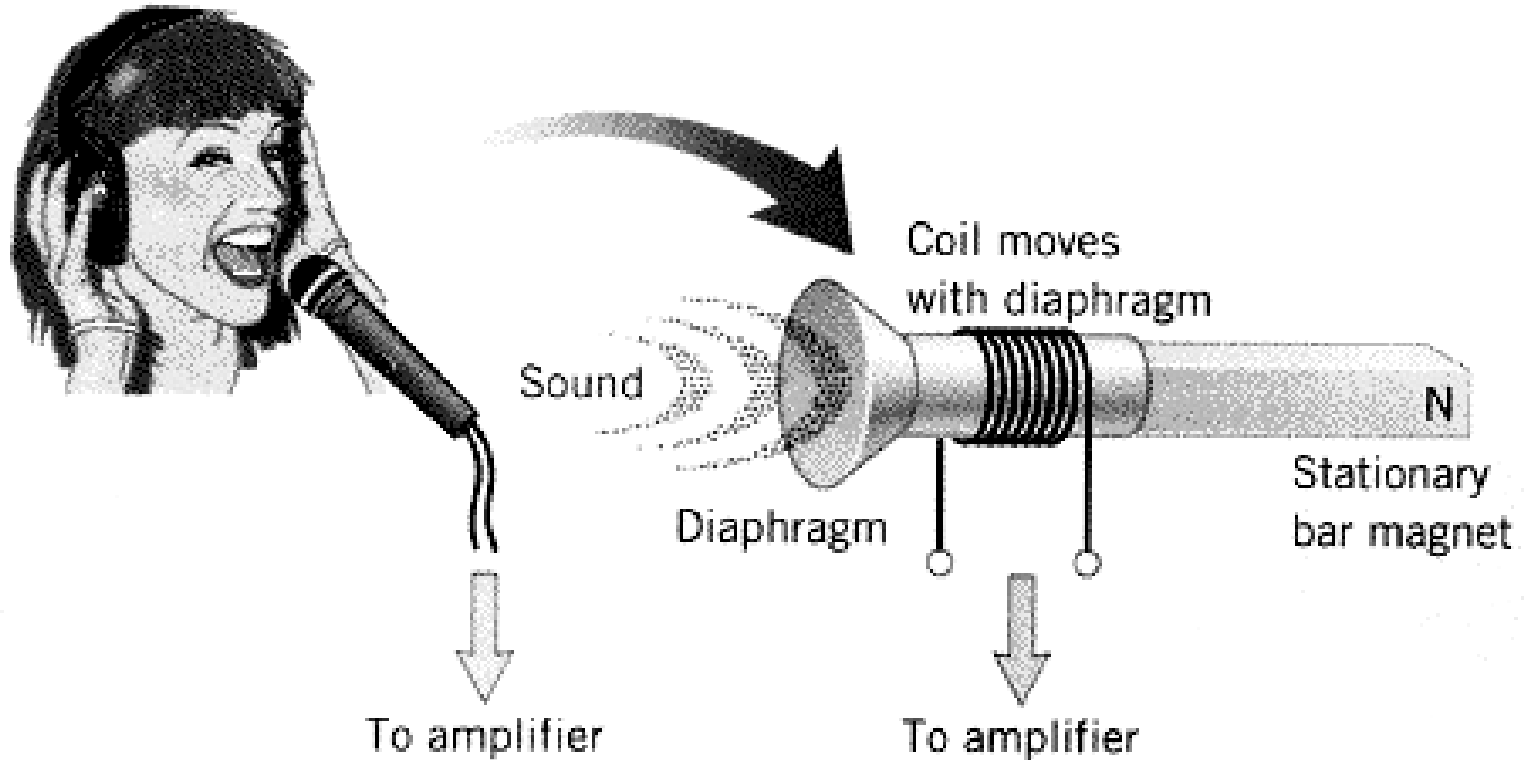


# Chapter 5

## Electromagnetic Induction



# Overview

In the last chapter, we studied how a current produces a magnetic field. Here we will study the reverse effect: A magnetic field can produce an electric field that can drive a current. This link between a magnetic field and the electric field it produces (induces) is now called *Faraday's law of induction*.

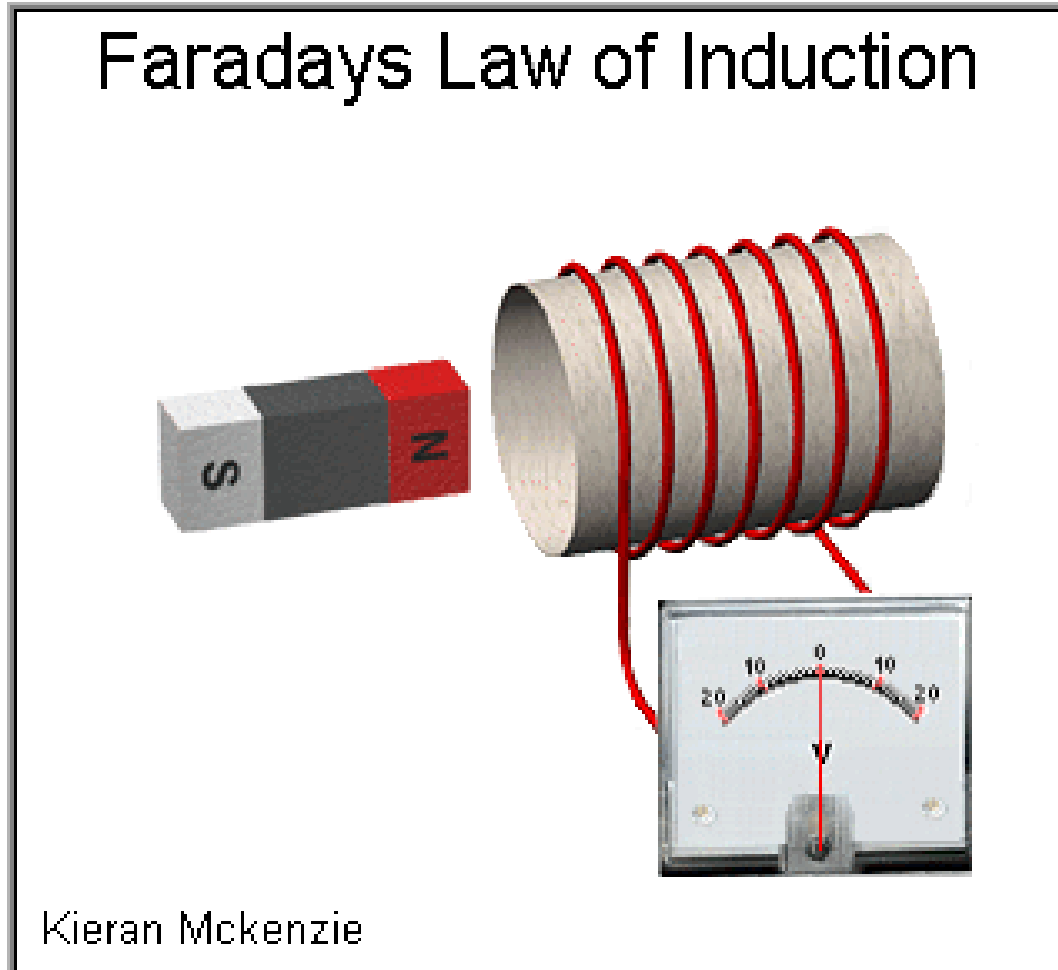
# Contents

- 5.1. Faraday's Law of Induction
- 5.2. Lenz's Law
- 5.3. Induction and Energy Transfers
- 5.4. Induced Electric Fields
- 5.5. Inductors and Inductance
- 5.6. Self-Induction
- 5.7. RL Circuits
- 5.8. Energy Stored in a Magnetic Field
- 5.9. Mutual Induction

