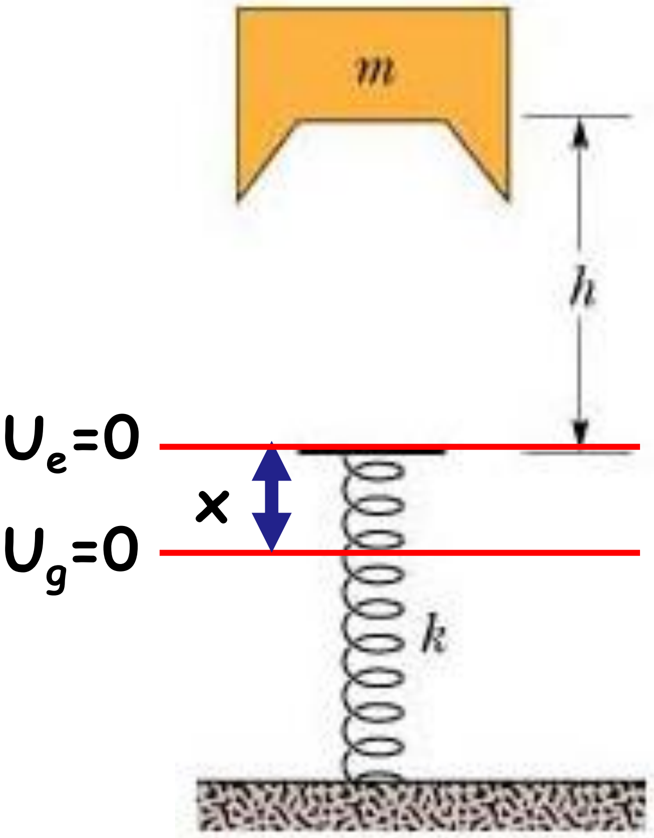
Homework: 24, 56 (pages 191-194);
2, 5, 13, 14, 22, 25, 38 (pages 230-233)

24. A block of mass m = 2.0 kg is dropped from height h = 50 cm onto a spring of spring constant k = 1960 N/m. Find the maximum distance the spring is compressed.



Gravitational potential energy:

$$U_g = mgh$$

Elastic potential energy:

$$U_e = \frac{1}{2}kx^2$$

Kinetic energy:

$$K = \frac{1}{2}mv^2$$

Conservation of mechanical energy:

$$K_i + U_i = K_f + U_f$$

$$mg(h + x) = \frac{1}{2}kx^2 \Rightarrow x = \frac{mg \pm \sqrt{(mg)^2 + 2mghk}}{k}$$

We select x > 0, so:

$$x = 0.11(\text{m})$$

56. You push a 2.0 kg block against a horizontal spring, compressing the spring by 12 cm. Then you release the block, and the spring sends it sliding across a tabletop. It stops 75 cm from where you released it. The spring constant is 170 N/m. What is the block-table coefficient of kinetic friction?

At the beginning:

$$U_e = \frac{1}{2}kx^2$$

When the block stops, elastic potential energy is completely transferred to thermal energy (work done by friction):

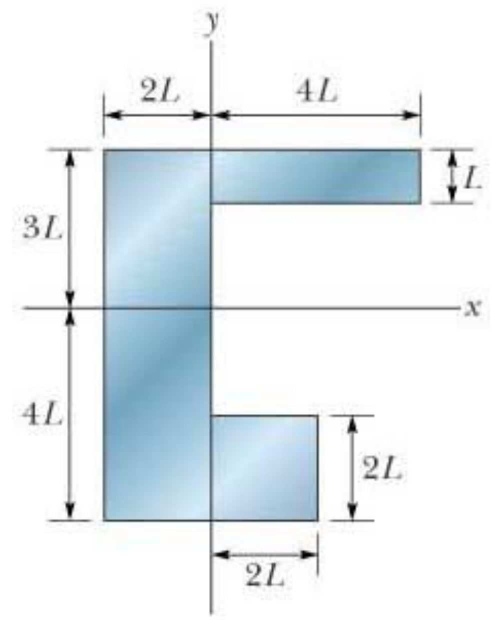
$$U_e = \frac{1}{2}kx^2 = \Delta E_{\text{thermal}} = f_k . d$$

f_k is the kinetic frictional force: $f_k = \mu_k mg$

$$\Rightarrow \mu_k = \frac{kx^2}{2mgd} = \frac{170 \times 0.12^2}{2 \times 2.0 \times 9.8 \times 0.75} = 0.083$$

5. What are (a) the x coordinate and (b) the y coordinate of the center of mass for the uniform plate shown in the figure below if L=5.0 cm?

$$x_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i, \quad y_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i$$



three pieces:
1. L x 4L; 2. 2L x 7L; and
3. 2L x 2L.

