Homework (lecture 11):

3, 5, 9, 13, 21, 25, 29, 31, 40, 45, 49, 51, 57, 62

3. An electron that has velocity: $\vec{v} = (2.0\times10^{6}\,m/s)\hat{i} + (3.0\times10^{6}\,m/s)\hat{j}$ moves through the uniform magnetic field $\vec{B} = (0.030T)\hat{i} - (0.15T)\hat{j}$ (a) Find the force on the electron. (b) Repeat your calculation for a proton having the same velocity.

Recall:

$$\vec{c} = \vec{a}\times\vec{b}$$

$$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$$

(a) The force acts on the electron: $\vec{F} = q\vec{v}\times\vec{B}$

$$\vec{F} = q(v_x B_y - v_y B_x)\hat{k}$$

$$\vec{F} = (-1.6\times10^{-19})[2\times10^{6}\times(-0.15) - (3\times10^{6}\times0.03)]\hat{k}$$

$$\vec{F} = 6.24\times10^{-14}\hat{k}$$

(b) The proton has a positive charge:

$$\vec{F}_p = -6.24\times10^{-14}\hat{k}$$

So, the force acting on the proton has the same magnitude but opposite in direction to the force acting on the electron

5. An electron moves through a uniform magnetic field given by $\vec{B} = B_x \hat{i} + (3.0 B_x)\hat{j}$. At a particular instant, the electron has velocity $\vec{v} = (2.0\hat{i} + 4.0\hat{j}) m/s$ and the magnetic force acting on it is $(6.4 \times 10^{-19} N)\hat{k}$ Find B_x.

$$\vec{F} = q(v_x B_y - v_y B_x)\hat{k} = F_z \hat{k}$$

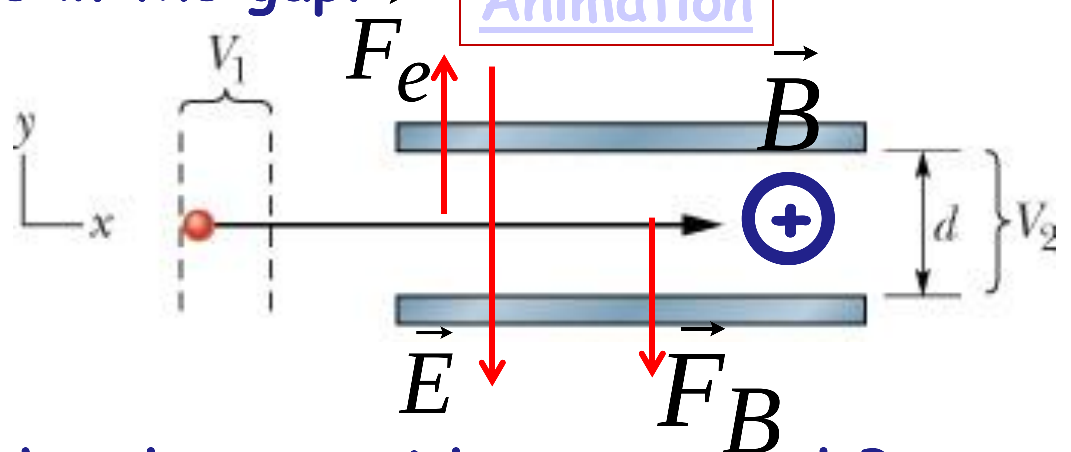
$$B_x = \frac{F_z}{q(3v_x - v_y)} \quad (\mathrm{T})$$

9. In Fig. 28-33, an electron accelerated from rest through potential difference V_1 = 1.00 kV enters the gap between two parallel plates having separation d = 20.0 mm and potential difference V_2 = 100 V. The lower plate is at the lower potential. Neglect fringing and assume that the electron's velocity vector is perpendicular to the electric field vector between the plates. In unit-vector notation, what uniform magnetic field allows the electron to travel in a straight line in the gap?

Animation

The electric force: $\vec{F}_e = q\vec{E}$

$$E = \frac{V_2}{d} \Rightarrow F_e = \frac{eV_2}{d}$$



The magnetic field force acts on the electron with an external B:

$$\vec{F}_B = q\vec{v} \times \vec{B} \Rightarrow F_B = evB$$

$$\frac{1}{2}mv^2 = eV_1 \Rightarrow v = \sqrt{\frac{2eV_1}{m}}$$

To travel in a straight line:

$$\vec{F}_e = \vec{F}_B$$

$$\frac{eV_2}{d} = evB \Rightarrow B = \frac{V_2}{vd}$$

$$B = \frac{V_2}{d\sqrt{\frac{2eV_1}{m}}} = \frac{100}{20\times10^{-3}\sqrt{\frac{2\times1.6\times10^{-19}\times1000}{9.11\times10^{-31}}}}$$

$$B = 2.67\times10^{-4}(T)$$

$$\boxed{\vec{B} = (-2.67\times10^{-4}T)\hat{k}}$$

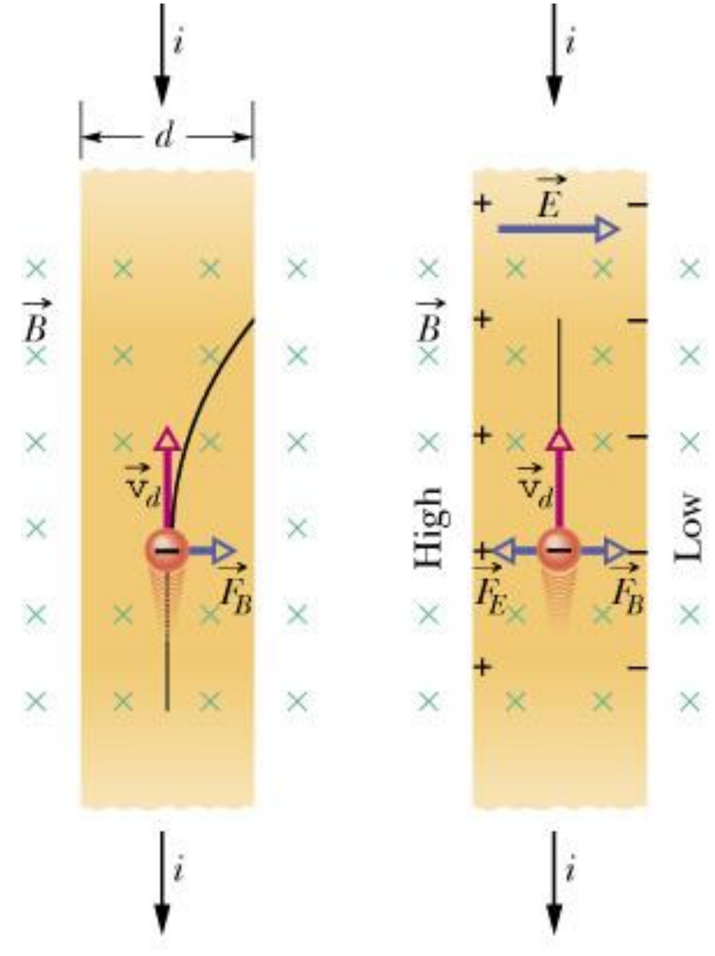
13. A strip of copper 150 μm thick and 4.5 mm wide is placed in a uniform magnetic field B of magnitude 0.65 T, with B perpendicular to the strip. A current i = 23 A is then sent through the strip such that a Hall potential difference V appears across the width of the strip. Calculate V. (The number of charge carriers per unit volume for copper is 8.47×10^{28} electron/m^3).

$$n = \frac{Bi}{Vle}$$

l: thickness of the strip

$$V = \frac{0.65 \times 23}{8.47 \times 10^{28} \times 150 \times 10^{-6} \times 1.6 \times 10^{-19}}$$

$$V = 7.4 \times 10^{-6} (V)$$



(a) (b)

21. An electron of kinetic energy 1.20 keV circles in a plane perpendicular to a uniform magnetic field. The orbit radius is 25.0 cm. Find (a) the electron's speed, (b) the magnetic field magnitude, (c) the circling frequency, and (d) the period of the motion.