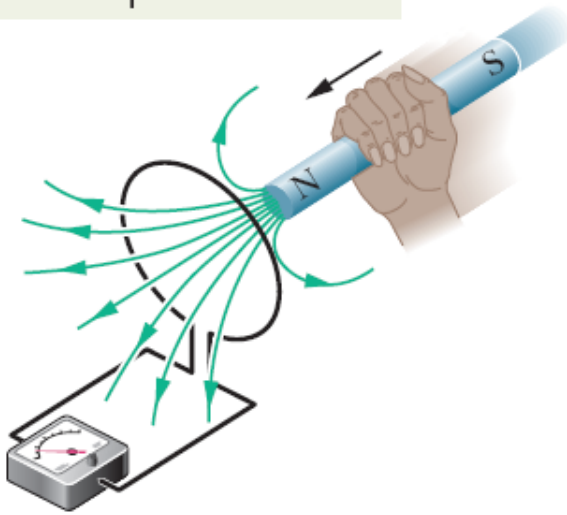
## 5.1. Faraday's Law of Induction

### 5.1.1. First Experiment:

If we move a bar magnet toward/away from the loop, a current appears in the circuit:



The magnet's motion creates a current in the loop.



**Fig. 30-1** An ammeter registers a current in the wire loop when the magnet is moving with respect to the loop.

1. A current appears only if there is relative motion between the loop and the magnet.
2. Faster motion produces a greater current.
3. If moving the magnet's north pole toward the loop causes, say, clockwise current, then moving the north pole away causes counterclockwise current. Moving the south pole toward or away from the loop also causes currents, but in the reversed directions.

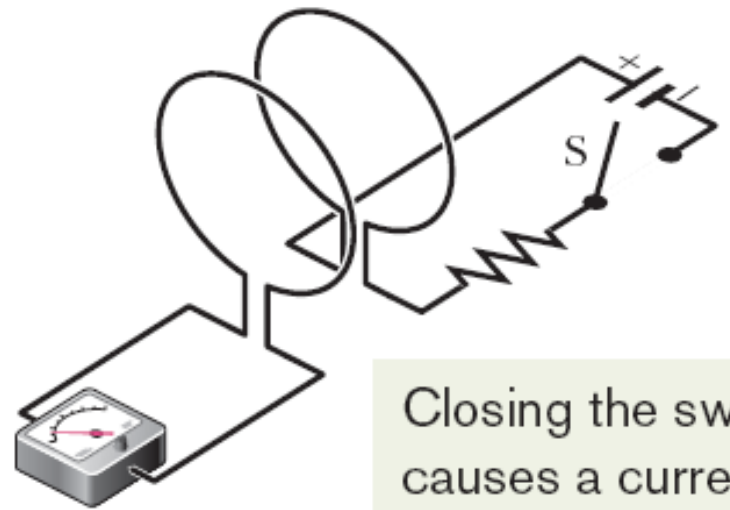
**Induced current:** the current thus produced in the loop is called.

**Induced emf (electromotive force):** the work done per unit charge to produce the induced current

**Induction:** the process of producing the current

### 5.1.2. Second Experiment:

- Two conducting loops close to each other but not touching
- If we close switch  $S$ , to turn on a current in the right-hand loop, the meter suddenly and briefly registers a current—**an induced current**—in the left-hand loop
- If we then open the switch, another sudden and brief induced current appears in the left hand loop, but in the opposite direction



Closing the switch causes a current in the left-hand loop.

*We get an induced current (and thus an induced emf) only when the current in the right-hand loop is changing (either turning on or turning off) and not when it is constant (even if it is large)*

### 5.1.3. Faraday's Law of Induction (the magnitude of induced emf):

An emf is induced in a loop when the number of magnetic field lines that pass through the loop is changing.

- Suppose a loop enclosing an area  $A$  is placed in a magnetic field  $B$ . Then the *magnetic flux through the loop* is

$$\Phi_B = \int \vec{B} d\vec{A} \quad (\text{magnetic flux through area } A)$$

$d\vec{A}$ : is a vector of magnitude  $dA$  perpendicular to a differential area  $dA$

- If the loop lies in a plane and the magnetic field is perpendicular to the plane of the loop, and if the magnetic field is constant, then

$$\Phi_B = BA \quad (\vec{B} \perp \text{area } A, \vec{B} \text{ uniform})$$

The SI unit for magnetic flux is the tesla-square meter, which is called the *weber* (abbreviated  $Wb$ ):

$$1 \text{ weber} = 1 \text{ Wb} = 1 \text{ Tm}^2$$

The magnitude of the emf  $\varepsilon$  induced in a conducting loop is equal to the rate at which the magnetic flux  $\Phi_B$  through that loop changes with time.

$$\varepsilon = -\frac{d\Phi_B}{dt} \text{ (Faraday's law)}$$

- If a coil has  $N$  turns (closely packed):

$$\varepsilon = -N\frac{d\Phi_B}{dt} \text{ (coil of } N \text{ turns)}$$

Here are the general means by which we can change the magnetic flux through a coil:

1. Change  $B$  within the coil.

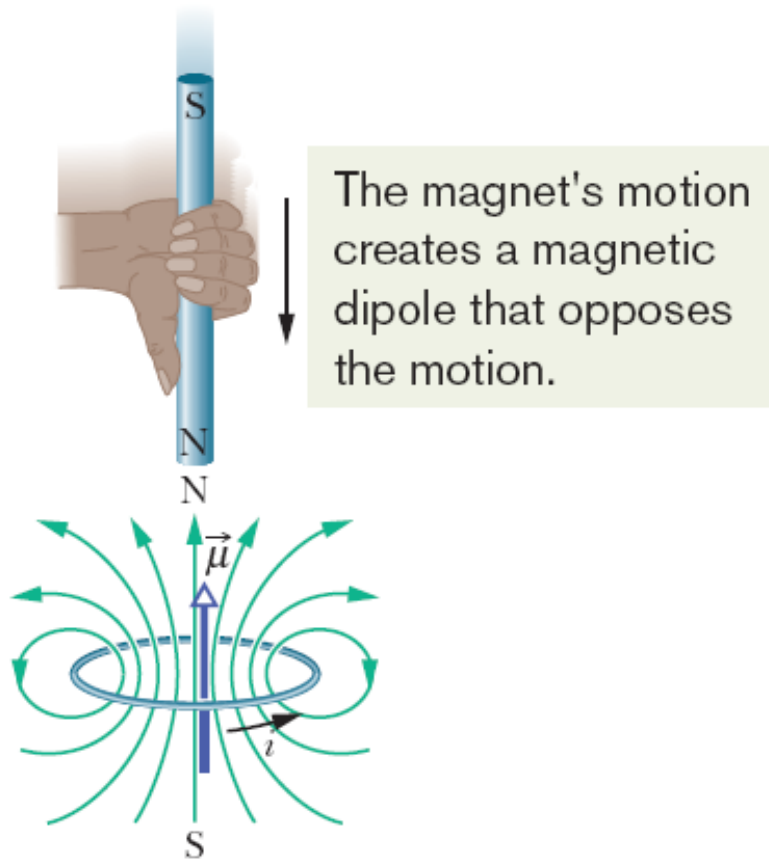
2. Change either the total area of the coil or the portion of that area that lies within the magnetic field (e.g., by expanding or sliding the coil into or out of the field).