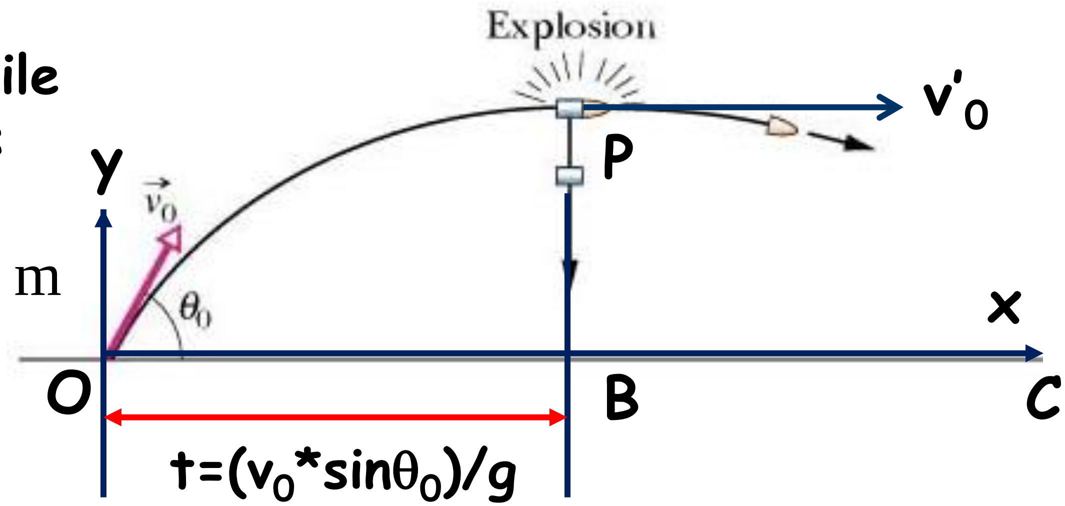
x_1=4L/2=10 cm; y_1=2.5L=12.5 cm;
x_2=-1L=-5 cm; y_2=-0.5L=-2.5 cm;
x_3=1L=5 cm; y_3=-3.0L=-15 cm;

$$\frac{m_1}{M} = \frac{m_1}{m_1+m_2+m_3} = \frac{\rho \times thickness \times area_1}{\rho \times thickness \times (area_1 + area_2 + area_3)} = \frac{4}{4+14+4} = 0.182$$

$$\frac{m_2}{M} = 0.636; \frac{m_3}{M} = 0.182$$

$$x_{com} = -0.45\,\text{cm};\ y_{com} = -2.01\,\text{cm}$$

13. A shell is shot with an initial velocity v_0 of 20 m/s, at an angle of $\theta_0 = 60^0$ with the horizontal. At the top of the trajectory, the shell explodes into 2 fragments of equal mass (see figure). One fragment, whose speed immediately after the explosion is zero, falls vertically. How far from the gun does the other fragment land, assuming that the terrain is level and that air drag is negligible?



- First stage, from O to P is a projectile motion with mass M, v_0 and angle = θ_0:

$$x_B = (v_0 \cos\theta_0)t = \frac{v_0^2}{g}\sin\theta_0\cos\theta_0 = 17.7\text{ m}$$

$$y_P = \frac{v_0^2}{2g}\sin^2\theta_0$$

- Second stage, from P to C is a projectile motion with mass M/2, v'_0 and angle = 0^0:

$$Mv_0\cos\theta_0 = \frac{1}{2}M\times 0 + \frac{1}{2}Mv_0' \Rightarrow v_0' = 2v_0\cos\theta_0$$

