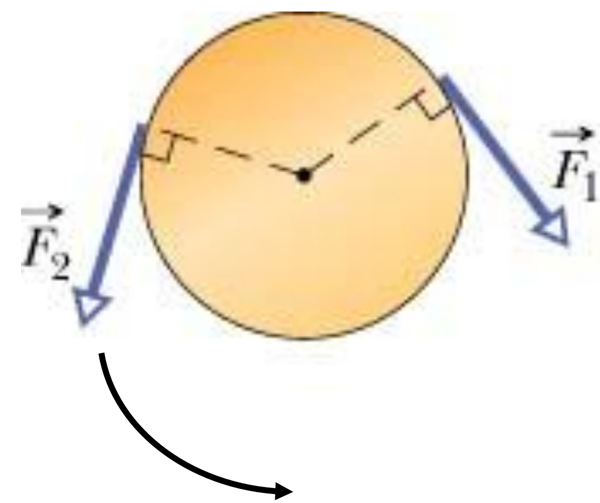
Homework: 53, 56 (p. 270-271); 26, 38 (p. 299-300)

Final exam: Chapter 7, 8, 9, 10, 11 (textbook)

Read 11-12 Precession of a Gyroscope

53. The figure below shows a uniform disk that can rotate around its center like a merry-go-round. The disk has a radius of 2.0 cm and a mass of 20.0 grams and is initially at rest. Starting at time t=0, two forces are to be applied tangentially to the rim as indicated, so that at time t=1.25 s the disk has an angular velocity of 250 rad/s counterclockwise. Force F_1 has a magnitude of 0.1 N. What is magnitude F_2?



$$\tau_{net} = I\alpha$$

$$\tau_{net} = F_2R - F_1R = I\alpha$$

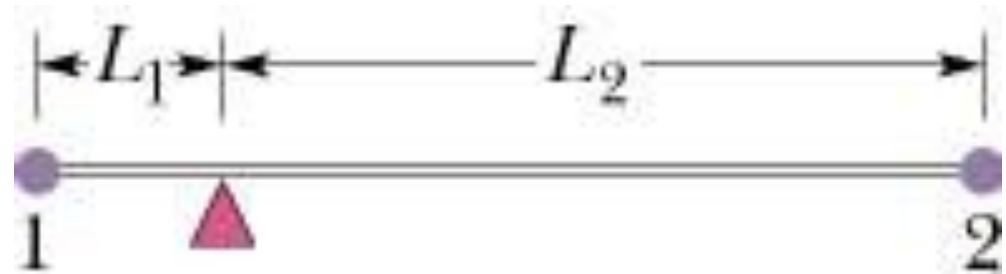
$$F_2 = \frac{I\alpha}{R} + F_1$$

For a uniform disk: $I = \frac{1}{2}MR^2$

For rotation: $\omega = \omega_0 + \alpha t = \alpha t \Rightarrow \alpha = \frac{\omega}{t}$

$$F_2 = \frac{MR\omega}{2t} + F_1 = \frac{(20.0\times10^{-3})(2.0\times10^{-2})(250)}{2\times1.25} + 0.1 = 0.14\ (\text{N})$$

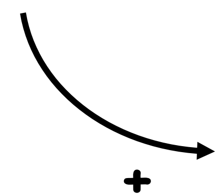
56. The figure shows particles 1 and 2, each of mass m, attached to the ends of a rigid massless rod of length L_1+L_2, with $L_1=20$ cm and $L_2=80$ cm. The rod is held horizontally on the fulcrum and then released. What are the magnitudes of the initial accelerations of (a) particle 1 and (b) particle 2?



