planck constant.

- When S_z is parallel to the z axis, m_S is +1/2 and the electron is said to be *spin up*.
- When S_z is antiparallel to the z axis, m_S is -1/2 and the electron is said to be *spin down*.

The spin magnetic dipole moment $\vec{\mu}_S$ of an electron also cannot be measured, only its component along any axis can be measured. and that component is quantized too.

$$\mu_{S,z} = -\frac{e}{m} S_z$$

$$\Rightarrow \mu_{S,z} = \pm \frac{eh}{4\pi m}$$

"+" and "-" correspond to $\mu_{S,z}$ being parallel and antiparallel to the z axis, respectively.

$$\boxed{\mu_B = \frac{eh}{4\pi m} = 9.27\times10^{-24}\,\text{J/T (Bohr magneton)}}$$

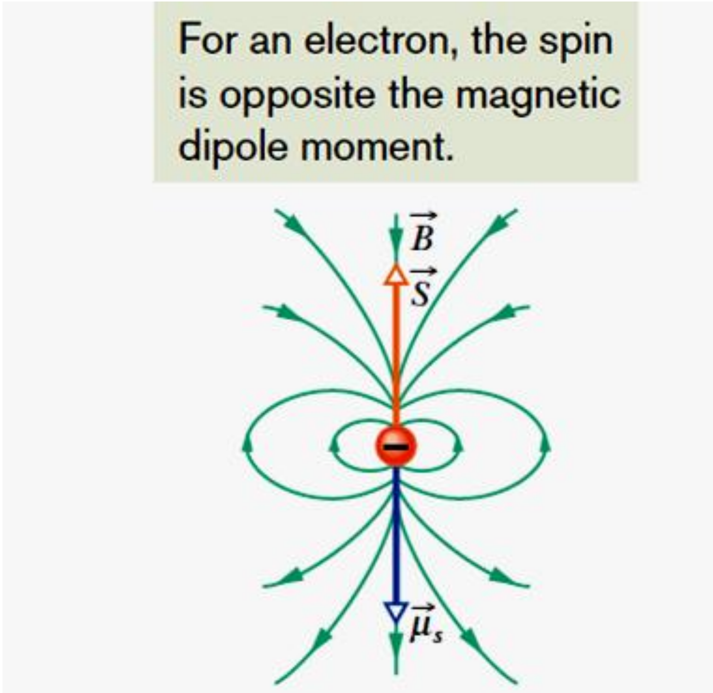


Fig. 32-10 The spin $\vec{S}$, spin magnetic dipole moment $\vec{\mu}_s$, and magnetic dipole field $\vec{B}$ of an electron represented as a microscopic sphere.

When an electron in an external B field, a potential energy U can be associated with the orientation of the electron's $\vec{\mu}_S$ when B_{ext} is the exterior magnetic field aligned along the z-axis.

$$U = -\vec{\mu}_S \cdot \vec{B}_{ext} = -\mu_{S,z} B_{ext}$$

Note: We do not examine the contribution of the magnetic dipole moments of protons and neutrons to the magnetic field of atoms because they are about a thousand times smaller than that due to an electron, due to their much larger mass.

Orbital Magnetic Dipole Moment:

- When an electron is in an atom, it has an additional angular momentum called its orbital angular momentum, $\vec{L}_{\mathrm{orb}}$. Associated with it is an orbital magnetic dipole moment, $\vec{\mu}_{\mathrm{orb}}$; the two are related by

$$\vec{\mu}_{\mathrm{orb}} = -\frac{e}{2m}\vec{L}_{\mathrm{orb}}$$

- Only the component along any axis of the orbital angular momentum can be measured, and that component is quantized

$$L_{\mathrm{orb},z} = m_l \frac{h}{2\pi} \text{ for } m_l = 0, \pm 1, \pm 2, \ldots, \pm(\text{limit})$$

in which is m_l called the orbital magnetic quantum number and "limit" refers to its largest allowed integer value.