(a) The speed:

$$K = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2K}{m}}$$

(b) The centripetal force here is the magnetic force:

$$F = ma = m\frac{v^2}{R} = qvB \Rightarrow B = \frac{mv}{qR}$$

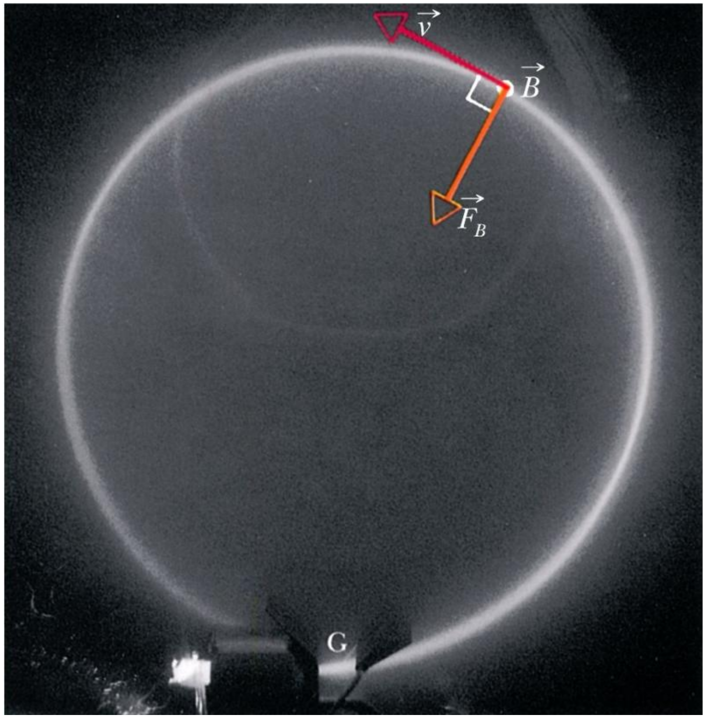
(c) The frequency:

$$f = \frac{1}{T} = \frac{v}{2\pi R} \quad (Hz)$$

(d) The period:

$$T = \frac{1}{f} \quad (s)$$

25. (a) Find the frequency of revolution of an electron with an energy of 100 eV in a uniform magnetic field of magnitude 35.0 μT. (b) Calculate the radius of the path of this electron if its velocity is perpendicular to the magnetic field.



(a) the frequency: $f = \dfrac{qB}{2\pi m}$

For the electron case: $f = \dfrac{eB}{2\pi m_e}$

$$f = \frac{1.6\times10^{-19}\times35.0\times10^{-6}}{2\times3.14\times9.11\times10^{-31}}$$

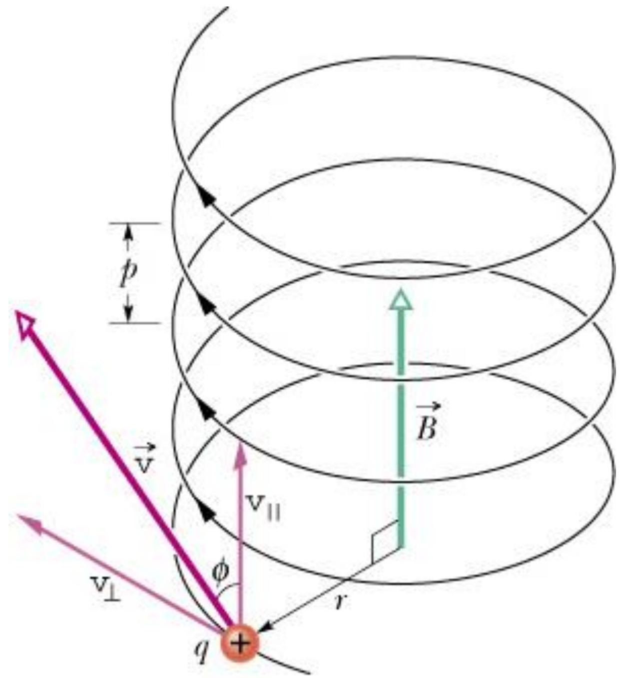
$$f = 9.8\times10^{5}(Hz)$$

(b) the radius:

$$r = \frac{mv}{qB} = \frac{m\sqrt{2K/m}}{qB} = \frac{\sqrt{2Km_e}}{eB} = 0.96(m)$$

29. An electron follows a helical path in a uniform magnetic field of magnitude 0.300 T. The pitch of the path is 6.00 μm, and the magnitude of the magnetic force on the electron is 2.00 x 10^{-15} N. What is the electron's speed?

$$v_{\|} = v\cos\phi \text{ and } v_{\perp} = v\sin\phi$$



$$F_B = qv_{\perp}B$$

$$\Rightarrow v_{\perp} = \frac{F_B}{eB} = \frac{2\times10^{-15}}{1.6\times10^{-19}\times0.3} = 4.17\times10^{4}(m/s)$$

The period:

$$T = \frac{2\pi r}{v_{\perp}} = \frac{2\pi m}{qB}; T = \frac{p}{v_{\|}}$$

$$v_{\|} = \frac{qBp}{2\pi m} = \frac{eBp}{2\pi m_e} = \frac{1.6\times10^{-19}\times0.3\times6\times10^{-6}}{2\times3.14\times9.11\times10^{-31}}$$