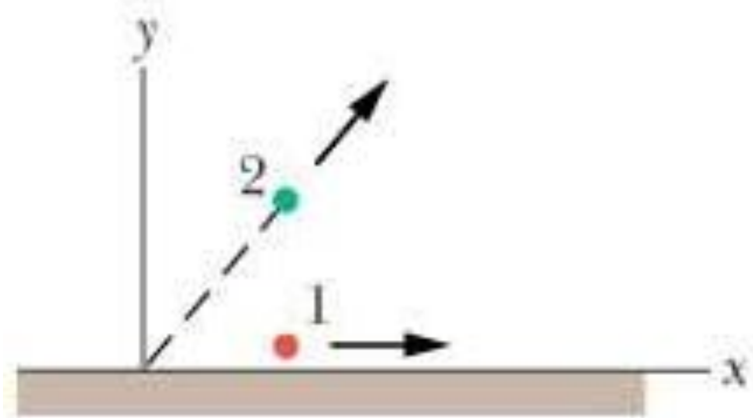
- time for the other fragment flies from P to C:

$$t_{PC} = \sqrt{\frac{2y_B}{g}} = \frac{v_0\sin\theta_0}{g} = t_{OP}$$

$$x_C = (v_0\cos\theta_0)t + (2v_0\cos\theta_0)t = 3x_B = 53{,}1\text{ m}$$

14. In the figure below, two particles are launched from the origin of the coordinates system at time t=0. Particle 1 of mass m_1=5.0 g is shot directly along the x axis, where it moves with a constant speed of 10 m/s. Particle 2 of mass m_2=3.0 g is shot with a velocity of magnitude 20.0 m/s, at an upward angle such that it always stays directly above particle 1 during its flight. (a) What is the maximum height H_{max} reached by the com of the two particle system? In unit-vector notation, what are the (b) velocity and (c) acceleration of the com when the com reaches H_{max}?

$$v_{2,y}^2 - v_{2,y0}^2 = -2gy$$



(a) At the maximum height:

$$-v_{2,y0}^2 = -2gy_{\max}$$

Particle 2 always stays directly above P.1:

$$v_{2,x} = v_{1,x}$$

$$\Rightarrow v_{2,y0} = \sqrt{v_2^2 - v_{2,x}^2} = \sqrt{v_2^2 - v_{1,x}^2} = 17.3\,(\text{m/s})$$

$$y_{\max} = 15.3\,(\text{m}) \Rightarrow H_{\max} = \frac{m_2 y_{\max}}{m_1 + m_2} = 5.74\,(\text{m})$$

(b)

$$\vec{v}_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i\vec{v}_i = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

At the maximum height, $v_{2,y}$=0:

$$v_{com,y} = 0;\ v_{com,x} = \frac{m_1 v_{1,x} + m_2 v_{2,x}}{m_1 + m_2} = v_{1,x}$$

$$\vec{v}_{com} = (10\ \text{m/s})\,\hat{\text{i}}$$

(c)

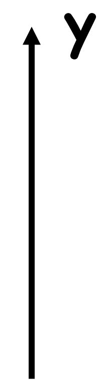
$$M\vec{a}_{com} = \sum_{i=1}^{n} m_i\vec{a}_i$$

$$\vec{a}_{com} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$$

$$a_{com} = \frac{m_2 g}{m_1 + m_2} = 3.68\,(\text{m/s}^2)$$

$\vec{a}_{com}$ is downward, hence : $\vec{a}_{com} = (-3.68\ \text{m/s}^2)\ \hat{\text{j}}$

25. A 1.2 kg ball drops vertically onto a floor, hitting with a speed of 25 m/s. It rebounds with an initial speed of 10 m/s. (a) What impulse acts on the ball during the contact? (b) If the ball is in contact with the floor for 0.020 s, what is the magnitude of the average force on the floor from the ball?