- Similarly, only the component of the magnetic dipole moment of an electron along an axis can be measured, and that component is quantized.

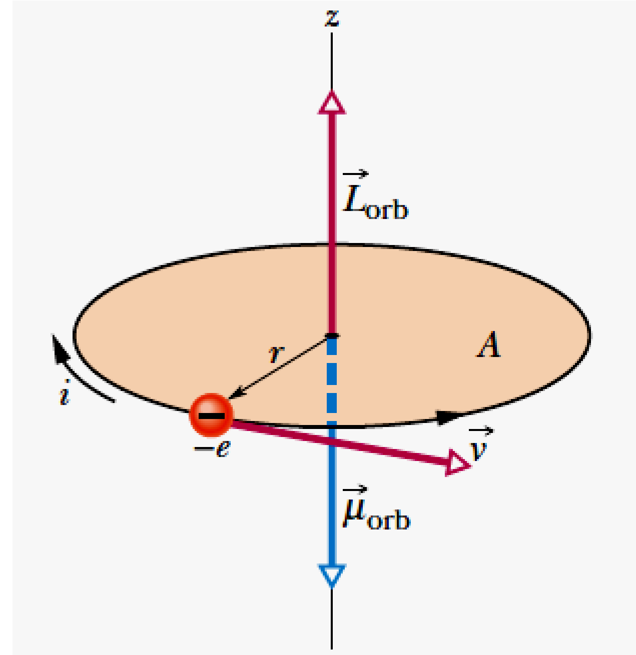
$$\Rightarrow \mu_{\mathrm{orb},z} = -m_l \frac{eh}{4\pi m} = -m_l \mu_B \ (\mu_B : \text{Bohr magneton})$$

- In an external B field, a potential energy U:

$$U = -\vec{\mu}_{\mathrm{orb}} \cdot \vec{B}_{\mathrm{ext}} = -\mu_{\mathrm{orb},z} B_{\mathrm{ext}}$$

where the z axis is taken in the direction of B_{ext}

Loop Model for Electron Orbits:



We can obtain the relationship between μ_{orb} and L_{orb} with the non-quantum derivation as follows:

- We imagine an electron moving in a circular path as shown
- The motion of the electron is equivalent to a current i
- The magnitude of the orbital magnetic dipole moment of such a current loop is:

$$\mu_{orb} = iA$$

where A is the area enclosed by the loop

The direction of $\vec{\mu}_{orb}$ is downward

The current:

$$i = \frac{\text{charge}}{\text{time}} = \frac{e}{2\pi r / v} \Rightarrow \mu_{orb} = \frac{e}{2\pi r / v}\pi r^2 = \frac{evr}{2} \quad (1)$$

The electron's orbital angular momentum (using $\vec{l} = \vec{r} \times \vec{p}$):

$$L_{orb} = mrv \sin 90^0 = mrv \quad (2)$$

The direction of $\vec{L}_{orb}$ is upward

(1) & (2):

$$\vec{\mu}_{orb} = -\frac{e}{2m}\vec{L}_{orb}$$

Loop Model for Electron Orbits in a Nonuniform Field:

• We study an electron orbit as a current loop but now in a nonuniform magnetic field. This helps us understand forces acting in magnetic materials in a nonuniform magnetic field.