$$\tau_{net} = mgL_1 - mgL_2 = I\alpha$$

$$I = mL_1^2 + mL_2^2$$

$$\Rightarrow \alpha = \frac{g(L_1 - L_2)}{L_1^2 + L_2^2}$$



$$\alpha = \frac{9.8(0.2-0.8)}{0.2^2+0.8^2} = -8.65\,(\text{rad/s}^2)$$

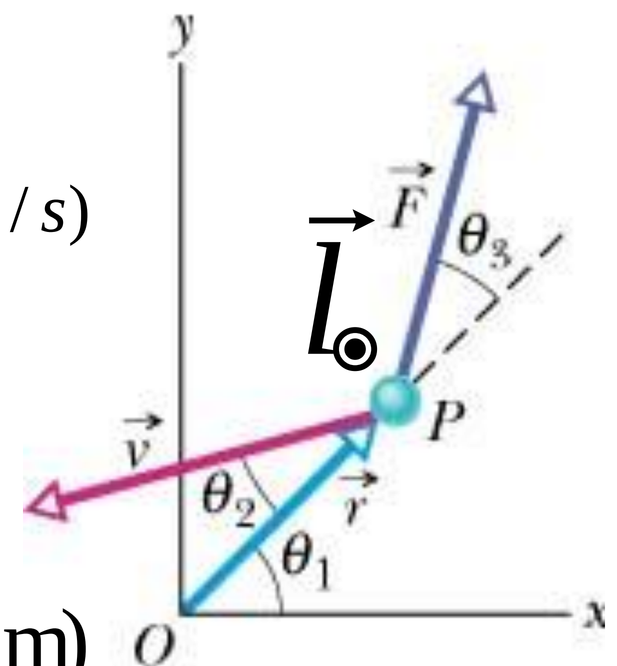
$$\vec{a} = \vec{a}_r + \vec{a}_t;\, a_r = \frac{v^2}{r};\, a_t = \alpha\, r$$

(a) $a_{1t} = \alpha\, L_1 = -8.65 \times 0.2 = -1.73\,(\text{m/s}^2)$

(b) $a_{2t} = \alpha\, L_2 = -8.65 \times 0.8 = -6.92\,(\text{m/s}^2)$

26. At the instant of the figure below, a 2.0 kg particle P has a position vector $\vec{r}$ of magnitude 5.0 m and angle θ_1=45^0 and a velocity vector $\vec{V}$ of magnitude 4.0 m/s and angle θ_2=30^0. Force $\vec{F}$, of magnitude 2.0 N and angle θ_3=30^0, acts on P. All three vectors lie in the xy plane. About the origin, what are the (a) magnitude and (b) direction of the angular momentum of P and the (c) magnitude and (d) direction of the torque acting on P?

$$\vec{l} = \vec{r} \times \vec{p}$$



(a) $l = rmv\sin\theta_2 = 5.0\times2.0\times4.0\times\sin(30) = 20\ (\text{kg m}^2/s)$

(b) Using the right-hand rule, $\vec{l}$ points out of the page and it is perpendicular to the figure plane.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

(c) $\tau = rF\sin\theta_3 = 5.0\times2.0\times\sin(30) = 5\,(\text{N m})$

(d) $\vec{\tau}$ points out of the page and it is perpendicular to the figure plane.

38. A sanding disk with rotational inertia 8.6×10^{-3} kg.m^2 is attached to an electric drill whose motor delivers a torque of magnitude 16 N.m about the central axis of the disk. About that axis and with the torque applied for 33 ms (milliseconds), what is the magnitude of the (a) angular momentum and (b) angular velocity of the disk?

(a) Using Newton's second law for rotation:

$$\tau_{avg} = \frac{\Delta L}{\Delta t}$$

$$\Rightarrow \Delta L = L - L_0 = L = \tau_{avg} \Delta t$$

$L = 16 \times 33 \times 10^{-3} = 0.528$ (Nms) or 0.528 (kg m^2/s)

(b)

$$L = I\omega \Rightarrow \omega = \frac{L}{I}$$

$$\omega = \frac{0.528}{8.6 \times 10^{-3}} = 61.4 \text{ (rad/s)}$$

Chapter 6 Equilibrium and Elasticity

6.1. Equilibrium
6.2. The Center of Gravity and Conditions for Stable Equilibrium
6.3. Elasticity

6.1. Equilibrium

Examples: (1) a book resting on a table, (2) a hockey puck sliding with constant velocity across a frictionless surface, (3) the rotating blades of a ceiling fan, and (4) the wheel of a bicycle traveling along a straight path at constant speed.

a. The requirements of Equilibrium of the objects above:

1. The linear momentum $\vec{P}$ of its center of mass is constant.

$$\vec{F}_{net} = \frac{d\vec{P}}{dt} = 0$$

2. Its angular momentum $\vec{L}$ about its COM, or about any other point, is also a constant.

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt} = 0$$

b. Static equilibrium: Objects are not moving either in translation or in rotation. In these examples above, the book resting on the table is in static equilibrium.