3. Change the angle between the direction of the magnetic field  $B$  and the plane of the coil (e.g., by rotating the coil)



## 5.2. Lenz's Law (the direction of induced current):

An induced current has a direction such that the magnetic field due to the current opposes the change in the magnetic flux that induces the current.

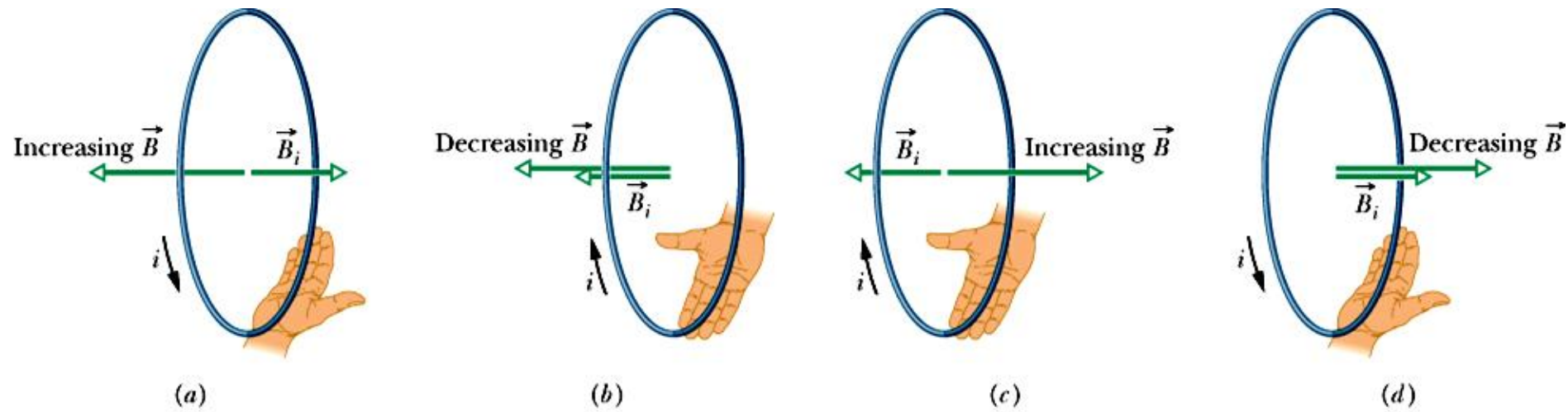


### Animation

#### *Opposition to Pole Movement:*

- The approach of the magnet's north pole increases the magnetic flux through the loop, inducing a current in the loop.
- To oppose the magnetic flux increase being caused by the approaching magnet, the loop's north pole (and the magnetic moment  $m$ ) must face toward the approaching north pole so as to repel it.
- The current induced in the loop must be **counterclockwise**.

• If we next pull **the magnet away** from the loop, a current will again be induced in the loop. Now, the loop will have a south pole facing the retreating north pole of the magnet, so as to oppose the retreat. Thus, the induced current will be **clockwise**.

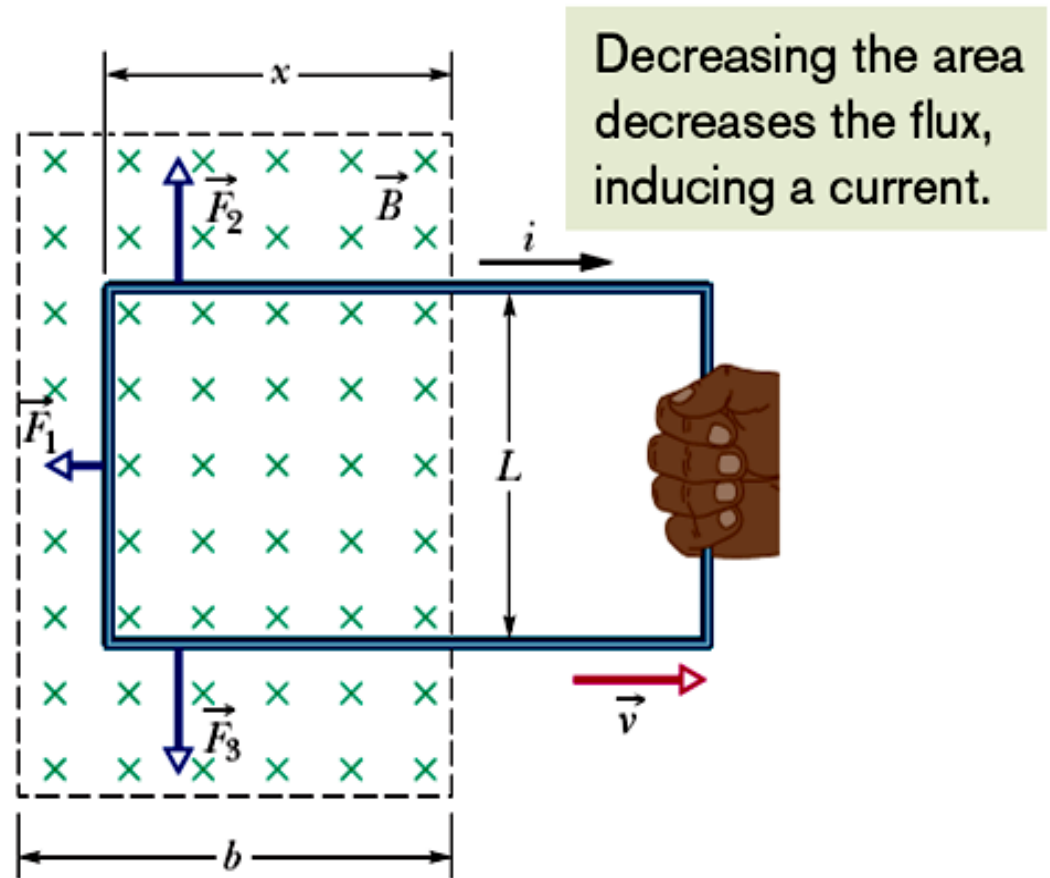


The direction of the current  $i$  induced in a loop is such that the current's magnetic field  $B_{ind}$  opposes the change in the magnetic field inducing  $i$ . The field is always directed opposite an increasing field (a, c) and in the same direction (b, d) as a decreasing field  $B$ .