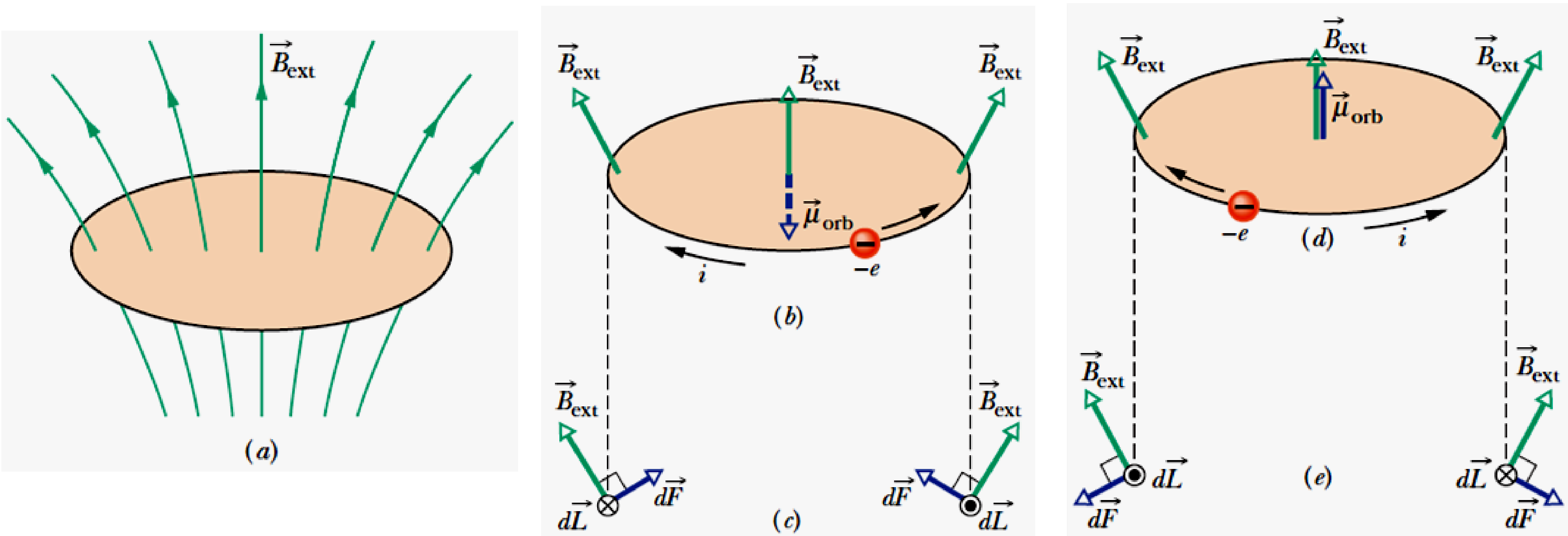


Fig. 32-12 (a) A loop model for an electron orbiting in an atom while in a nonuniform magnetic field $\mathbf{B}_{ext}$. (b) Charge e moves counterclockwise; the associated conventional current *i* is clockwise. (c) The magnetic forces ***dF*** on the left and right sides of the loop, as seen from the plane of the loop. The net force on the loop is upward. (d) Charge e now moves clockwise. (e) The net force on the loop is now downward.

$$d\vec{F} = i d\vec{L} \times \vec{B}_{ext}$$

7.2.3. Magnetic Materials:

Each electron in an atom has an orbital magnetic dipole moment and a spin magnetic dipole moment. The resultant of these two vectors combines with similar resultants for all other electrons in the atom, and the resultant for each atom combines with those for all the other atoms in a sample of a material. In a magnetic material the combination of all these magnetic dipole moments produces a magnetic field. There are three general types of magnetism:

1. **Diamagnetism:** In diamagnetism, weak magnetic dipole moments are produced in the atoms of the material when the material is placed in an external magnetic field $\mathbf{B}_{ext}$; the combination gives the material as a whole only a feeble net magnetic field.
2. **Paramagnetism:** Each atom of such a material has a permanent resultant magnetic dipole moment, but the moments are randomly oriented in the material and the material lacks a net magnetic field. An external magnetic field $\mathbf{B}_{ext}$ can partially align the atomic magnetic dipole moments to give the material a net magnetic field.
3. **Ferromagnetism:** Some of the electrons in these materials have their resultant magnetic dipole moments aligned, which produces regions with strong magnetic dipole moments. An external field $\mathbf{B}_{ext}$ can align the magnetic moments of such regions, producing a strong magnetic field for the material.