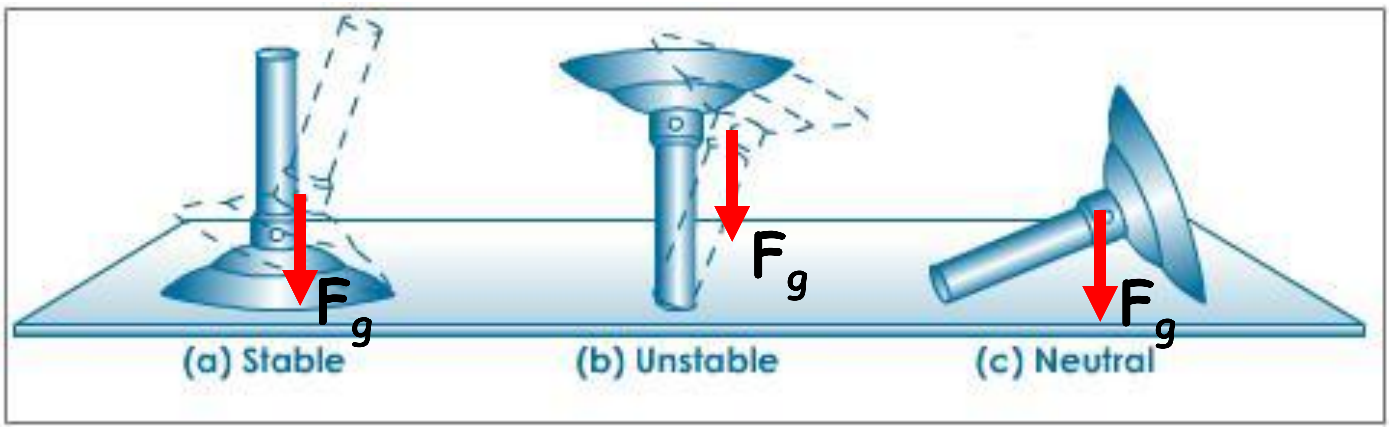
A body may be in one of three states of static equilibrium: neutral, stable, and unstable:

- **Stable Equilibrium:** A body is in stable equilibrium if it returns to its equilibrium position after it has been displaced *slightly*.

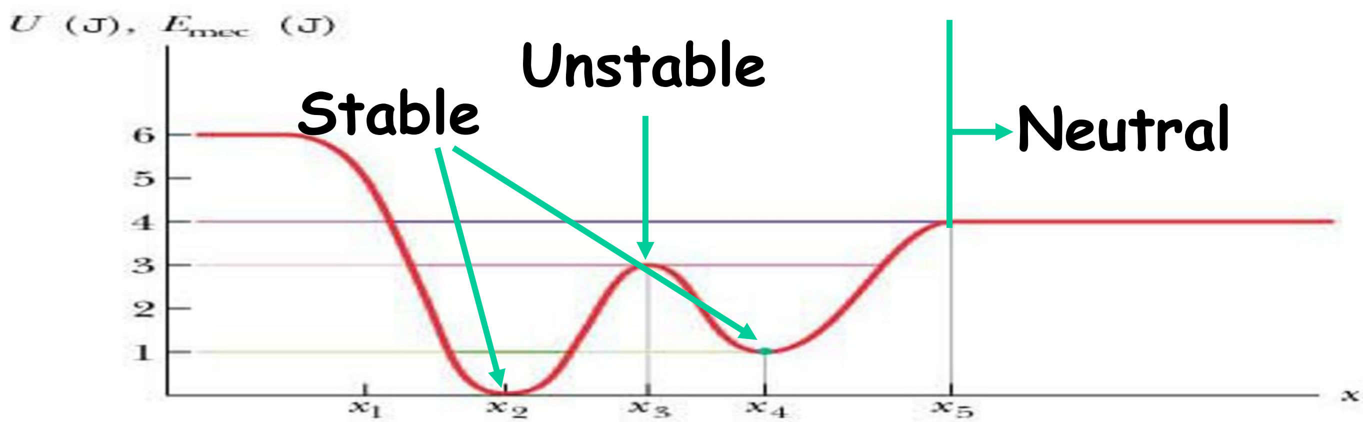
- **Unstable Equilibrium:** A body is in unstable equilibrium if it does not return to its equilibrium position and does not remain in the displaced position after it has been displaced slightly.

- **Neutral Equilibrium:** A body is in neutral equilibrium if it stays in the displaced position after it has been displaced slightly.

Equilibrium of a Bunsen burner



Potential Energy Curve



6.2. The Center of Gravity and Conditions for Stable Equilibrium

•**Center of Gravity (COG):** The point of a body at which the gravitational force can be considered to act and which undergoes no internal motion.

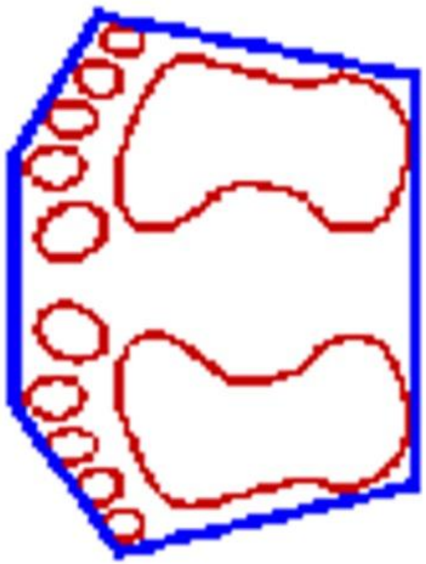
•If the gravitational acceleration g is the same for all elements of a body, then the body's COG is coincident with its COM.

