Chapter 3: Current and Resistance. Direct Current Circuits

3.1. Electric Current

3.2. Resistance and Resistivity

3.3. Ohm's Law and a Microscopic View of Ohm's Law

3.4. Semiconductors and Superconductors

3.5. Work, Energy, and Emf

3.6. Kirchhoff's Rules

3.7. Resistors in Series and in Parallel

3.8. RC Circuits

3.5. Work, Energy, and Emf

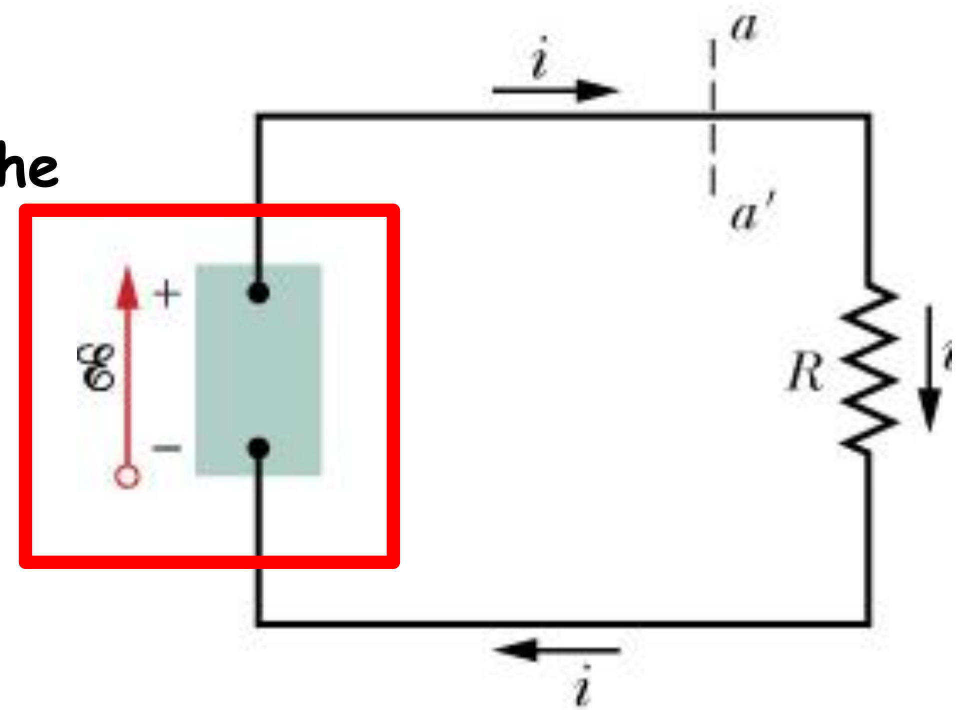
3.5.1. Concepts:

- To make charge carriers flow through a device we must establish a potential difference between the ends of the device, e.g., connecting the device to a charged capacitor but the duration time is very short
- To produce a steady flow, we need a *charge pump*, an **emf** device ε (emf: electromotive force, outdated phrase)
- Some examples for emf devices: battery, electric generators (solar cells, fuel cells, thermopiles), living systems (electric eels)

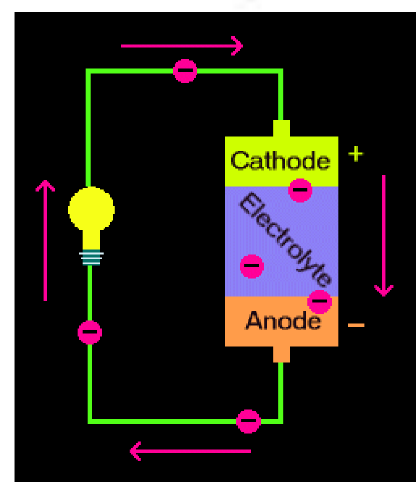

• When an emf is connected (in a closed circuit), within the emf device, positive charge carriers move from a region of low electric potential (low electric potential energy, negative terminal) to a region of higher potential (higher potential energy, positive terminal)