The curled-straight right-hand rule gives the direction of the induced current based on the direction of the induced field.

### 5.3. Induction and Energy Transfers:

- If the loop is pulled at a constant velocity  $v$ , one must apply a constant force  $F$  to the loop since an equal and opposite magnetic force acts on the loop to oppose it. The power is  $P = Fv$ .
- As the loop is pulled, the portion of its area within the magnetic field, and therefore the magnetic flux, decrease.
- According to Faraday's law, a current is produced in the loop. The magnitude of the flux through the loop is  $\Phi_B = BA = BLx$ .



The induced emf magnitude:

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d}{dt} BLx = BL \frac{dx}{dt} = BLv$$

The induced current:

$$i = \frac{BLv}{R}$$

The net deflecting force:

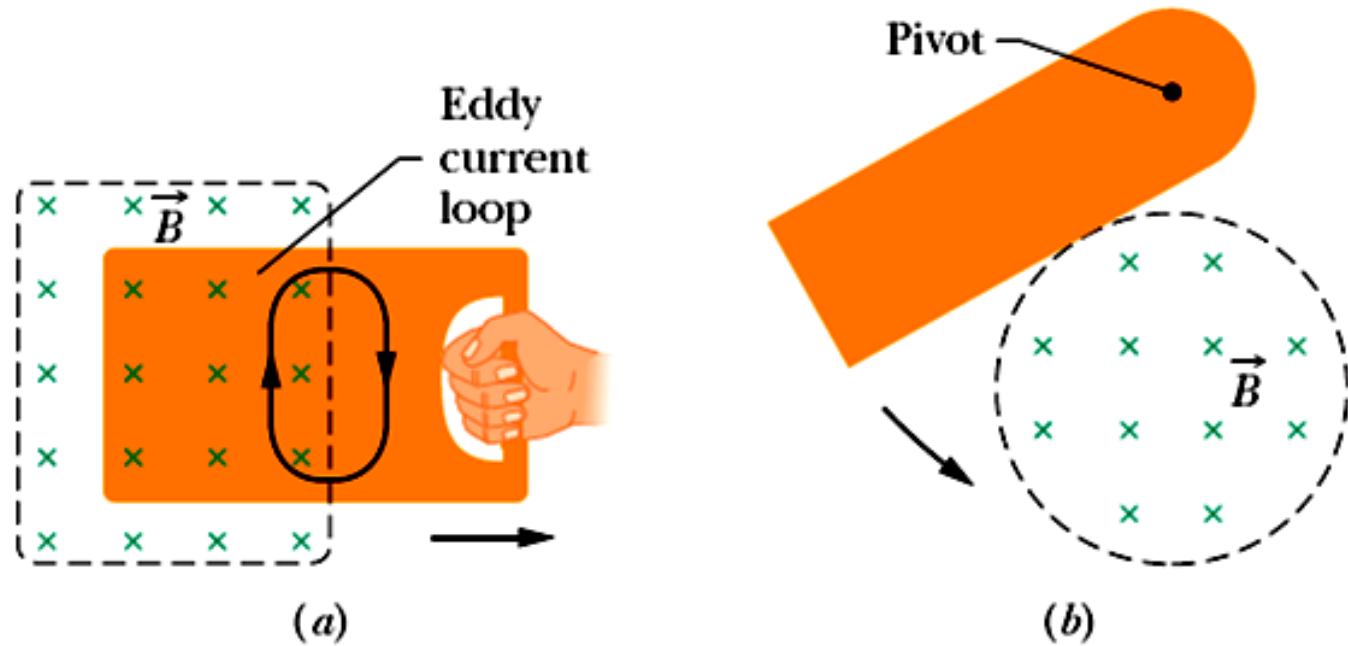
$$F = F_1 = iLB \sin 90^\circ = iLB = \frac{B^2 L^2 v}{R}$$

The power:

$$P = Fv = \frac{B^2 L^2 v^2}{R}$$

→ Therefore, the work that you do in pulling the loop through the magnetic field appears as thermal energy in the loop. Regardless of how current is induced in the loop, energy is always transferred to thermal energy during the process because of the electrical resistance of the loop.

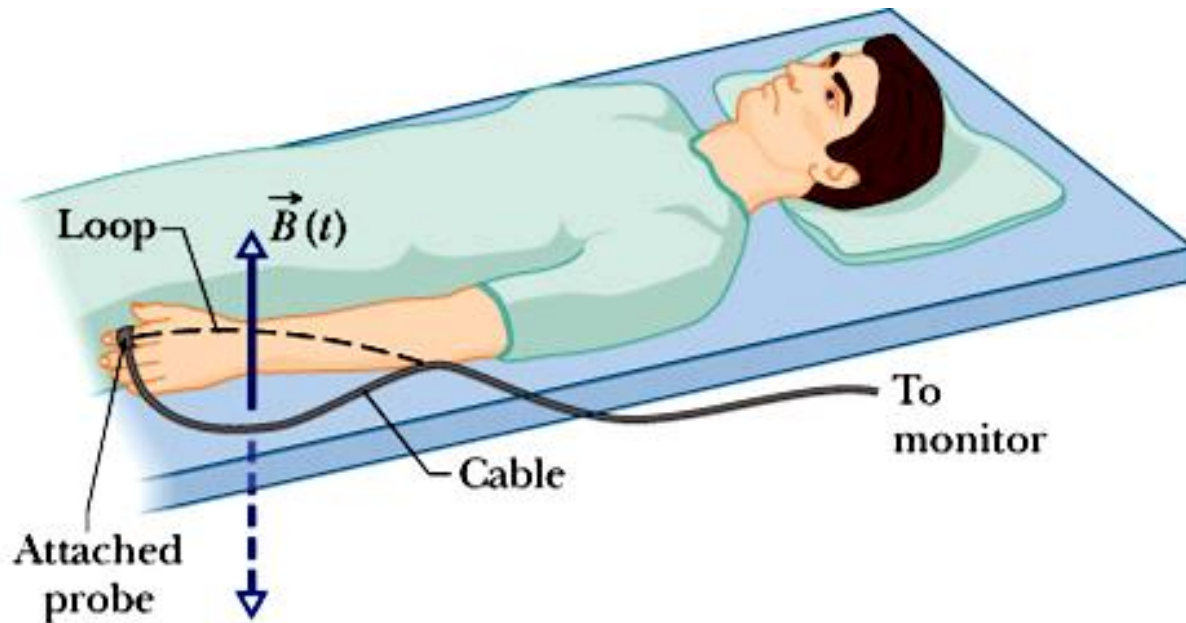
## Induction and Energy Transfers: Eddy Currents



(a) As you pull a solid conducting plate out of a magnetic field, eddy currents are induced in the plate. A typical loop of eddy current is shown (in fact, the conduction electrons swirl about within the plate as if they were caught in an eddy (whirlpool) of water).