•Recall Center of Mass (COM): $\vec{r}_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i$

•**Base of Support:** It is the area formed by a perimeter around the supporting parts of an object in balance on a surface.

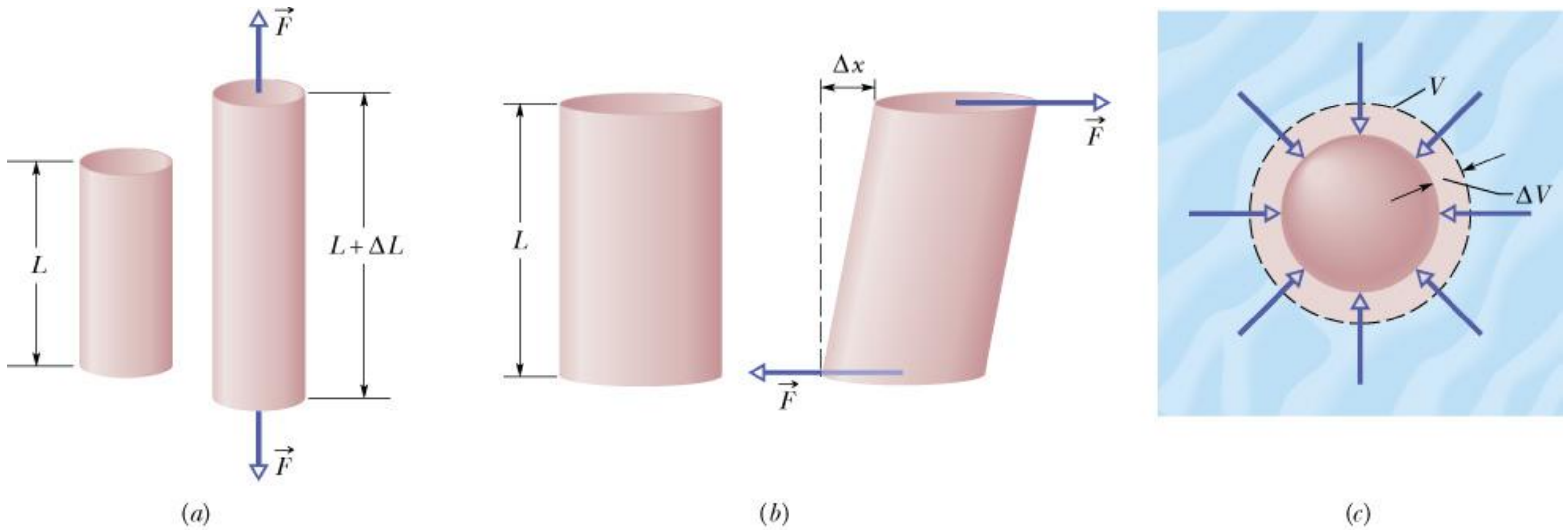
•**Conditions for Stable Equilibrium:**

- •The body should have a broad base of support.
- •Center of gravity of the body should be as low as possible.
- •Vertical line drawn from the center of gravity should fall within the base of support.



6.3. Elasticity

It is the physical property of a material that returns to its original shape once deforming forces (stress) are removed.



Tensile stress **Shearing stress** **Hydraulic stress**

Read text (p. 315-318)