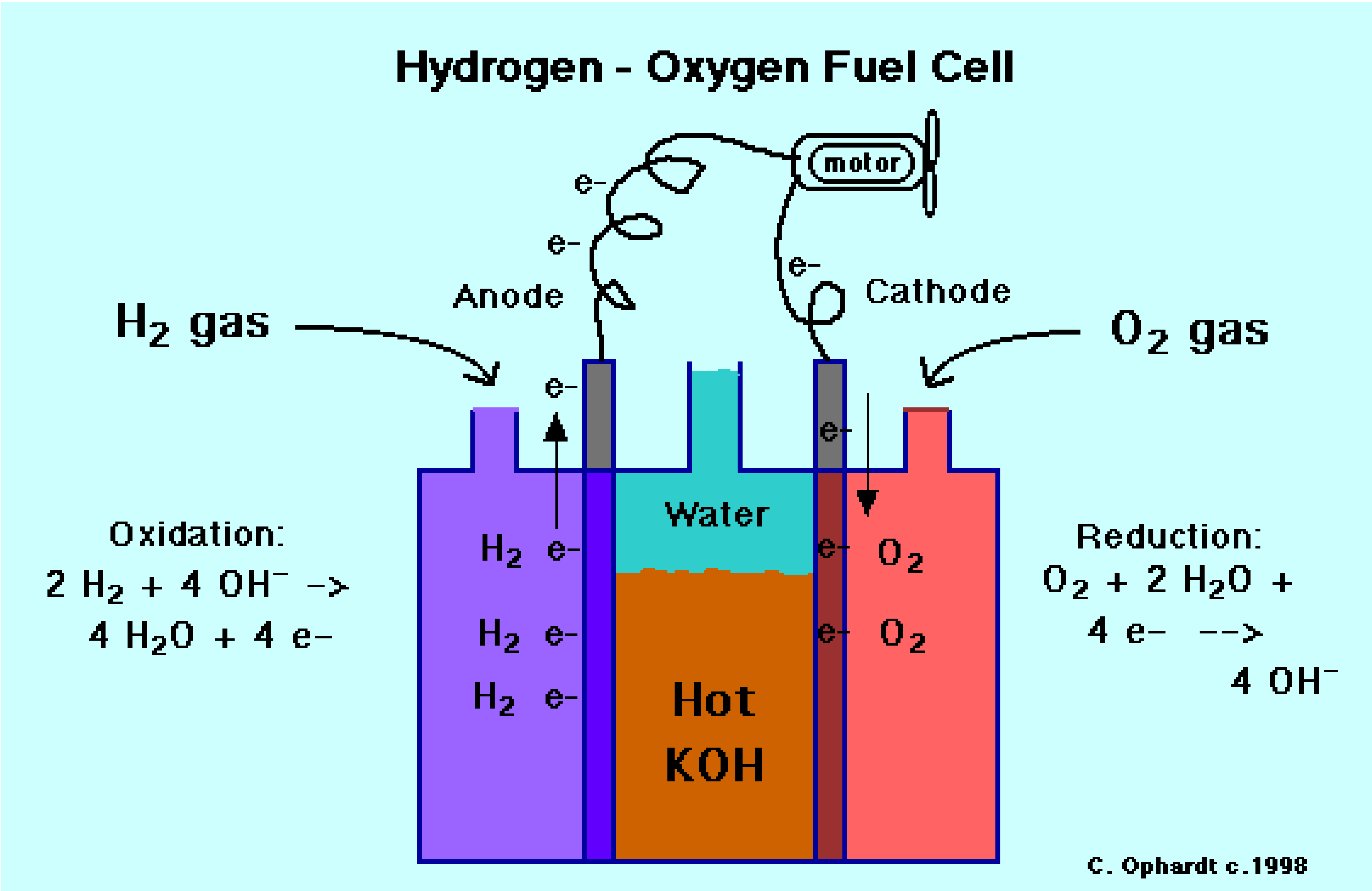
• This flow of positive charge and the current have the same direction