(b) A conducting plate swings like a pendulum about a pivot and into a region of magnetic field. As it enters and leaves the field, eddy currents are induced in the plate  $\rightarrow$  mechanical energy transferred to thermal energy  $\rightarrow$  the warmed-up plate just hangs from its pivot.

## Induction and Energy Transfers: Burns During MRI Scans



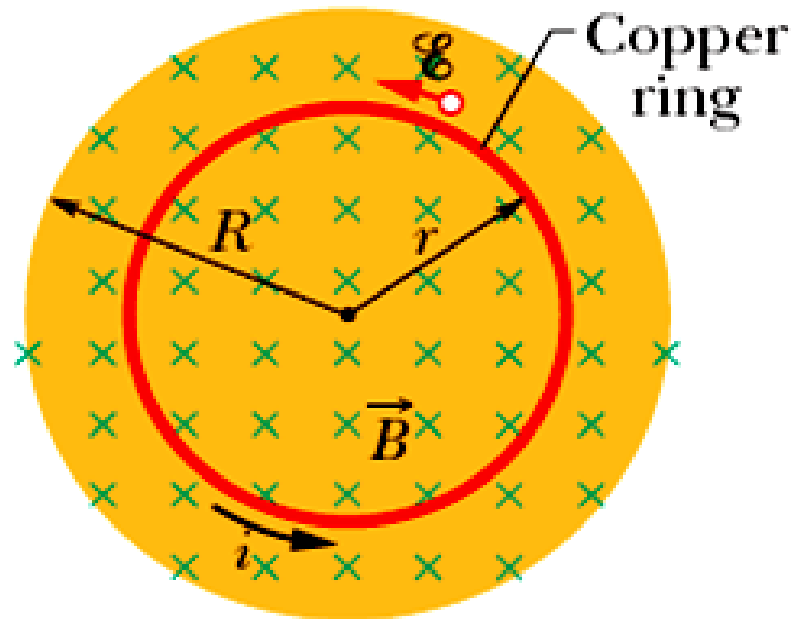
- A patient undergoing an MRI (Magnetic Resonance Imaging) scan lies in apparatus with two magnetic fields: a large constant field  $B_{con}$  and a small sinusoidally varying field  $B(t)$ . The scan requires the patient to lie motionless for a long time.
- Any patient unable to lie motionless, e.g. a child, is sedated. This requires a general anesthetic and the patient must be carefully monitored with a pulse oximeter.
- If the oximeter cable touches the patient's arm and a closed loop is formed.  $B(t)$  varies with  $t$ , causing the finger and the skin were burned.

## 5.4. Induced Electric Fields:

- Place a copper ring of radius  $r$  in a uniform external magnetic field
- We then increase  $B$  at a steady state
- According to Faraday's law, there is an induced emf, hence a current is counterclockwise in the ring (use Lenz's law).
- A current in the ring requires an electric field to be present along the ring and it is called the **induced electric field**.

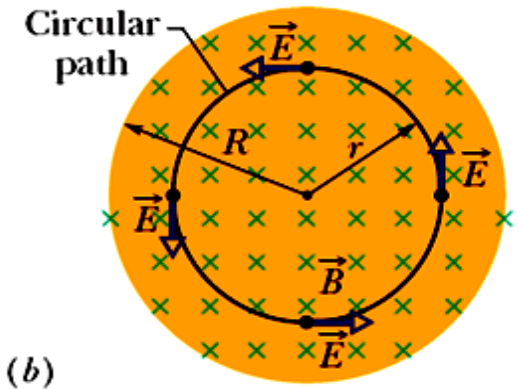
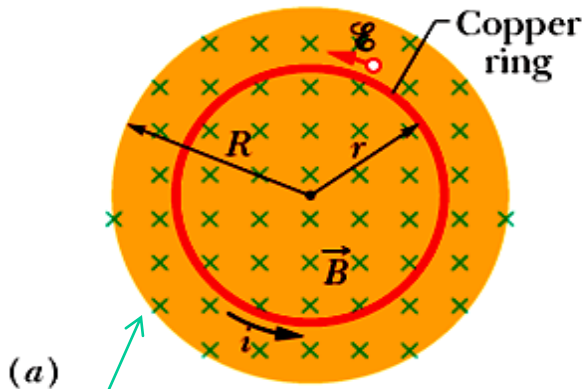


A changing magnetic field produces an electric field.

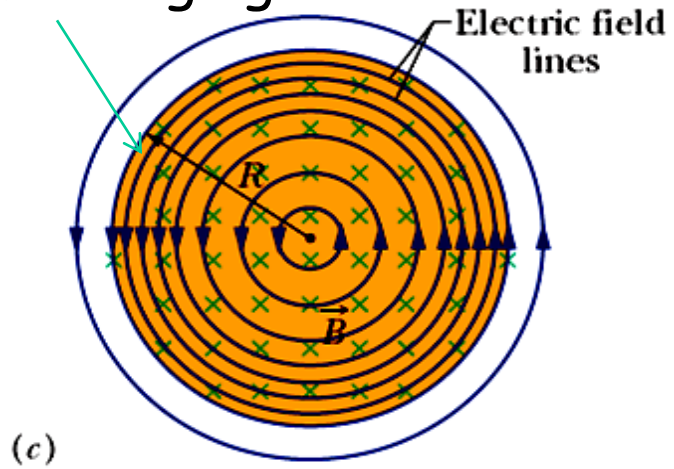




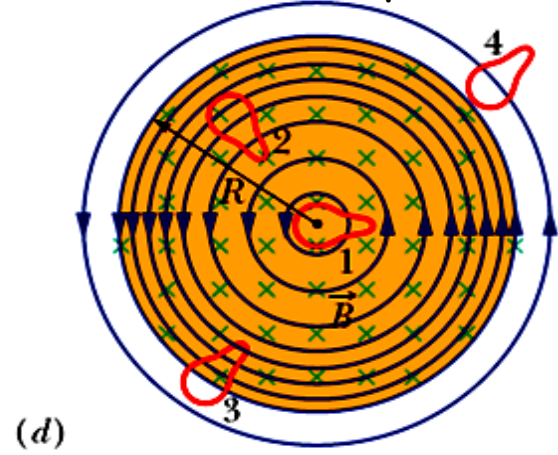
- *Even if there is no copper ring, the electric field is still induced*
- In Figure b, the electric field induced at various points around the circular path must be tangent to the circle



region of changing B



loops 1, 2: equal emfs  
 loop 3: smaller emf  
 loop 4: no net emf





## Induced Electric Fields, Reformulation of Faraday's Law:

Consider a particle of charge  $q_0$  moving around the circular path in Figure b. The work  $W$  done on it in one revolution by the induced electric field is  $W = \varepsilon q_0$ , where  $\varepsilon$  is the induced emf.