Chapter 7 Gravitation

7.1. Newton's Law of Gravitation
7.2. Kepler's Laws
7.3. Gravitational Lensing Effect

Formation of the solar system

Asteroid Hit in Russia on Feb 15 2013

7.1. Newton's Law of Gravitation

$$F = G\frac{m_1 m_2}{r^2}$$

$G = 6.67 \times 10^{-11}$ N m^2/kg^2: the gravitational constant

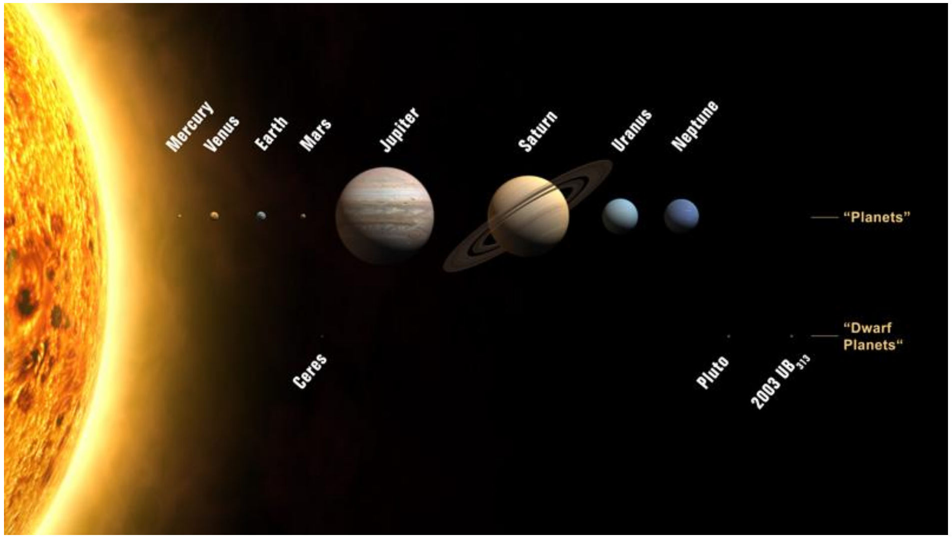
- **Gravitational Potential Energy:**

$$U = -\frac{GMm}{r}$$

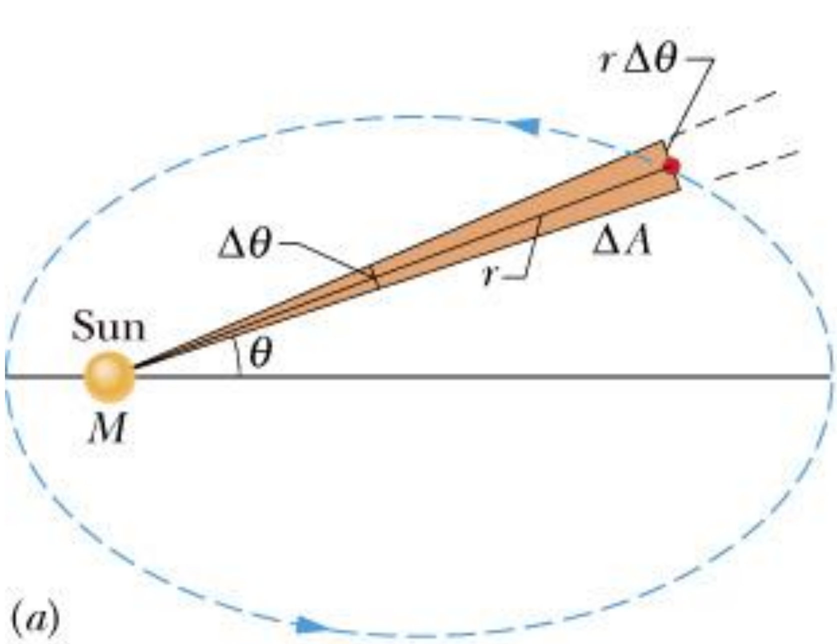
$$U = 0 \text{ for } r = \infty$$

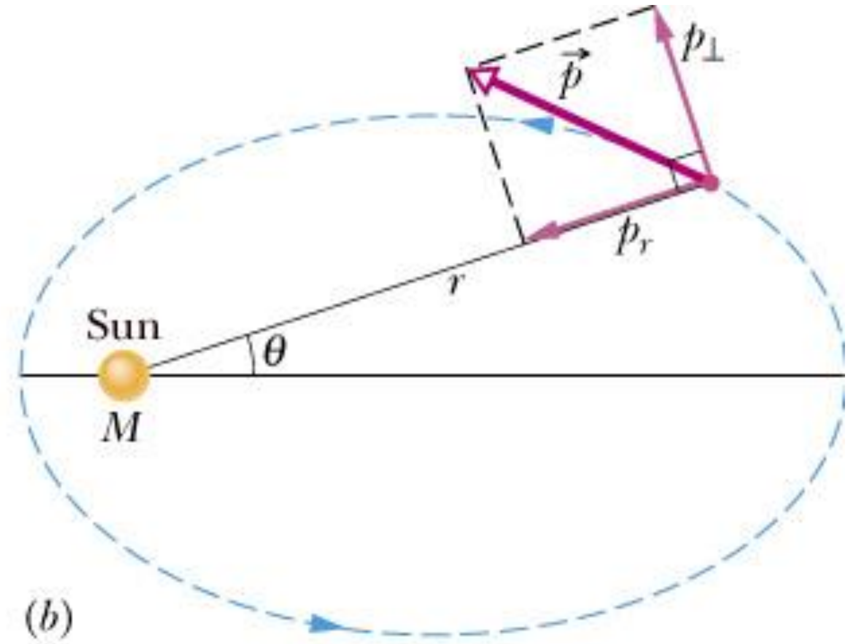
7.2. Kepler's Laws

1. The law of orbits: **All planets move in elliptical orbits, with the Sun at one focus.**



2. The law of areas: A line that connects a planet to the Sun sweeps out equal areas in the plane of the planet's orbit in equal time intervals; that is, the rate dA/dt at which it sweeps out area A is constant.