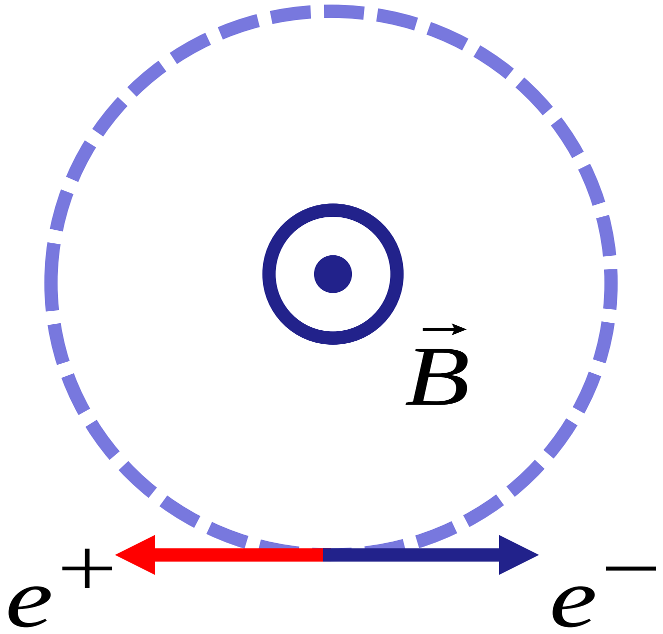
$$v_{\|} = 5.03\times10^{4}(m/s)$$

$$\boxed{v = \sqrt{v_{\|}^{2} + v_{\perp}^{2}} = 6.53\times10^{4}(m/s)}$$

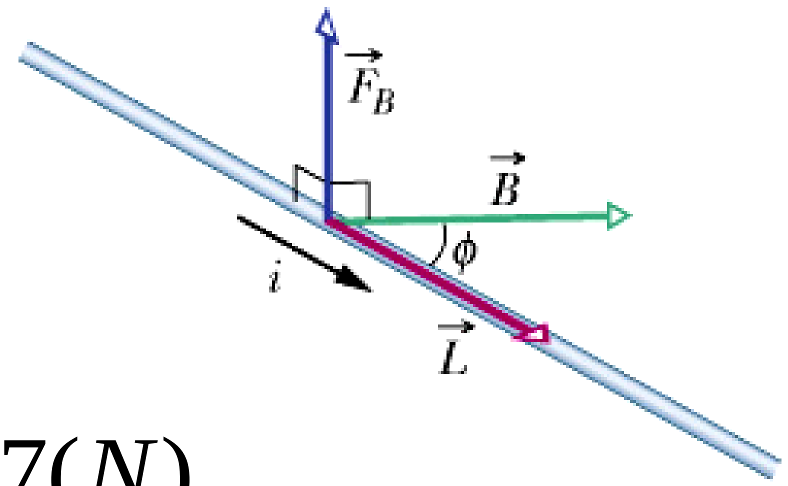
31. A particular type of fundamental particle decays by transforming into an electron e^- and a positron e^+. Suppose the decaying particle is at rest in a uniform magnetic field B of magnitude 3.53 mT and the e^- and e^+ move away from the decay point in paths lying in a plane perpendicular to B. How long after the decay do the e^- and e^+ collide?

When the electron and the positron travel half of the circular path, they will collide:

$$t = \frac{1}{2}T = \frac{\pi m_e}{eB} = 5.1 \times 10^{-9} (s)$$



40. A wire 2.30 m long carries a current of 13.0 A and makes an angle of 35.0^0 with a uniform magnetic field of magnitude B = 1.5 T. Calculate the magnetic force on the wire.



$$\vec{F}_B = i\vec{L} \times \vec{B}$$

$$F_B = iLB\sin\phi$$

$$F_B = 13 \times 2.3 \times 1.5 \times \sin 35 = 25.7(N)$$

45. A wire 50.0 cm long carries a 0.500 A current in the positive direction of an x axis through a magnetic field: $\vec{B} = (3.0mT)\hat{j} + (10.0mT)\hat{k}$. In unit-vector notation, what is the magnetic force on the wire?

$$\vec{F}_B = i\vec{L} \times \vec{B}$$

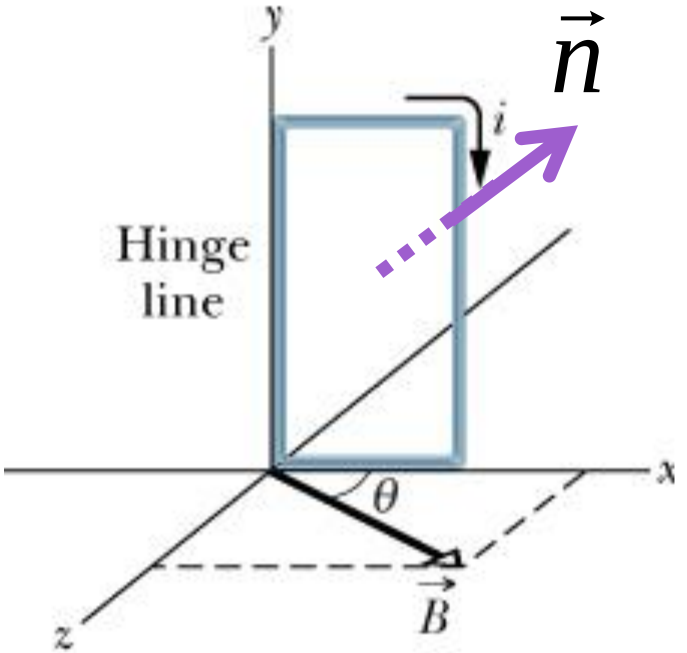
$$\vec{L} = (0.5m)\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\vec{B} = 0\hat{i} + (3.0\times10^{-3}T)\hat{j} + (10^{-2}T)\hat{k}$$

$$\Rightarrow \vec{F}_B = i[(-0.5\times10^{-2})\hat{j} + (0.5\times3.0\times10^{-3})\hat{k}]$$

$$\vec{F}_B = (-2.5\times10^{-3})\hat{j} + (0.75\times10^{-3})\hat{k}$$

49. Figure 28-41 shows a rectangular 20-turn coil of wire, of dimensions 10 cm by 5.0 cm. It carries a current of 0.10 A and is hinged along one long side. It is mounted in the xy plane, at angle θ = 30⁰ to the direction of a uniform magnetic field of magnitude 0.50 T. In unit-vector notation, what is the torque acting on the coil about the hinge line.



$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$\tau = (NiA)B\sin\theta'$$

θ' is the angle between $\vec{n}$ and $\vec{B}$

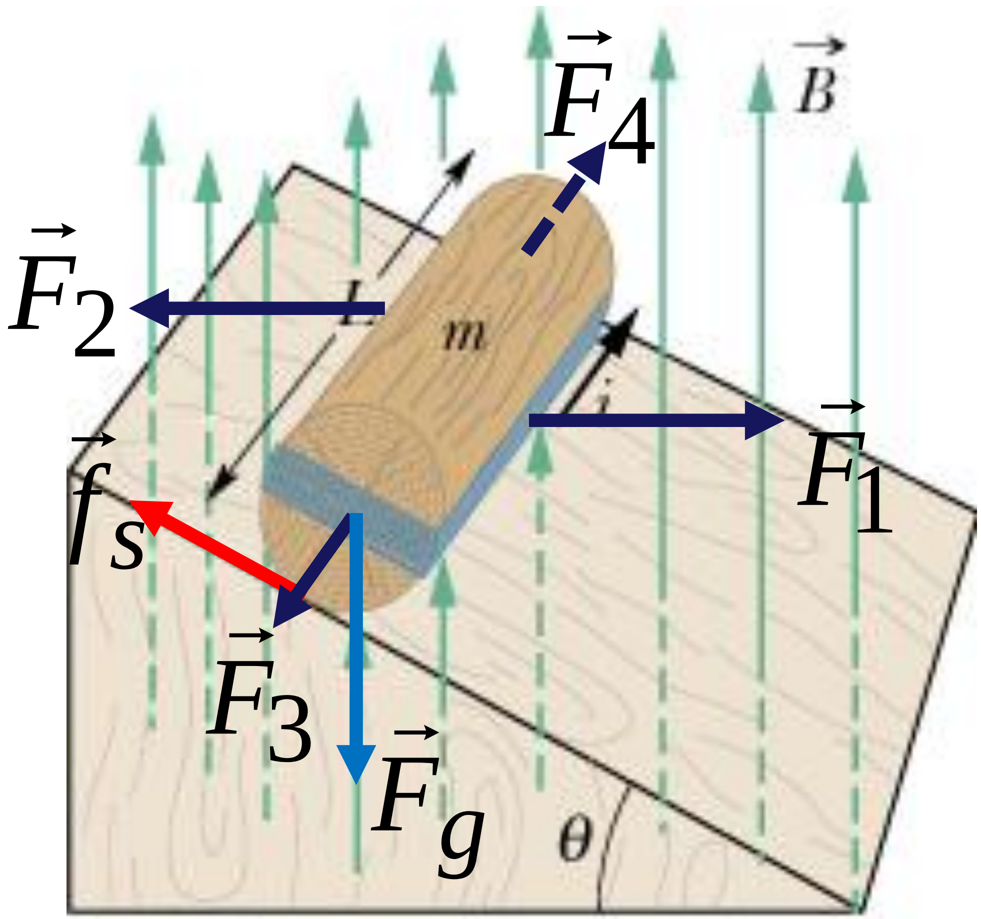
$$\theta' = 120^0$$

$$\tau = 20 \times 0.1 \times (50 \times 10^{-4}) \times 0.5 \times \sin 120^0 = 4.33 \times 10^{-3} (N.m)$$

Using the right hand rule to determine the torque direction:

$$\vec{\tau} = (-4.33 \times 10^{-3} N.m)\hat{j}$$

51. Figure 28-44 shows a wood cylinder of mass m = 0.250 kg and length L = 0.100 m, with N = 10.0 turns of wire wrapped around it longitudinally, so that the plane of the wire coil contains the long central axis of the cylinder. The cylinder is released on a plane inclined at an angle θ to the horizontal, with the plane of the coil parallel to the incline plane. If there is a vertical uniform magnetic field of magnitude 0.500 T, what is the least current i through the coil that keeps the cylinder from rolling down the plane?