From another point of view, the work is  $W = \int \vec{F} d\vec{s} = (q_0 E)(2\pi r)$

Here where  $q_0 E$  is the magnitude of the force acting on the test charge and  $2\pi r$  is the distance over which that force acts.

$$\Rightarrow \varepsilon = 2\pi r E$$

In general,  $W = \oint \vec{F} d\vec{s} = q_0 \oint \vec{E} d\vec{s} \Rightarrow \varepsilon = \oint \vec{E} d\vec{s}$



$$\oint \vec{E} d\vec{s} = -\frac{d\Phi_B}{dt} \text{ (Faraday's law)}$$

→ This equation shows that a changing magnetic field induces an electric field.

## Induced Electric Fields, A New Look at Electric Potential:

- Induced electric fields are produced not by static charges but by a changing magnetic flux.
- There is an important difference between them: The field lines of induced electric fields form closed loops. The field lines produced by static charges do not, leading us to a more formal sense:



Electric potential has meaning only for electric fields that are produced by static charges; it has no meaning for electric fields that are produced by induction.

We can understand this statement qualitatively:

- For electric fields produced by charges:  $V_f - V_i = - \int_i^f \vec{E} d\vec{s}$  ,  
for a closed loop:  $V_f = V_i \Rightarrow \oint \vec{E} d\vec{s} = 0$
- For induced electric fields:  $\oint \vec{E} d\vec{s} = - \frac{d\Phi_B}{dt} \neq 0$

Thus, assigning electric potential to an induced electric field leads us to a contradiction. We conclude that electric potential has no meaning for electric fields associated with induction.

## 5.5. Inductors and Inductance:



The crude inductors with which Michael Faraday discovered the law of induction.

An inductor (symbol  $\text{Ⓛ}$ ) can be used to produce a desired magnetic field. If we establish a current  $i$  in the windings (turns) of the solenoid which can be treated as our inductor, the current produces a magnetic flux  $\Phi_B$  through the central region of the inductor.

The inductance of the inductor is then

$$L = \frac{N\Phi_B}{i} \text{ (inductance defined)}$$

$N\Phi_B$ : *the magnetic flux linkage*

The SI unit of inductance is the tesla-square meter per ampere ( $\text{T m}^2/\text{A}$ ). We call this the henry (H), after American physicist Joseph Henry,

## *Inductance of a Solenoid:*

Consider a long solenoid of cross-sectional area  $A$ , with number of turns  $N$ , and of length  $l$ . The flux is  $N\Phi_B = (nl)(BA)$

Here  $n$  is the number of turns per unit length.

The magnitude of  $B$  is given by:  $B = \mu_0 in$

$$\text{Therefore, } L = \frac{N\Phi_B}{i} = \frac{(nl)(BA)}{i} = \frac{(nl)(\mu_0 in)(A)}{i} = \mu_0 n^2 l A$$

The inductance per unit length near the center is therefore:

$$\frac{L}{l} = \mu_0 n^2 A \text{ (solenoid)}$$

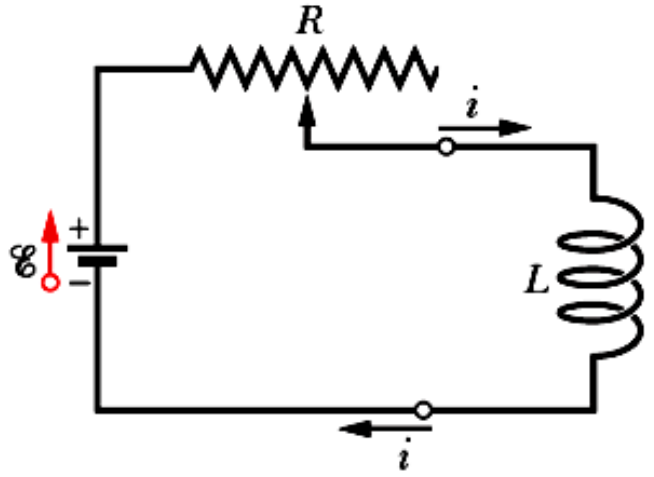
Here,

$$\begin{aligned} \mu_0 &= 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A} \\ &= 4\pi \times 10^{-7} \text{ H/m.} \end{aligned}$$

# 5.6. Self-Induction:

An induced emf  $\epsilon_L$  appears in any coil in which the current is changing

- If  $i$  is changed by varying  $R$ , a self-induced emf  $\epsilon_L$  will appear in the coil.



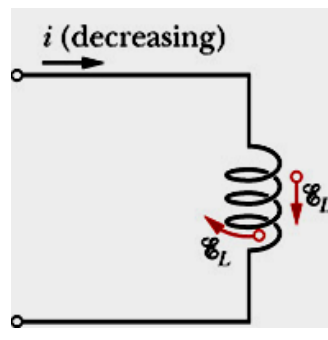
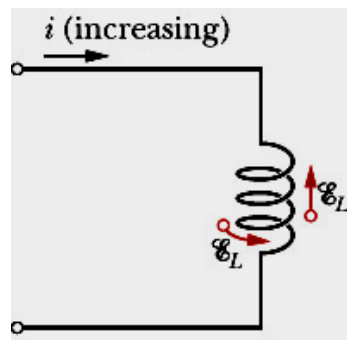
- This process is called **self-induction**, and the emf is called a self-induced emf. It also obeys Faraday's law of induction:

$$\epsilon_L = -\frac{d(N\Phi_B)}{dt}$$

$$N\Phi_B = Li$$

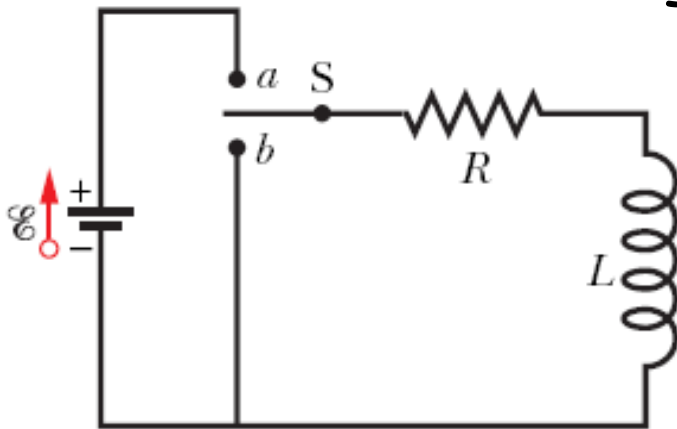
$$\epsilon_L = -L \frac{di}{dt} \text{ (self - induced emf)}$$

- The self-induced emf has the orientation such that it opposes the change in current  $i$



## 5.7. RL Circuits:

- Consider a circuit below, if  $S$  is on  $a$ :



There are **two** emfs in the circuit:

- A constant emf  $\varepsilon$  due to the battery
- A variable  $\varepsilon_L = -Ldi/dt$  (Faraday's law), from Lenz's law this emf opposes the rise of the current, which means that it opposes  $\varepsilon$  in polarity.