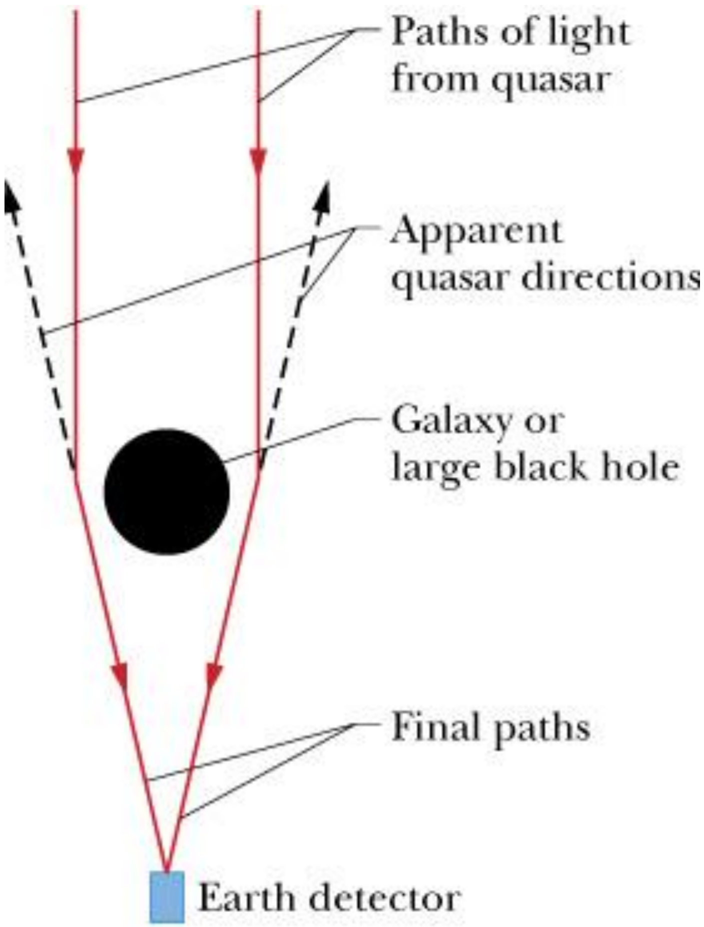


3. The law of periods: The square of the period of any planet is proportional to the cube of the semimajor axis of its orbit.

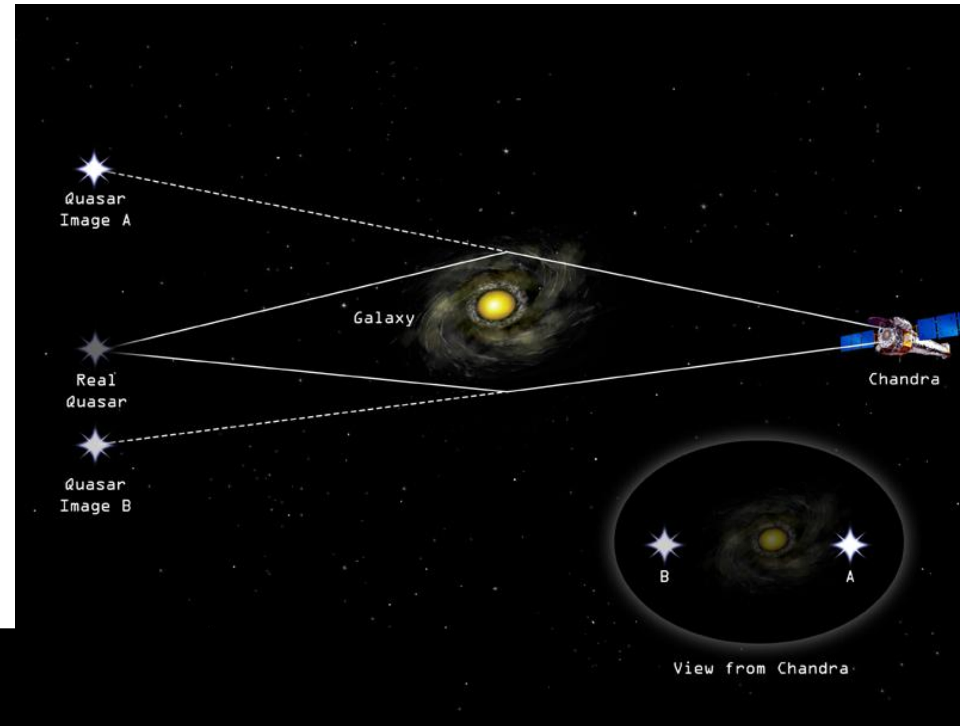
$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