• As this motion is the opposite of what the electric field between the terminals would cause the positive charges to move, therefore there must be a energy source to do work on the charges



• The source may be chemical, or involves mechanical forces, temperature differences, or the Sun

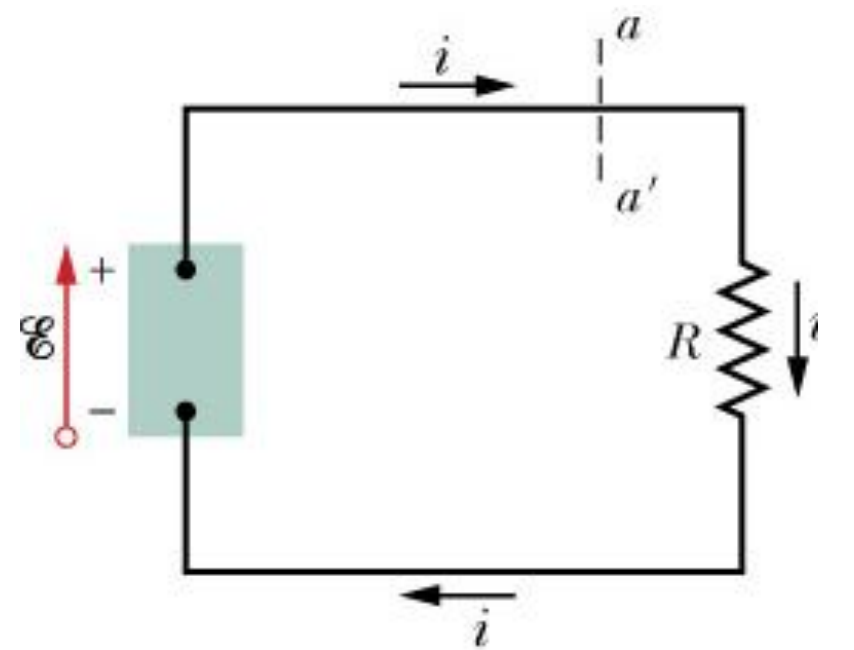




Hydrogen - Oxygen Fuel Cell
motor
e-
e-
e-
Anode
Cathode
H2 gas
O2 gas
e-
e-
Water
H2 e-
H2 e-
H2 e-
e- O2
e- O2
Hot
KOH
Oxidation:
2 H2 + 4 OH⁻ ->
4 H2O + 4 e-
Reduction:
O2 + 2 H2O +
4 e- -->
4 OH⁻
C. Ophardt c.1998

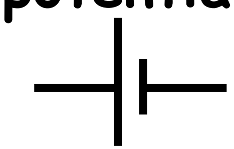
3.5.2. Work, Energy, and Emf:

Emf ε:



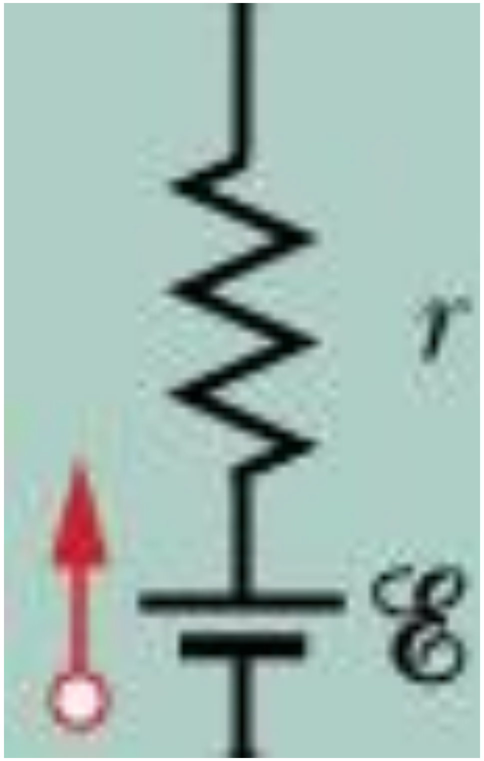
- We analyze a circuit as shown: in any time interval dt, a charge dq passes through any cross section of the circuit
- The emf device does an amount of work dW:, we define the emf of the emf device as follows:

$$\varepsilon = \frac{dW}{dq}$$

- So, the emf of an emf device is the work per unit charge that the device does in moving charge from its low-potential terminal to its high-potential terminal (symbol: )
- SI unit: 1 Volt (V) = 1 Joule / 1 Coulomb