Diamagnetism:

A diamagnetic material placed in an external magnetic field $\vec{B}_{ext}$ develops a magnetic dipole moment directed opposite $\vec{B}_{ext}$. If the field is nonuniform, the diamagnetic material is repelled *from* a region of greater magnetic field *toward* a region of lesser field.

If a magnetic field is applied, the diamagnetic material develops a magnetic dipole moment and experiences a magnetic force. When the field is removed, both the dipole moment and the force disappear.

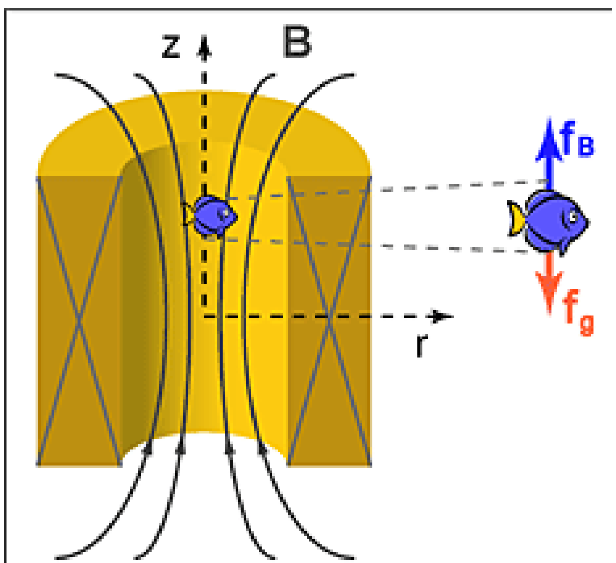
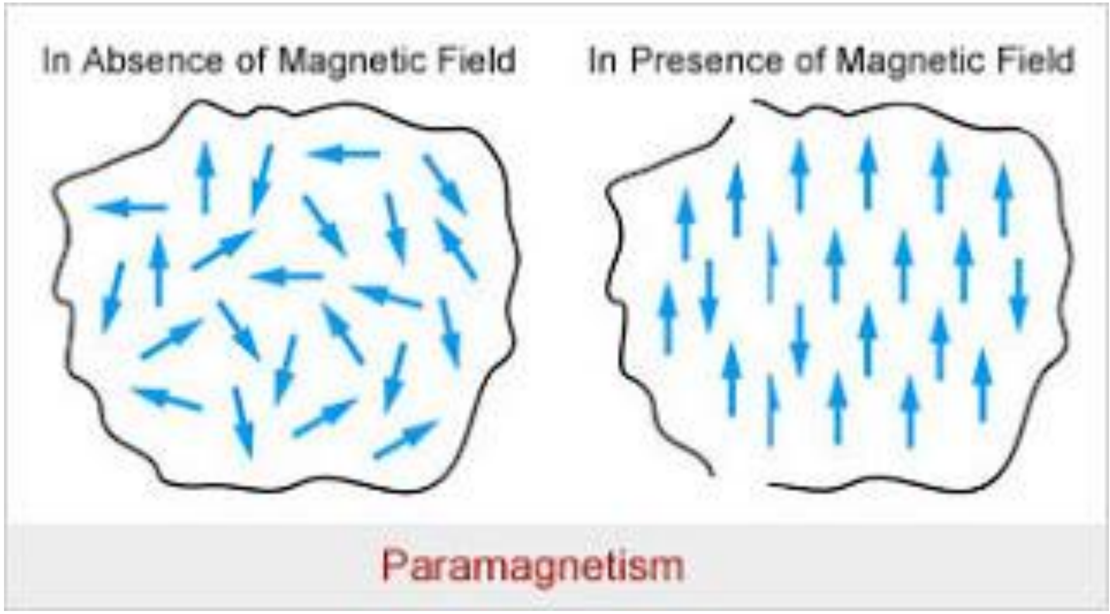


Fig. 32-13 An overhead view of a frog that is being levitated in a magnetic field produced by current in a vertical solenoid below the frog. *(Courtesy A. K. Gein, High Field Magnet Laboratory, University of Nijmegen, The Netherlands)*

clip for diamagnetic levitation

Paramagnetism:

A paramagnetic material placed in an external magnetic field $\vec{B}_{ext}$ develops a magnetic dipole moment in the direction of $\vec{B}_{ext}$. If the field is nonuniform, the paramagnetic material is attracted *toward* a region of greater magnetic field *from* a region of lesser field.