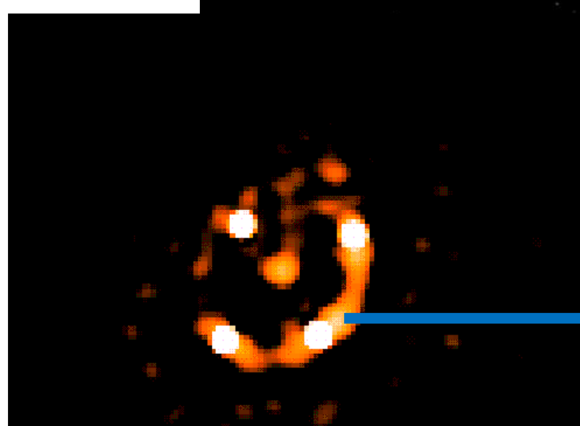
7.3. Gravitational Lensing Effect

According to Einstein's general theory of relativity, when light passes near a massive object (e.g., Earth), the path of the light bends slightly because of the curvature of space there. This effect is called Gravitational Lensing.



(a)





Deconvolved image of H 1413+117

Courtesy of P. Magain

Einstein's ring

Review

- **Work and Kinetic Energy:** $$\Delta K = K_f - K_i = W$$
- **Conservative and Non-conservative Forces**
- **Mechanical Energy:** $$E_{mec} = K + U$$

$$\mathrm{U(y) = mgy} \quad \text{(gravitational potential energy)}$$

$$\mathrm{U} = \frac{1}{2}kx^2 \quad \text{(elastic potential energy)}$$

For an isolated system: $$K_1 + U_1 = K_2 + U_2$$

$$\Delta E_{mec} = 0$$

- **Center of mass:** $$\vec{r}_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i$$
- **Collision and Impulse:** $$\Delta\vec{p} = \vec{J} \text{ (linear momentum - impulse theorem)}$$

$$J = F_{avg}\Delta t$$

- **Conservation of Linear Momentum:**

For an isolated system: $\vec{P} = \text{constant}$

- **Inelastic Collisions:**

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

$$KE \neq \text{constant}$$

- **Elastic Collisions:**

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

$$K_{1i} + K_{2i} = K_{1f} + K_{2f}$$

- **Rotation:**

$$\omega = \omega_0 + \alpha t$$

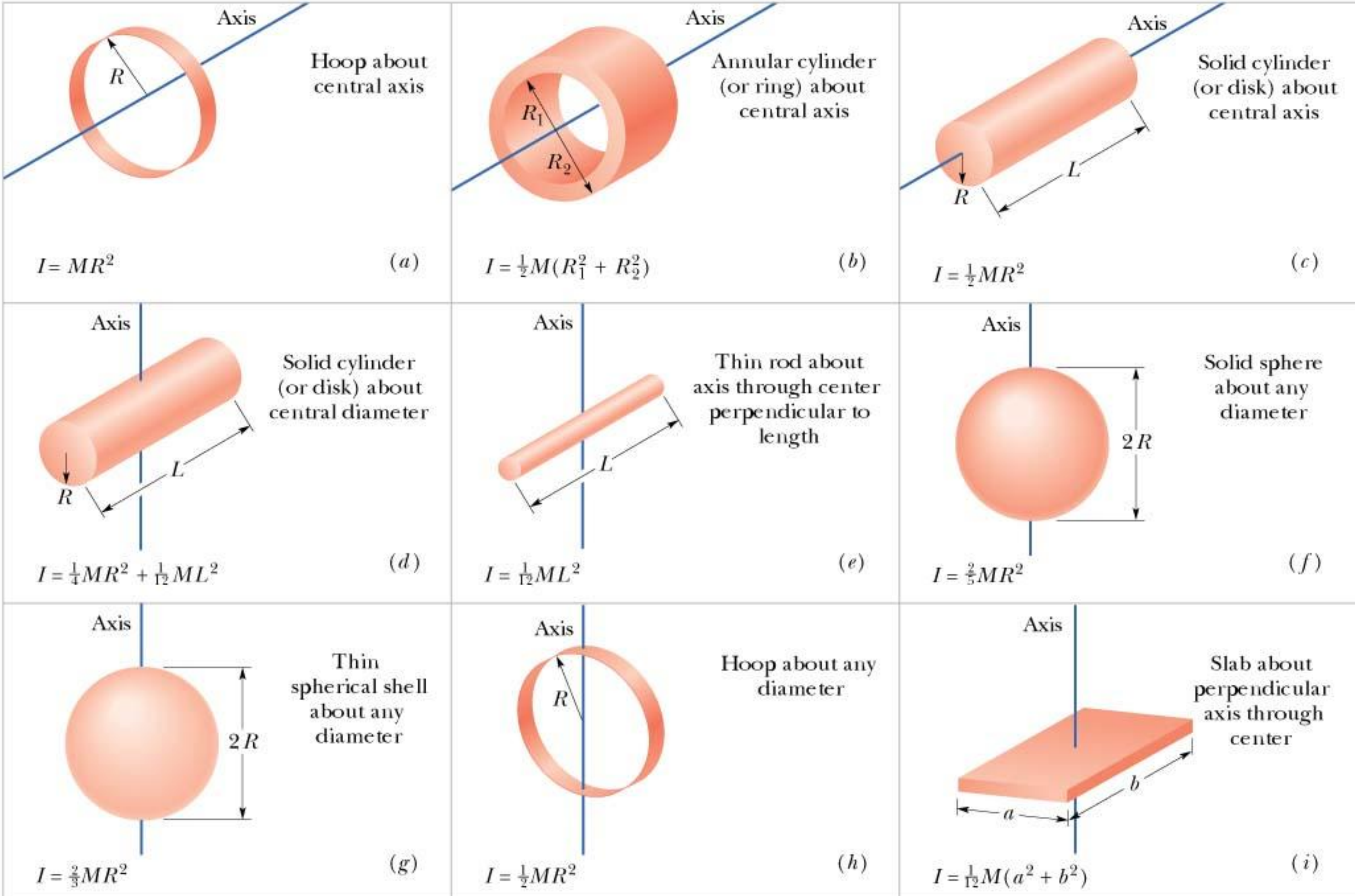
$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 - \omega_0^2 = 2\alpha(\theta - \theta_0)$$