- To keep the cylinder from rolling down:

(1) The net force is zero:

$$mg\sin\theta - f_s = ma = 0$$

f_s: frictional force

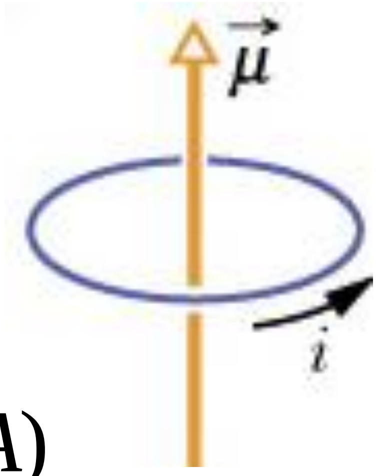
AND

(2) The net torque acting on the cylinder about the cylinder axis must be zero:

$$\tau = \tau_f + \tau_B = \mu B\sin\theta - f_s r = I\alpha = 0;\ \mu = NiA$$

$$\Rightarrow NiAB\sin\theta - mg\sin\theta r = 0 \Rightarrow i = \frac{mgr}{NAB} = \frac{mgr}{NL2rB} = \frac{mg}{2NLB}$$

57. A circular coil of 160 turns has a radius of 1.90 cm. (a) Calculate the current that results in a magnetic dipole moment of magnitude 2.30 A.m^2. (b) Find the maximum magnitude of the torque that the coil, carrying this current, can experience in a uniform 35.0 mT magnetic field.



(a) The magnetic dipole moment:

$$\mu = NiA$$

$$i = \frac{\mu}{NA} = \frac{2.3}{160 \times 3.14 \times 0.019^2} = 12.7(A)$$

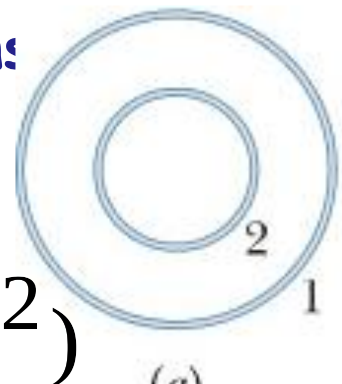
(b) The torque acts on the coil:

$$\tau = \mu B \sin\theta$$

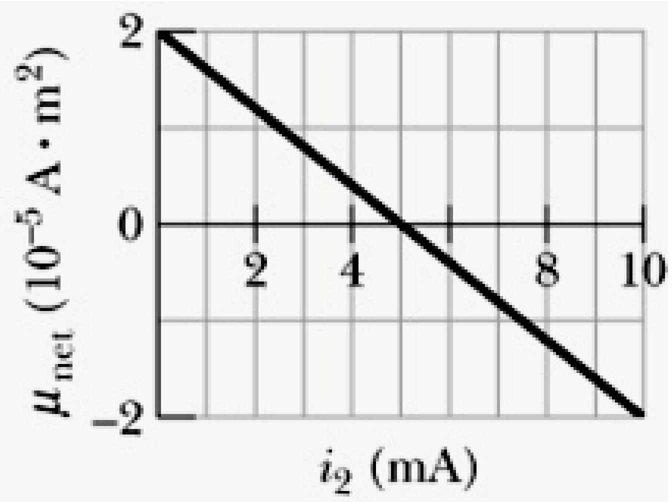
τ is maximum if $\theta = 90^0$:

$$\tau_{\max} = \mu B = 2.3 \times 35.0 \times 10^{-3} = 0.081(N.m)$$

62. In Fig. 28-47a, two concentric coils, lying in the same plane, carry currents in opposite directions. The current in the larger coil i is fixed. Current i_2 in coil 2 can be varied. Figure 28-47b gives the net magnetic moment of the two-coil system as a function of i_2. If the current in coil 2 is then reversed, what is the magnitude of the net magnetic moment of the two-coil system when i_2 = 7.0 mA?



(a)



(b)

- i_1 and i_2 are in opposite directions

$$\mu_{net} = \mu_1 - \mu_2$$

At i_2 = 0:

$$\mu_{net} = \mu_1 = 2\times10^{-5}(A.m^2)$$

At i_2 = 5 mA:

$$\mu_{net} = \mu_1 - N_2 i_2 A_2 = 2\times10^{-5} - N_2 A_2 \times 5\times10^{-3} = 0$$

$$\Rightarrow N_2 A_2 = 4.0\times10^{-3}$$

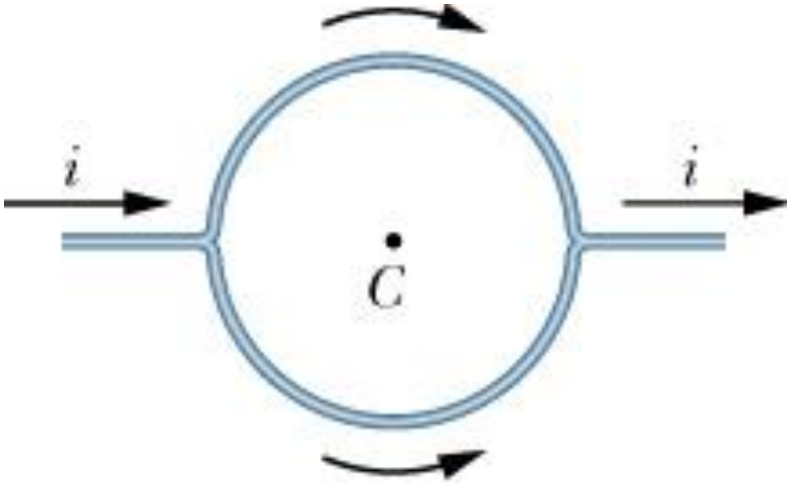
- if i_2 is inversed:

$$\mu_{net} = \mu_1 + N_2 i_2 A_2 = 2\times10^{-5} + 4.0\times10^{-3} \times 7\times10^{-3} = 4.8\times10^{-5}(A.m^2)$$

Homework (lecture 12):

4, 7, 12, 16, 18, 22, 35, 38, 43, 46, 49, 50, 57, 62

4. A straight conductor carrying current i = 5.0 A splits into identical semicircular arcs as shown in Fig. 29-34. What is the magnetic field at the center C of the resulting circular loop?