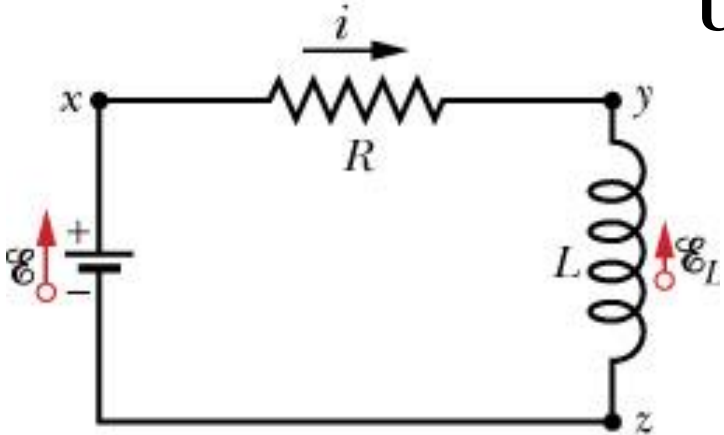
- As time goes on,  $i$  increases less rapid,  $\varepsilon_L$  becomes smaller and therefore current  $i$  approaches  $\varepsilon/R$



Initially, an inductor acts to oppose changes in the current through it. A long time later, it acts like ordinary connecting wire.

We now analyze the statements quantitatively:

Using the loop rule:



$$-iR - L \frac{di}{dt} + \varepsilon = 0$$

or

$$L \frac{di}{dt} + Ri = \varepsilon$$

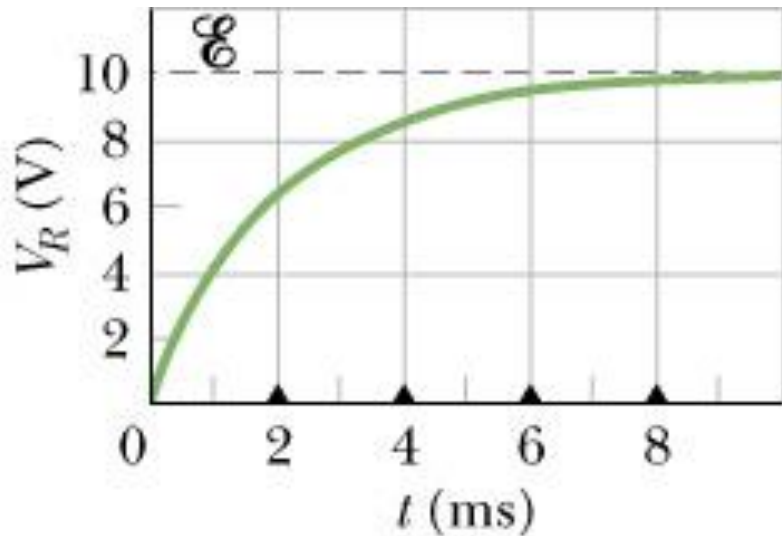
the solution is:

$$i = \frac{\varepsilon}{R} \left( 1 - e^{-Rt/L} \right)$$

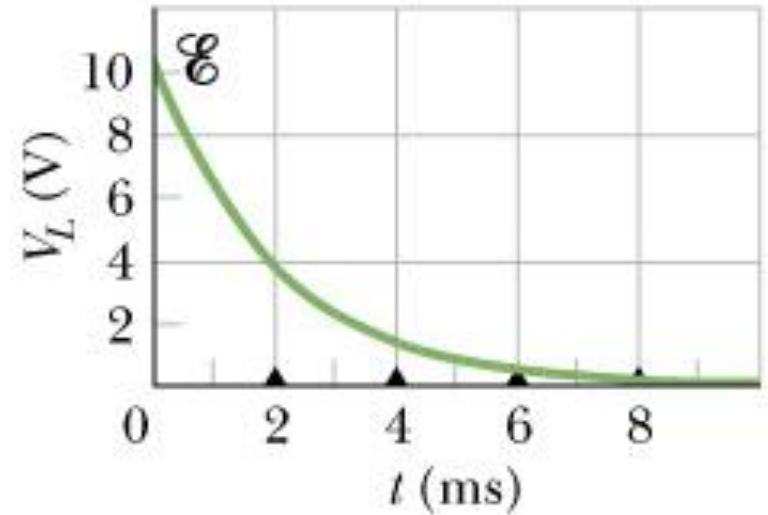
we define the *inductive time constant*:

$$\tau_L = \frac{L}{R} \text{ (inductive time constant)}$$

$$i = \frac{\varepsilon}{R} \left( 1 - e^{-t/\tau_L} \right) \text{ (rise of current)}$$



(a)



(b)

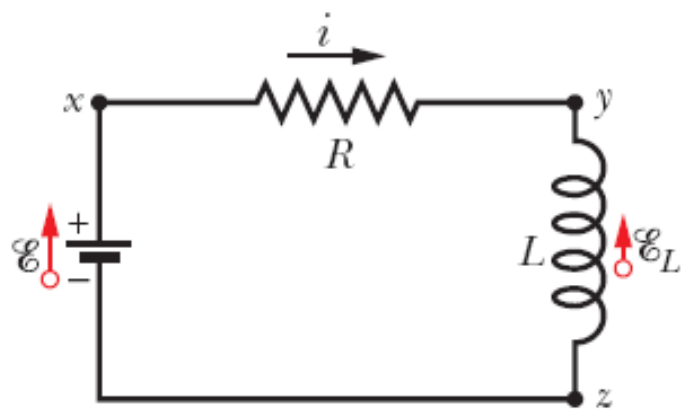
If we suddenly remove the emf from this same circuit, the current does not immediately fall to zero but approaches zero in an exponential fashion:

$$L \frac{di}{dt} + Ri = 0$$

$$i = \frac{\varepsilon}{R} e^{-t/\tau_L} = i_0 e^{-t/\tau_L} \text{ (decay of current)}$$



# 5.8. Energy Stored in a Magnetic Field:



$$\mathcal{E} = L \frac{di}{dt} + iR$$

$$\mathcal{E}i = Li \frac{di}{dt} + i^2 R$$

$\mathcal{E}i$ : the rate of at which the emf device delivers energy to the rest of the circuit  
 $i^2R$ : the rate of thermal energy in the resistor

$\frac{dU_B}{dt} = Li \frac{di}{dt}$  : This is the rate at which magnetic potential energy  $U_B$  is stored in the magnetic field.

$$\int_0^{U_B} dU_B = \int_0^i Lidi$$

$$U_B = \frac{1}{2} Li^2 \text{ (magnetic energy)}$$

This represents the total energy stored by an inductor  $L$  carrying a current  $i$ .

## Energy Density of a Magnetic Field:

Consider a length  $l$  near the middle of a long solenoid of cross-sectional area  $A$  carrying current  $i$ ; the volume associated with this length is  $Al$ .