(a)

$$v_i = -25\ \text{m/s};\ v_f = 10\ \text{m/s}$$

Impulse J:

$$\vec{J} = \Delta\vec{p}$$

The ball is dropping vertically → one dimensional motion:

$$J = \Delta p_y = m(v_f - v_i) = 1.2 \times [10 - (-25)] = 42\ (\text{kg.m/s})$$

$$\vec{J} = (42\ \text{kg.m/s})\,\hat{j}$$

(b)

$$J = F_{avg}\Delta t \Rightarrow F_{avg} = \frac{J}{\Delta t} = \frac{42}{0.02} = 2100\ (\text{N})$$

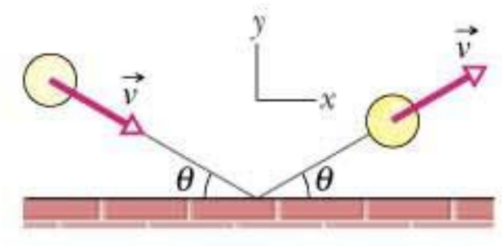
38. In the overhead view of the figure below, a 300g ball with a speed v of 8.0 m/s strikes a wall at an angle θ of 30⁰ and then rebounds with the same speed and angle. It is in contact with the wall for 10 ms. In unit-vector notation, what are (a) the impulse on the ball from the wall and (b) the average force on the wall from the ball?

$$\vec{J} = \Delta\vec{p}$$

$$\Delta\vec{p} = m(\vec{v}_f - \vec{v}_i)$$

$$\vec{v}_i = +6.9\hat{i} - 4.0\hat{j};\ \vec{v}_f = +6.9\hat{i} + 4.0\hat{j}$$

$$\vec{J} = (0.3\,\text{kg})(8.0\,\text{m/s})\,\hat{j} = (2.4\,\text{kg.m/s})\,\hat{j}$$



The average force on the ball from the wall:

$$\vec{F}_{avg} = \frac{\vec{J}}{\Delta t} = \left(\frac{2.4\,\text{kg.m/s}}{0.01\,\text{s}}\right)\hat{j} = (240\ \text{N})\,\hat{j}$$

According to Newton's third law, the force on the wall from the ball:

$$-\vec{F}_{avg} = (-240\ \text{N})\,\hat{j}$$

Chapter 4 Linear Momentum and Collisions

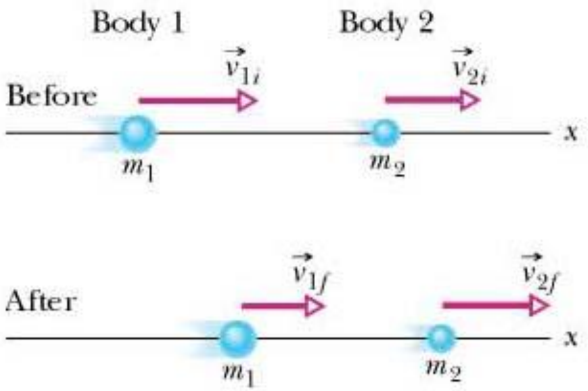
4.1. The Center of Mass. Newton's Second Law for a System of Particles
4.2. Linear Momentum and Its Conservation
4.3. Collision and Impulse
4.4. Momentum and Kinetic Energy in Collisions

4.4. Momentum and Kinetic Energy in Collisions

Three types of collisions: We consider a system of 2 bodies

1. Inelastic collision:

$$\text{total momentum} : \vec{P} = \text{constant}$$



$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

$$KE \neq \text{constant}$$

Some energy (KE) is transferred to other forms, e.g. heat, sound.

$$\vec{v}_{com} = \frac{\vec{P}}{m_1 + m_2} = \frac{\vec{p}_{1i} + \vec{p}_{2i}}{m_1 + m_2} = \frac{\vec{p}_{1f} + \vec{p}_{2f}}{m_1 + m_2} = \text{constant}$$

2. Elastic collision: $\vec{p}$ and KE are conserved.

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

$$K_{1i} + K_{2i} = K_{1f} + K_{2f}$$

- **In one dimension:** $$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

Special cases:

- $v_{2i} = 0:$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

$+ m_1 = m_2 : \quad v_{1f} = 0; \quad v_{2f} = v_{1i}$

$+ m_2 >> m_1 : \quad v_{1f} \approx -v_{1i}; \quad v_{2f} \approx \left(\frac{2m_1}{m_2} \right) v_{1i}$