$$v = \omega r \qquad a_t = \alpha r \qquad a_r = \frac{v^2}{r} = \omega^2 r$$

- **Rotational KE:**

$$K = \frac{1}{2} I \omega^2$$



- Torque: $\vec{\tau} = \vec{r} \times \vec{F}$

- Newton's second law for rotation: $\tau_{net} = I\alpha$ (I: kg.m²)
  - For a point mass: $I = mr^2$
  - For a rigid body: I depends on the object shape

- Work – (rotational) KE Theorem:

$$\Delta K = K_f - K_i = \frac{1}{2} I\omega_f^2 - \frac{1}{2} I\omega_i^2 = W$$

- Rolling motion: $v_{com} = \omega R$ $a_{com} = \alpha R$ $K = \frac{1}{2} I_{com}\omega^2 + \frac{1}{2} M v_{com}^2$

Rotational KE

Translational KE

- Angular Momentum: $\vec{l} = \vec{r} \times \vec{p}$

$$l = rmv \sin\phi$$

- **Newton's Second Law in Angular Form:**

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$$

- **Angular Momentum of a Rigid Body:**

$$L = I\omega$$

- **Conservation of Angular Momentum:**

If no net external torque acts on the system:

$$\vec{L} = \text{constant}$$

$$\vec{L}_i = \vec{L}_f$$

$$I_i\omega_i = I_f\omega_f$$

More Corresponding Variables and Relations for Translational and Rotational

Translational		Rotational	
Force	$\vec{F}$	Torque	$\vec{\tau} = \vec{r} \times \vec{F}$
Translational momentum	$\vec{p}_{sys}$	Rotational momentum	$\vec{\ell} = \vec{r} \times \vec{p}$
Translational momentum[a]	$\vec{p}_{sys} = \Sigma \vec{p}_i$	Rotational momentum[a]	$\vec{L} = \Sigma \vec{\ell}_i$
Translational momentum[a]	$\vec{p}_{sys} = M\vec{v}_{com}$	Rotational momentum[b]	$\vec{L} = I\vec{\omega}$
Newton's Second Law[a]	$\Sigma \vec{F}^{ext} = \frac{d\vec{p}_{sys}}{dt}$	Newton's Second Law[a]	$\Sigma \vec{\tau}^{ext} = \frac{d\vec{L}}{dt}$
Conservation law[c]	$\vec{p}_{sys} =$ a constant	Conservation law[c]	$\vec{L} =$ a constant

[a] For systems of particles, including rigid bodies.
[b] For a rigid body about a fixed axis, with L being the component along that axis.
[c] For a closed, isolated system ($\vec{F}^{net} = 0$, $\vec{\tau}^{net} = 0$).