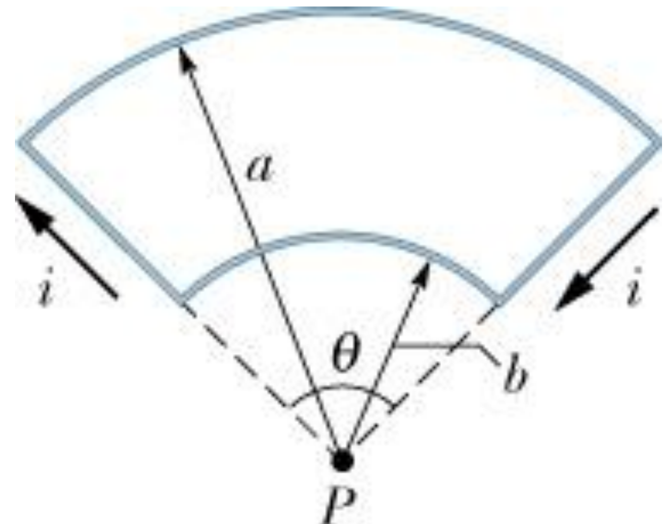


$$B = \frac{\mu_0 i \phi}{4\pi R}$$

We use the right hand rule to determine the B field direction of the two arcs:

$$B = B_{upper} - B_{lower} = 0$$

7. In Fig. 29-36, two circular arcs have radii a = 13.5 cm and b = 10.7 cm, subtend angle θ = 74.0^0, carry current i = 0.411 A, and share the same center of curvature P. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at P?



We choose the positive direction if B is directed into the page

The upper arc produces B directed into the page and the lower arc produces B out of the page, so:

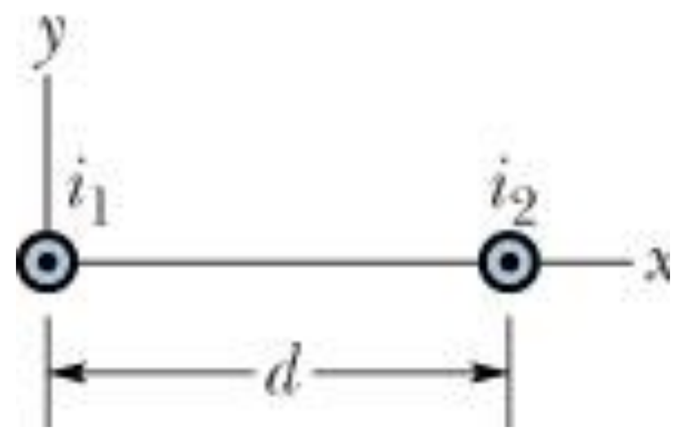
$$B_{net} = B_{upper} - B_{lower} = \frac{\mu_0 i\theta}{4\pi a} - \frac{\mu_0 i\theta}{4\pi b} = \frac{\mu_0 i\theta}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$B_{net} = \frac{4\pi\times10^{-7}\times0.411\times(74\times\pi/180)}{4\pi}\left(\frac{1}{13.5\times10^{-2}} - \frac{1}{10.7\times10^{-2}}\right)$$

$\Rightarrow B_{net} = -1.03\times10^{-7}\,(T)$ and the net field direction is out of the page

12. In Fig. 29-38, two long straight wires at separation d = 16.0 cm carry currents i_1 = 3.61 mA and $i_2 = 3.00 i_1$ out of the page. (a) At what point on the x axis shown is the net magnetic field due to the
currents equal to zero? (b) If the two currents are doubled, is the point of zero magnetic field shifted toward wire 1, shifted toward wire 2, or unchanged?

$$B = \frac{\mu_0 i}{2\pi d}$$



- i_1: B_1 is up (positive)
- i_2: B_2 is down (negative)

$$B_{net} = \frac{\mu_0 i_1}{2\pi d_1} - \frac{\mu_0 i_2}{2\pi d_2}$$

(a)

$$B_{net} = \frac{\mu_0 i_1}{2\pi}\left(\frac{1}{d_1} - \frac{3}{d_2}\right) = 0 \Rightarrow d_2 = 3d_1$$

$$\Rightarrow d_1 = 4cm; d_2 = 12cm$$

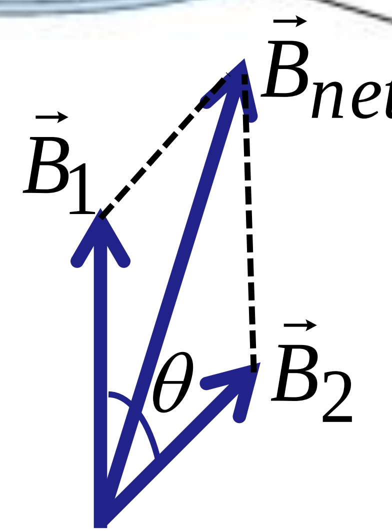
(b) unchanged

16. In Fig. 29-47, two concentric circular loops of wire carrying current in the same direction lie in the same plane. Loop L has radius 1.50 cm and carries 4.00 mA. Loop 2 has radius 2.50 cm and carries 6.00 mA. Loop 2 is to be rotated about a diameter while the net magnetic field B set up by the two loops at their common center is measured. Through what angle must loop 2 be rotated so that the magnitude of that net field is 100 nT?

$$B = \frac{\mu_0 i}{2R}$$

The net B field:

$$B^2 = B_1^2 + B_2^2 - 2B_1 B_2 \cos(180^0 - \theta)$$



$$\theta = \arccos\left(\frac{B_1^2 + B_2^2 - B^2}{-2B_1 B_2} \right)$$

$$\theta \approx 144^0$$

18. A current is set up in a wire loop consisting of a semi-circle of radius 4.5 cm, a smaller concentric semi-circle and two radial straight lengths, all in the same plane. Figure 29-48a shows the arrangement but is not drawn to scale. The magnitude of the magnetic field produced at the center of curvature is 47.25 μT. The smaller semicircle is then flipped over (rotated) until the loop is again entirely in the same plane (Fig. 29-48b). The magnetic field produced at the (same) center of curvature now has magnitude 15.75 μT and its direction is reversed. What is the radius of the smaller semicircle?

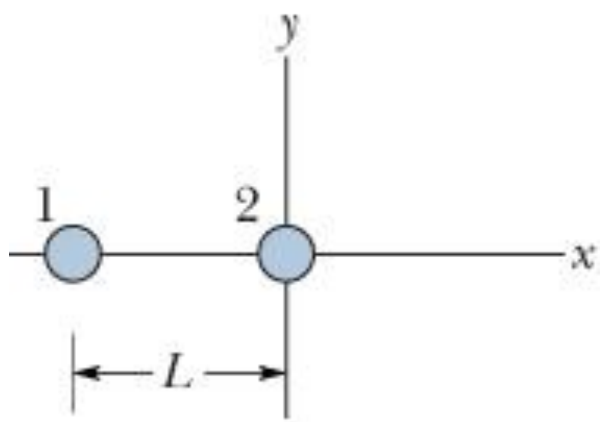
For a semi-circle: $B = \dfrac{\mu_0 i}{4R}$

Figure a: $B_a = \dfrac{\mu_0 i}{4R} + \dfrac{\mu_0 i}{4r} = \dfrac{\mu_0 i}{4}\left(\dfrac{1}{R} + \dfrac{1}{r}\right)$

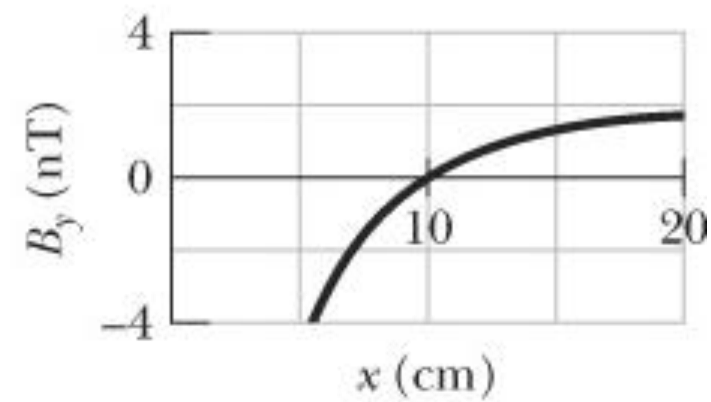
Figure b: $B_b = \dfrac{\mu_0 i}{4R} - \dfrac{\mu_0 i}{4r} = \dfrac{\mu_0 i}{4}\left(\dfrac{1}{R} - \dfrac{1}{r}\right)$

$$\frac{R+r}{r-R} = \frac{B_a}{B_b} = -\frac{47.25}{15.75} = -3 \Rightarrow r = \frac{R}{2} = 2.25(cm)$$