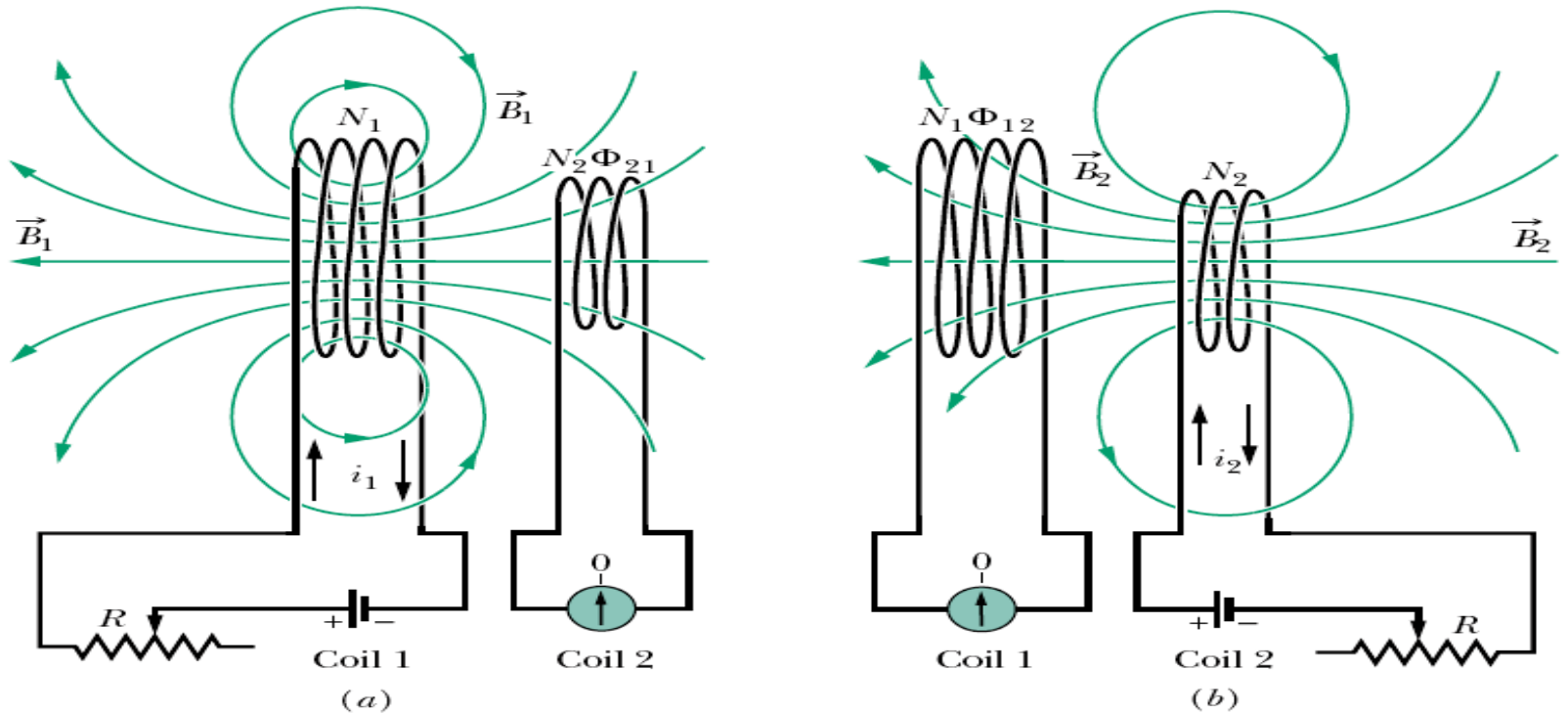
The energy  $U_B$  stored by the length  $l$  of the solenoid must lie entirely within this volume because the magnetic field outside such a solenoid is approximately zero. Also, the stored energy must be uniformly distributed within the solenoid because the magnetic field is (approximately) uniform everywhere inside.

Thus, the energy stored per unit volume of the field is

$$u_B = \frac{U_B}{Al}$$
$$U_B = \frac{1}{2} Li^2$$
$$u_B = \frac{Li^2}{2Al} = \frac{L}{l} \frac{i^2}{2A} = \frac{1}{2} \mu_0 n^2 i^2$$

$$B = \mu_0 in \Rightarrow u_B = \frac{B^2}{2\mu_0} \text{ (magnetic energy density)}$$

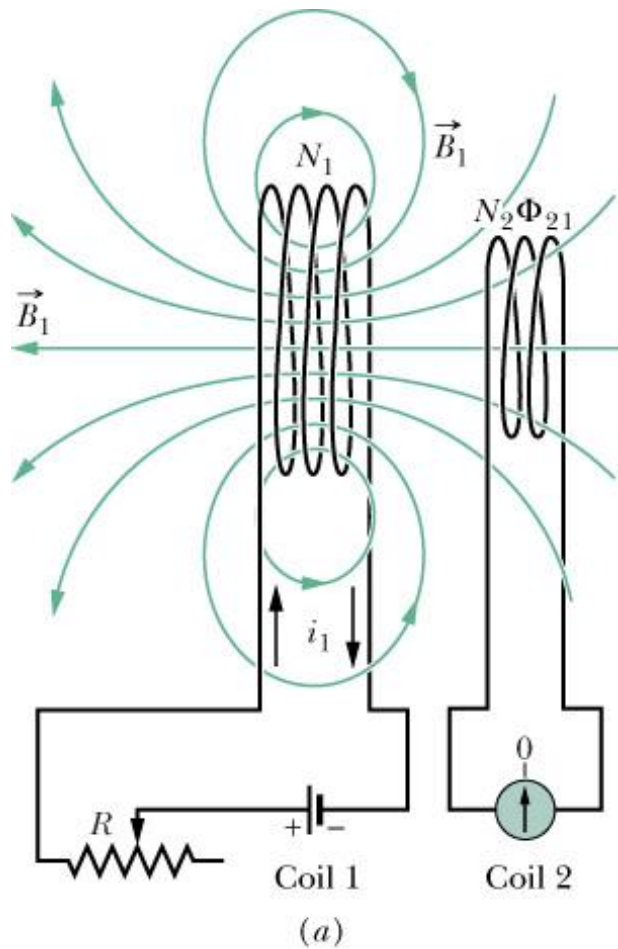
## 5.9. Mutual Induction:



If two coils are close together and if we change the current  $i$  in one coil, an emf  $\varepsilon$  appears in the second coil. We call this **mutual induction** to distinguish it from self-induction.

The mutual inductance  $M_{21}$  of coil 2 with respect to coil 1 is defined as

$$M_{21} = \frac{N_2 \Phi_{21}}{i_1}$$



$$\Rightarrow M_{21} \frac{di_1}{dt} = N_2 \frac{d\Phi_{21}}{dt}$$

The right side of this equation is, according to Faraday's law, just the magnitude of the emf  $\varepsilon_2$  appearing in coil 2 due to the changing current in coil 1.

$$\varepsilon_2 = -M_{21} \frac{di_1}{dt}$$

Similarly for case b,

$$\varepsilon_1 = -M_{12} \frac{di_2}{dt}$$

$M_{12}$  and  $M_{21}$  are in fact the same, so:

$$\Rightarrow M_{21} = M_{12} = M$$

**Unit for  $M$  is the henry (H)**

Homework: 1, 7, 8, 12, 14, 15, 20, 27, 29, 32, 37, 40,  
47, 55, 56, 63, 68, 73 (pages 818-824)