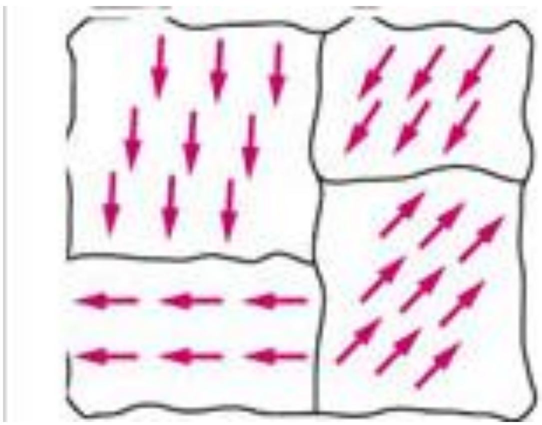


Liquid oxygen is suspended between the two pole faces of a magnet because the liquid is paramagnetic and is magnetically attracted to the magnet. *(Richard Megna/Fundamental Photographs)*

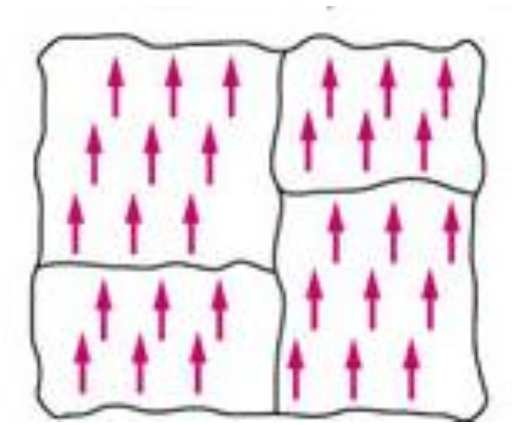
Ferromagnetism:

A ferromagnetic material placed in an external magnetic field $\vec{B}_{ext}$ develops a strong magnetic dipole moment in the direction of $\vec{B}_{ext}$. If the field is nonuniform, the ferromagnetic material is attracted *toward* a region of greater magnetic field *from* a region of lesser field.

In Absence of Magnetic Field



In Presence of Magnetic Field



- Ferromagnetism is very important in industry and modern technology, and is the basis for many electrical and electromechanical devices such as electric motors, generators, transformers, and magnetic storage such as hard disks.

- Every ferromagnetic substance has its own individual temperature, called the Curie temperature, or Curie point, above which it loses its ferromagnetic properties.

• The magnetization of a ferromagnetic material such as iron can be studied with an arrangement called a *Rowland ring (Fig. 32-15).*

• The material is formed into a thin toroidal core of circular cross section. A primary coil P having *n turns* per unit length is wrapped around the core and carries current i_P. If the iron core were not present, the magnitude of the magnetic field inside the coil would be:

$$B_0 = \mu_0 i_P n$$

• With the iron core present, the magnetic field inside the coil is greater than $\mathbf{B}_o$, usually by a large amount:

$$B = B_0 + B_M$$

Here $\boldsymbol{B_M}$ is the magnitude of the magnetic field contributed by the iron core.