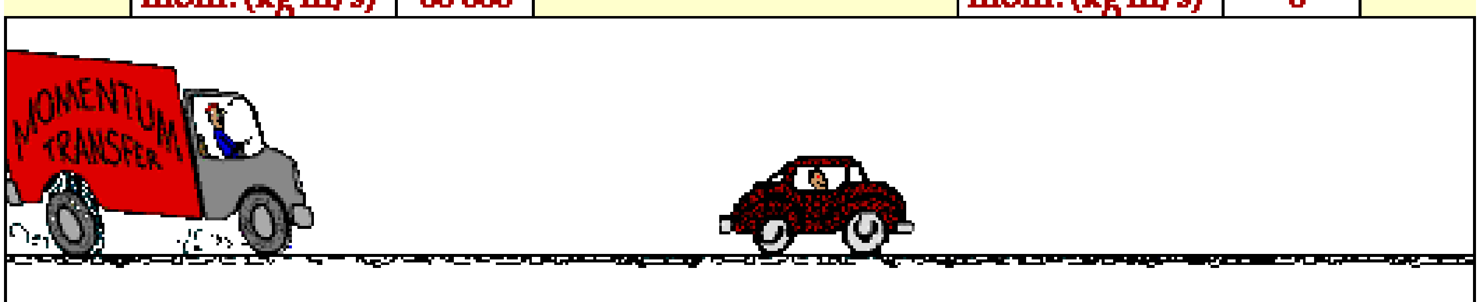
$+ m_1 >> m_2 : \quad v_{1f} \approx v_{1i}; \quad v_{2f} \approx 2v_{1i}$

Truck

mass (kg)	3000
vel. (m/s)	20.0
mom. (kg m/s)	60 000

Car

mass (kg)	1000
vel. (m/s)	0.0
mom. (kg m/s)	0



3. Perfectly inelastic collision: two bodies stick together after collision:

$\vec{p}$ conserved but not KE.

3.1. In one dimension:

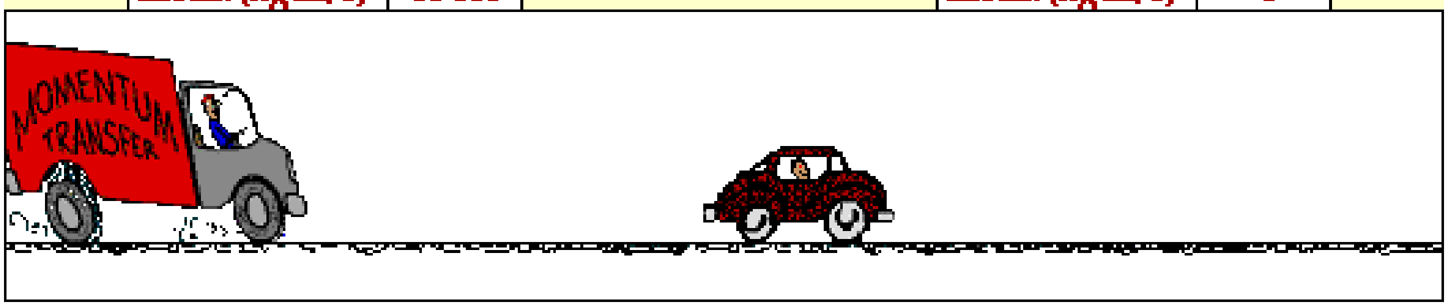
- $v_{1f} = v_{2f} = v_f$: $m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$