22. Figure 29-42a shows, in cross section, two long, parallel wires carrying current and separated by distance L. The ratio i_1/i_2 of their currents is 4.00; the directions of the currents are not indicated. Figure 29-42b shows the y component B_y of their net magnetic field along the x axis to the right of wire 2. (a) At what value of x > 0 is B_y maximum? (b) If i_2 = 3 mA, what is the value of that maximum? What is the direction (into or out of the page) of (c) i_1 and (d) i_2?



(a)



(b)

• B = 0 at x = 10 cm,
so i_1 and i_2 are in opposite directions:

$$B_{net} = \frac{\mu_0 i_1}{2\pi(L+x)} - \frac{\mu_0 i_2}{2\pi x} = \frac{\mu_0 i_2}{2\pi}\left(\frac{4}{L+x} - \frac{1}{x}\right)$$

- B is maximum when x matches the following equation:

$$f'(x) = \left(\frac{4}{L+x} - \frac{1}{x}\right)' = 0$$

$$-3x^2 + 2Lx + L^2 = 0$$

$$\Rightarrow x = L; x = -L/3(\text{discarded as } x < 0)$$

Now, we need to calculate L. At x = 10 cm, B_y = 0, so:

$$3x - L = 0 \Rightarrow L = 3x = 30(cm)$$

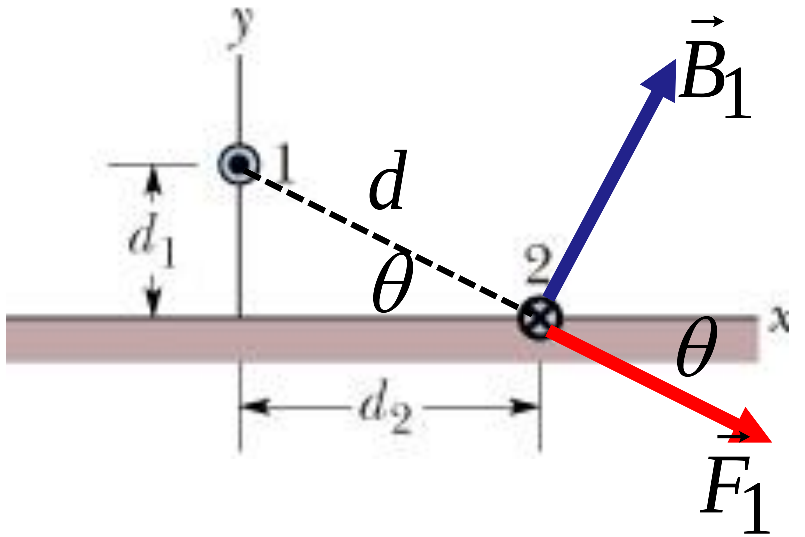
(b) if i_2 = 3 mA:

$$B_{net,\max} = \frac{\mu_0 i_2}{2\pi}\left(\frac{4}{L+L} - \frac{1}{L}\right) = \frac{\mu_0 i_2}{2\pi L}$$

$$B_{net,\max} = \frac{4\pi\times10^{-7}\times3\times10^{-3}}{2\pi\times0.3} = 2\times10^{-9}(T)$$

(c), (d): for x > 10 cm, B_{net} > 0, and B_1 > B_2: so B_1 > 0 and B_2 < 0, thus i_1 is directed out and i_2 into the page

35. Figure 29-55 shows wire 1 in cross section; the wire is long and straight, carries a current of 4.00 mA out of the page, and is at distance d_1 = 2.40 cm from a surface. Wire 2, which is parallel to wire 1 and also long, is at horizontal distance d_2 = 5.00 cm from wire 1 and carries a current of 6.80 mA into the page. What is the x component of the magnetic force per unit length on wire 2 due to wire 1?



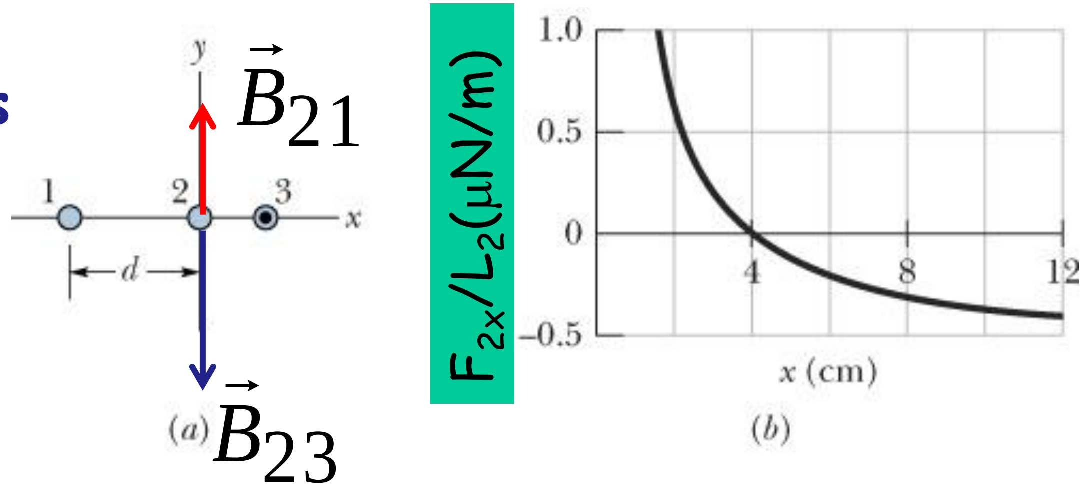
$$F_1 = \frac{\mu_0 L i_1 i_2}{2\pi d}$$

$$F_{1,x} = F_1 \cos\theta = \frac{\mu_0 L i_1 i_2}{2\pi d}\cos\theta$$

$$F_{1,x} = \frac{\mu_0 L i_1 i_2 d_2}{2\pi d^2} \Rightarrow \frac{F_{1,x}}{L} = \frac{\mu_0 i_1 i_2 d_2}{2\pi\left(d_1^2 + d_2^2\right)}$$

38. Figure 29-57a shows, in cross section, three current carrying wires that are long, straight, and parallel to one another. Wires 1 and 2 are fixed in place on an x axis, with separation d. Wire 1 has a current of 0.750 A, but the direction of the current is not given. Wire 3, with a current of 0.250 A out of the page, can be moved along the x axis to the right of wire 2. As wire 3 is moved, the magnitude of the net magnetic force F_2 on wire 2 due to the currents in wires 1 and 3 changes. The x component of that force is F_2, and the value per unit length of wire 2 is F_{2x}/L_2. Figure 29-57b gives F_{2x}/L_2 versus the position x of wire 3. The plot has an asymptote $F_{2x}/L_2 = -0.627\ \mu N/m$ as $x \to \infty$. What are the (a) size and (b) direction (into or out of the page) of the current in wire 2?