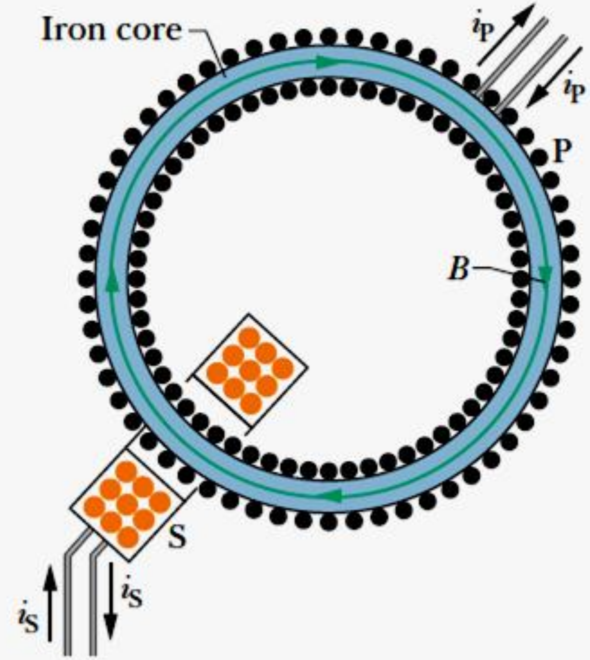


Fig. 32-15 A Rowland ring. A primary coil P has a core made of the ferromagnetic material to be studied (here iron). The core is magnetized by a current i_P sent through coil P. (The turns of the coil are represented by dots.) The extent to which the core is magnetized determines the total magnetic field $\vec{B}$ within coil P. Field $\vec{B}$ can be measured by means of a secondary coil S.

Hysteresis

Magnetization curves for ferromagnetic materials are not retraced as we increase and then decrease the external magnetic field B_0.

Figure 32-18 is a plot of B_M versus B_0 during the following operations with a Rowland ring:

I. Starting with the iron unmagnetized (point *a)*, increase the current in the toroid until B_0 $(=m_0 in)$ has the value corresponding to point *b;*

II. Reduce the current in the toroid winding (and thus B_0) back to zero (point *c);*

III. Reverse the toroid current and increase it in magnitude until B_0 has the value corresponding to point *d;*

IV. Reduce the current to zero again (point *e);*

V. Reverse the current once more until point *b* is reached again.

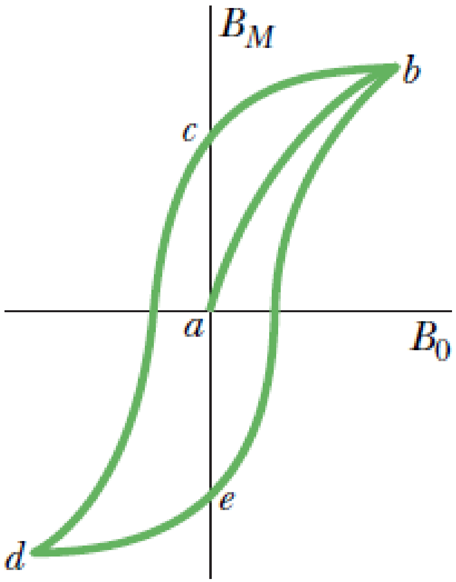


Fig. 32-18 A magnetization curve (*ab*) for a ferromagnetic specimen and an associated hysteresis loop (*bcdeb*).

The lack of retraceability shown in Fig. 32-18 is called **hysteresis**, and the curve *bcdeb* is called a ***hysteresis loop.***

• Hysteresis can be understood through the concept of magnetic domains, which are regions of a ferromagnetic material in which the magnetic dipole moments are aligned parallel:

- When the applied magnetic field B_0 is increased and then decreased back to its initial value, the domains do not return completely to their original configuration but retain some "memory" of their alignment after the initial increase. This memory of magnetic materials is applicable for the magnetic storage of information, as on magnetic tapes and disks.

(image source: wikipedia)

- This memory of the alignment of domains can also occur naturally. What is the origin of lodestones?

Lightning sends currents through the ground that produce strong magnetic fields. The fields can suddenly magnetize any ferromagnetic material in nearby rock. Because of hysteresis, such rock material retains some of that magnetization after the lightning strike.