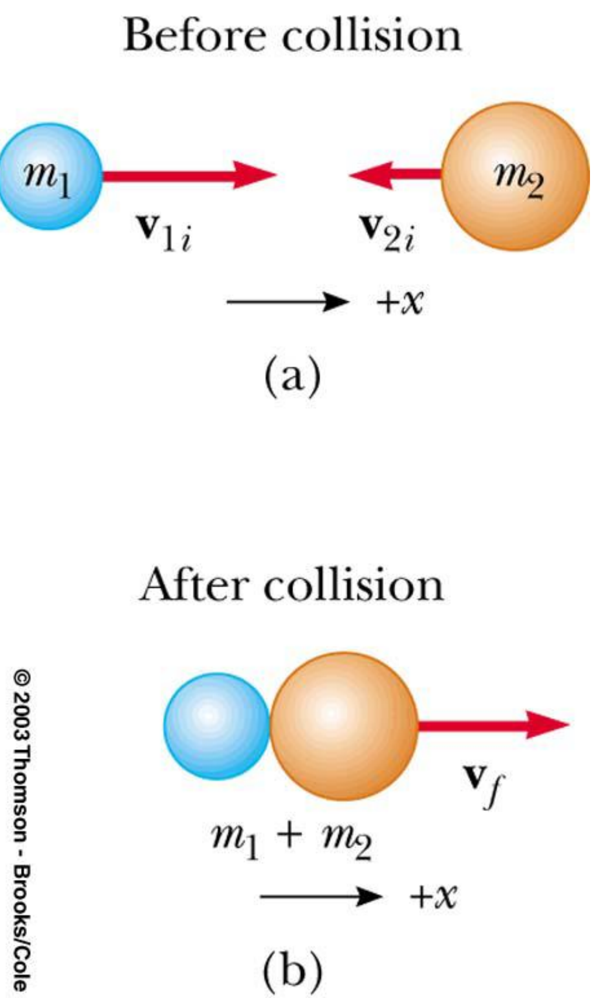
Truck

mass (kg)	3000
vel. (m/s)	20.0
mom. (kg m/s)	60 000

Car

mass (kg)	1000
vel. (m/s)	0.0
mom. (kg m/s)	0





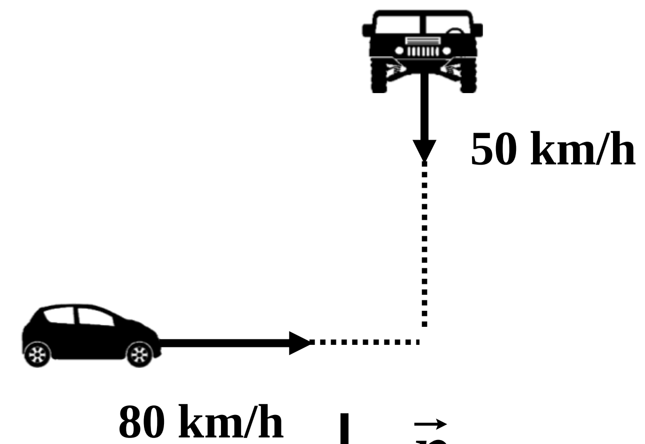
3.2. In two dimensions:

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = (m_1 + m_2) \vec{v}_f$$

Case 3

Example: (Perfectly inelastic collision)

A 1000-kg car travelling east at 80.0 km/h collides with a 3000 kg car traveling south at 50.0 km/h. The two cars stick together after the collision. What is the speed of the cars after the collision? (Final exam, June 2014)



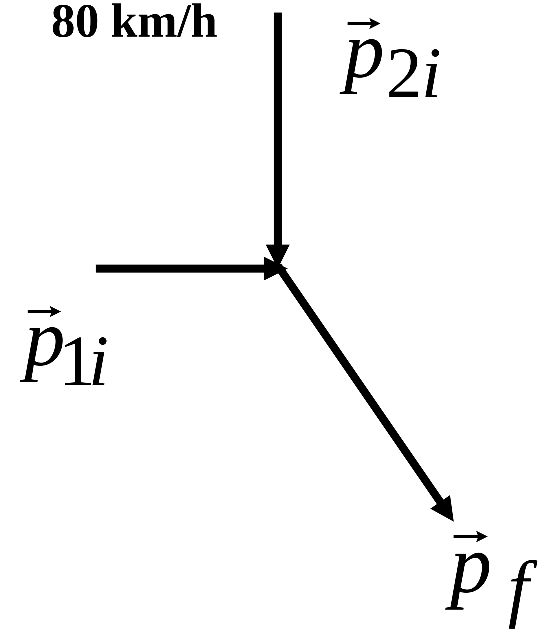
$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_f$$

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f$$

$$p_f = \sqrt{p_{1i}^2 + p_{2i}^2} = \sqrt{(1000 \times 80.0)^2 + (3000 \times 50.0)^2}$$

$$p_f = 170000 \ \text{(kg km/h)}$$

$$v_f = \frac{p_f}{(m_1 + m_2)} = 42.5 \ \text{(km/h) or } 11.8 \ \text{m/s}$$



Homework: 49, 56, 67, 60, 64, 74 (p. 234-237)