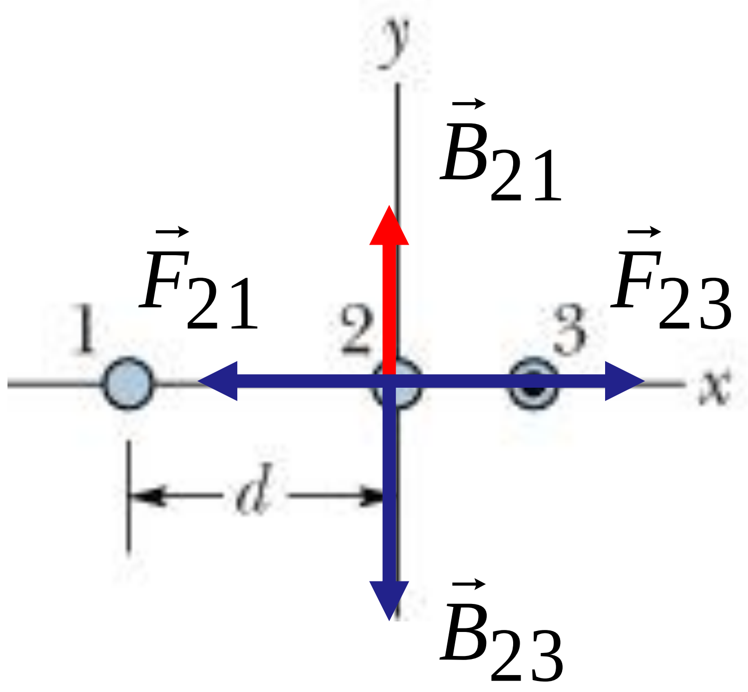
- At x = 4 cm, $F_{2x} = 0$, so i_1 is in the same direction as i_3.
- When i_3 approaches i_2, F_2 is positive and dominated by the force from i_3, so i_2 is out of the page.



At x = 4 cm:

$$\frac{F_{2,net}}{L_2} = \frac{\mu_0 i_2 i_3}{2\pi x} - \frac{\mu_0 i_1 i_2}{2\pi d} = 0$$

$$d = \frac{i_1}{i_3} x = \frac{0.75}{0.25} 4 = 12(cm)$$



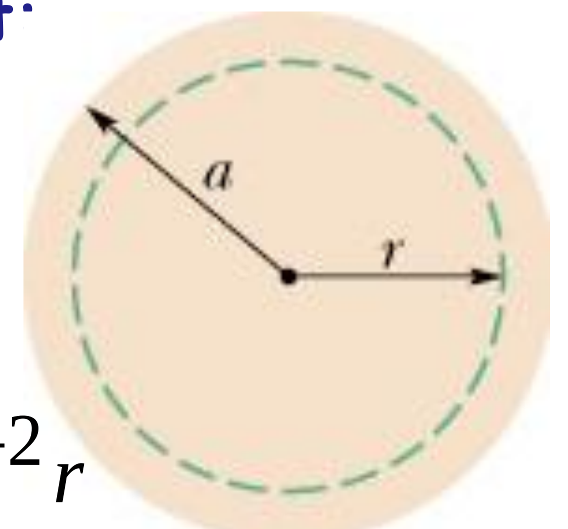
When x → ∞:

$$\frac{F_{2,net}}{L_2} = \frac{F_{21}}{L_2} = \frac{\mu_0 i_1 i_2}{2\pi d} = 0.627 \times 10^{-6}$$

$$\Rightarrow i_2 \approx 0.5(A)$$

43. Figure 29-62 shows a cross section across a diameter of a long cylindrical conductor of radius a = 2.00 cm carrying uniform current 170 A. What is the magnitude of the current's magnetic field at radial distance (a) 0, (b) 1.00 cm, (c) 2.00 cm (wire's surface), and (d) 4.00 cm

The magnetic field is inside a wire with current:



$$B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r$$

(a) B = 0;

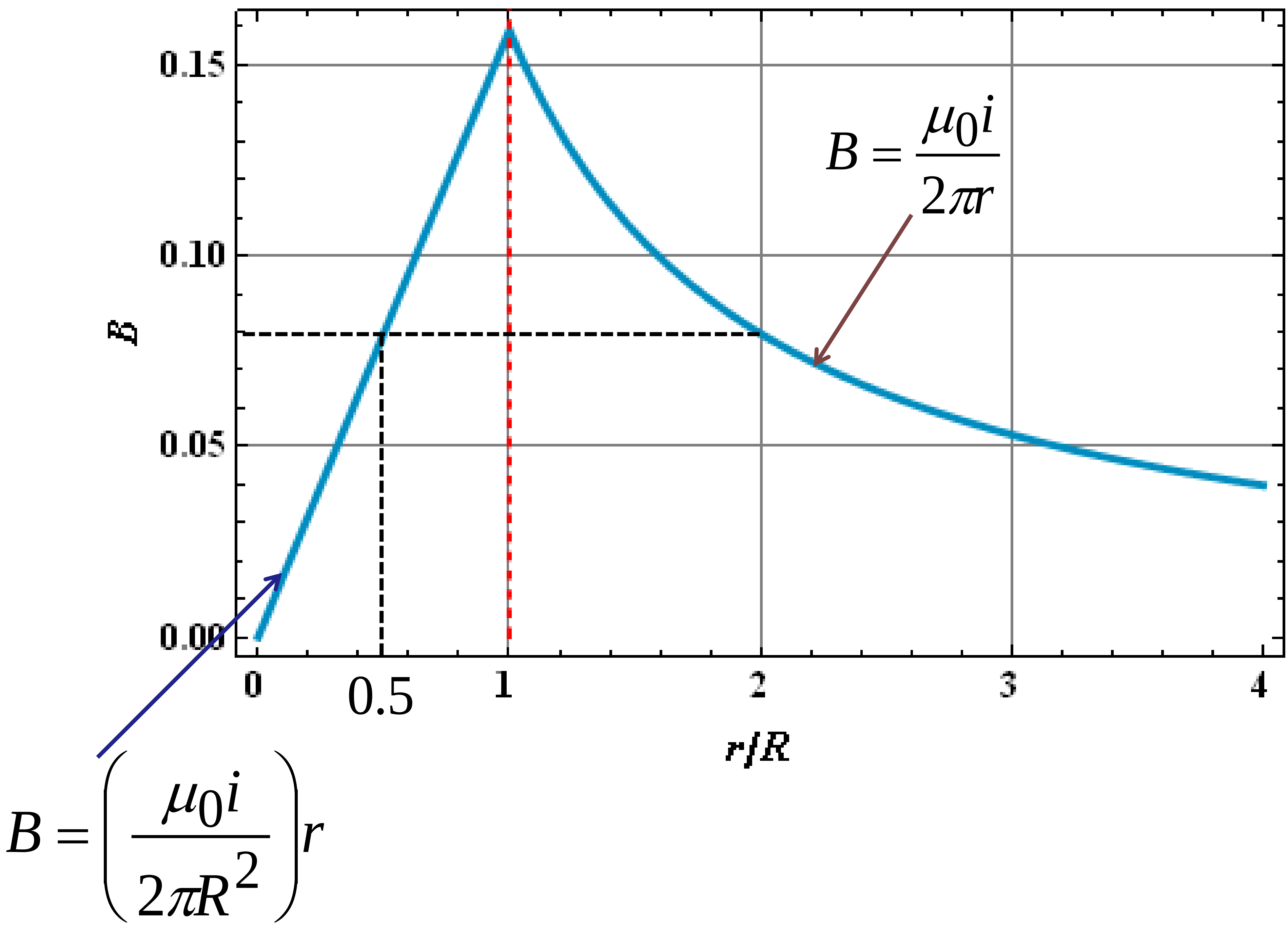
(b) $B = \left(\frac{\mu_0 i}{2\pi a^2}\right) r = \frac{4\pi \times 10^{-7} \times 170}{2\pi 4 \times 10^{-4}} r = 8.5 \times 10^{-2} r$

$$B = 8.5 \times 10^{-2} r = 8.5 \times 10^{-2} \times 1.0 \times 10^{-2} = 8.5 \times 10^{-4} (T)$$

(c)

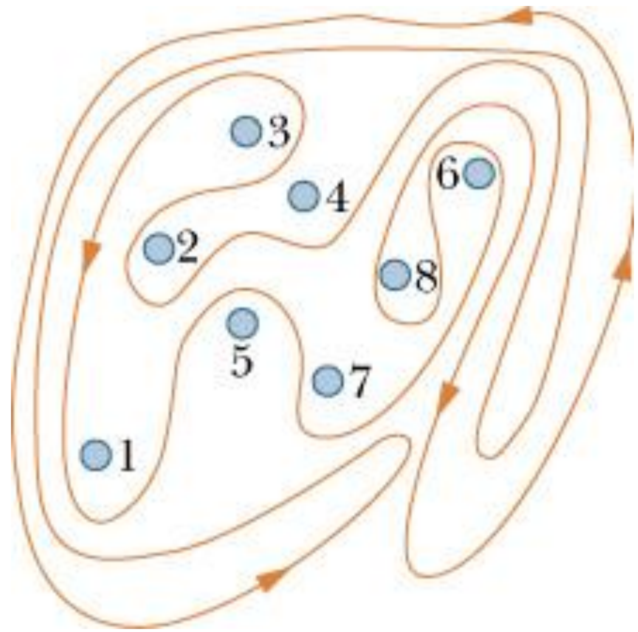
$$B = 17.0 \times 10^{-4} (T)$$

(d) $B = \frac{\mu_0 i}{2\pi r} \Rightarrow B = 8.5 \times 10^{-4} (T)$



0.15
0.10
0.05
0.00
B
0
0.5
1
2
3
4
r/R
$B = \frac{\mu_0 i}{2\pi r}$
$B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r$

46. Eight wires cut the page perpendicularly at the points shown in the figure below. A wire labeled with the integer k (k = 1, 2,…,8) carries the current ki, where i = 6.0 mA. For those wires with odd k, the current is out of the page; for those with even k, it is into the page. Evaluate $\oint \vec{B}.d\vec{s}$ along the closed path in the direction shown.



- Using Ampere's law:

$$\oint \vec{B}d\vec{s} = \mu_0 i_{enc} = \mu_0(i_1 + i_3 + i_6 + i_7)$$

- Using the curled-straight right hand rule:

i_1, i_3, i_7 are positive and i_6 is negative

$$\oint \vec{B}d\vec{s} = \mu_0(i + 3i - 6i + 7i) = 5i\mu_0$$

$$\oint \vec{B}d\vec{s} = 5 \times 6.0 \times 10^{-3} \times 4\pi \times 10^{-7} = 3.77 \times 10^{-8}(T.m)$$

49. A toroid having a square cross section, 5.00 cm on a side, and an inner radius of 15.0 cm has 500 turns and carries a current of 0.800 A. (It is made up of a square solenoid instead of a round one as in Fig. 29-17 -bent into a doughnut shape.) What is the magnetic field inside the toroid at (a) the inner radius and (b) the outer radius?

$$B = \frac{\mu_0 i N}{2\pi}\frac{1}{r}$$

(a) at the inner radius, r = 15 cm:

$$B = \frac{4\pi \times 10^{-7} \times 0.8 \times 500}{2\pi}\frac{1}{0.15} = 5.33 \times 10^{-4} (T)$$

(b) at the outer radius, r = 15 + 5 = 20 cm:

$$B = \frac{4\pi \times 10^{-7} \times 0.8 \times 500}{2\pi}\frac{1}{0.20} = 4.0 \times 10^{-4} (T)$$

50. A solenoid that is 95.0 cm long has a radius of 2.00 cm and a winding of 1500 turns; it carries a current of 3.60 A. Calculate the magnitude of the magnetic field inside the solenoid.

The magnetic field inside a solenoid is computed by:

$$B = \mu_0 i n$$

where n is the number of turns per unit length

$$n = \frac{N}{L} = \frac{1500}{0.95} = 1579\ (\text{turns/m})$$

So:

$$B = 4\pi \times 10^{-7} \times 3.6 \times 1579 = 7.14 \times 10^{-3} (T)$$

57. A student makes a short electromagnet by winding 300 turns of wire around a wooden cylinder of diameter d = 5.0 cm. The coil is connected to a battery producing a current of 4.0 A in the wire. (a) What is the magnitude of the magnetic dipole moment of this device? (b) At what axial distance z >> d will the magnetic field have the magnitude 5.0 μT (approximately one-tenth that of Earth's magnetic field)?

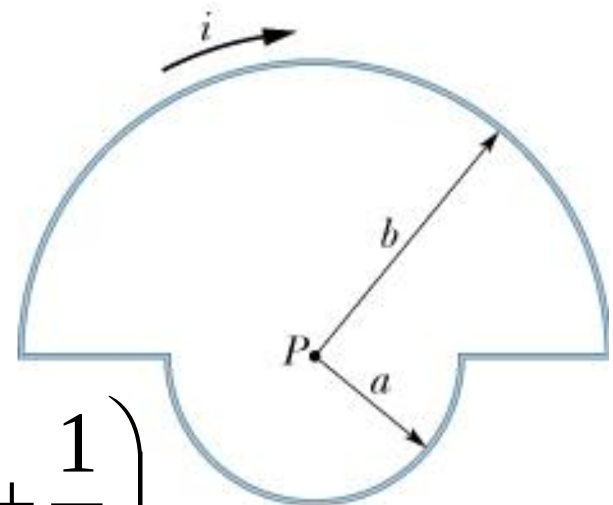
(a) The magnetic dipole moment: $\mu = NiA$

$$\mu = 300 \times 4.0 \times \pi (2.5 \times 10^{-2})^2 = 2.36 (\text{A.m}^2)$$

(b) if z >> d: $B(z) = \dfrac{\mu_0 NiA}{2\pi z^3} = \dfrac{\mu_0 \mu}{2\pi z^3} \Rightarrow z = \sqrt[3]{\dfrac{\mu_0 \mu}{2\pi B}}$

$$z = \sqrt[3]{\frac{4\pi \times 10^{-7} \times 2.36}{2\pi \times 5 \times 10^{-6}}} = 0.46(m)$$

62. In Fig. 29-66, current i = 56.2 mA is set up in a loop having two radial lengths and two semicircles of radii a = 5.72 cm and b = 8.57 cm with a common center P. What are the (a) magnitude and (b) direction (into or out of the page) of the magnetic field at P and the (c) magnitude and (d) direction of the loop's magnetic dipole moment?



(a) The B field due to a circular arc at its center:

$$B = \frac{\mu_0 i \phi}{4\pi R}$$

For the two semicirles:

$$B = B_a + B_b = \frac{\mu_0 i \pi}{4\pi a} + \frac{\mu_0 i \pi}{4\pi b} = \frac{\mu_0 i}{4}\left(\frac{1}{a} + \frac{1}{b}\right)$$

(b) Using the right hand rule, the B direction points into the page

(c) The magnetic dipole moment:

$$\mu = NiA = iA = i\,\frac{\pi(a^2 + b^2)}{2}$$

(d) $\vec{\mu}$ points into the page (the same direction as $\vec{n}$)