•An ideal emf device: the potential difference between the terminals is equal to the emf of the device (no internal resistance r = 0), V = ε (open or closed circuit)

• A real emf device: V = ε if there is no current through the device and V < ε if there is a current, it means the real devices have internal resistance (r ≠ 0)



A real emf device

Power of an emf device:

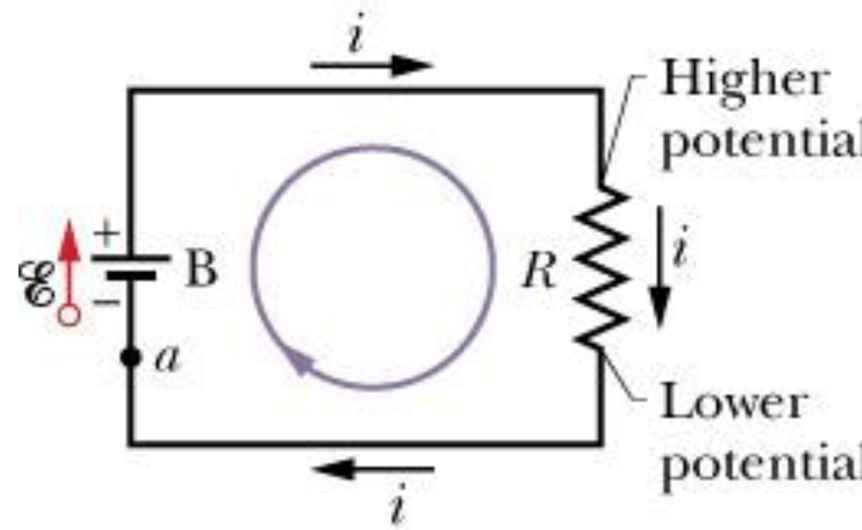
$$dW = \varepsilon dq = \varepsilon i dt = P dt$$

$$P = i\varepsilon$$

3.6. Kirchhoff's Rules

3.6.1. Loop Rule (Voltage Law): The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero

Example: We consider a circuit as shown, from point a we follow the clockwise direction, we have:



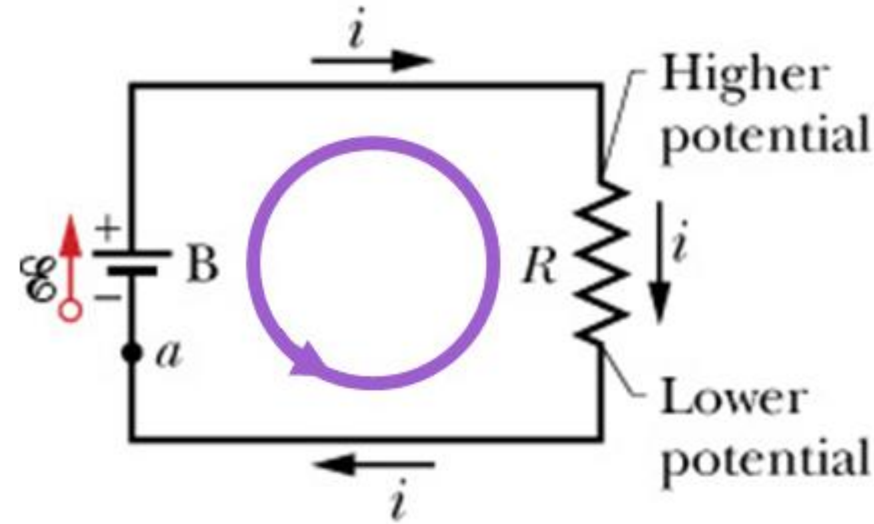
$$\varepsilon - iR = 0$$

Important Notes:

- For a move through a resistance in the direction of the current, the change in potential is −iR; in the opposite direction it is +iR (resistance rule)
- For a move through an ideal emf device in the direction of the emf arrow, the change in potential is +ε; in the opposite direction it is -ε (emf rule)

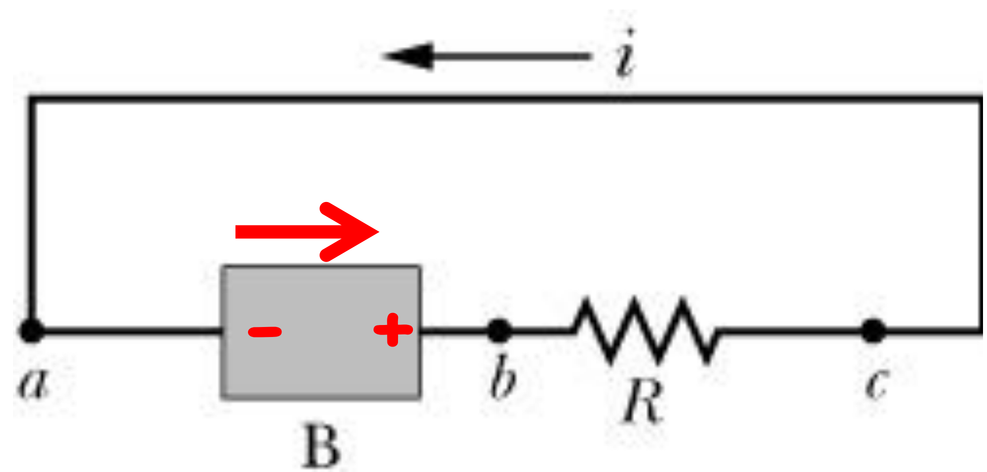
•If you follow the counterclockwise direction:

$$iR - \varepsilon = 0$$



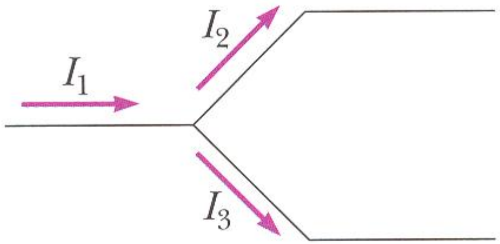
Checkpoint: The figure shows the current i in a single-loop circuit with a battery B and a resistance R (and wires of negligible resistance). (a) Should the emf arrow at B be drawn pointing leftward or rightward? At points a, b, and c, rank (b) the magnitude of the current, (c) the electric potential, and (d) the electric potential energy of the charge carriers, greatest first.

(a) rightward
(b) all tie
(c) V_b, V_a=V_c
(d) b, a=c



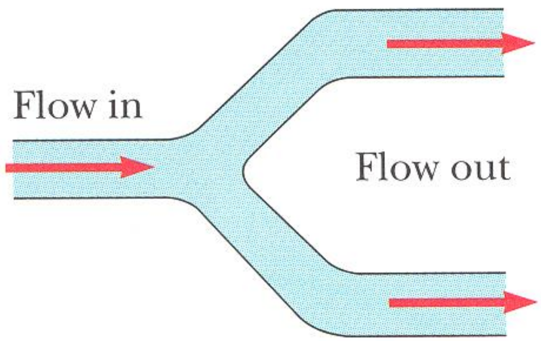
3.6.2. Junction Rule (Current Law): The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction

$$i_1 = i_2 + i_3$$



(a)

This rule is a statement of the conservation of charge for a steady flow of charge, there is neither a buildup nor a depletion of charge at a junction



(b)

3.7. Resistors in Series and in Parallel:

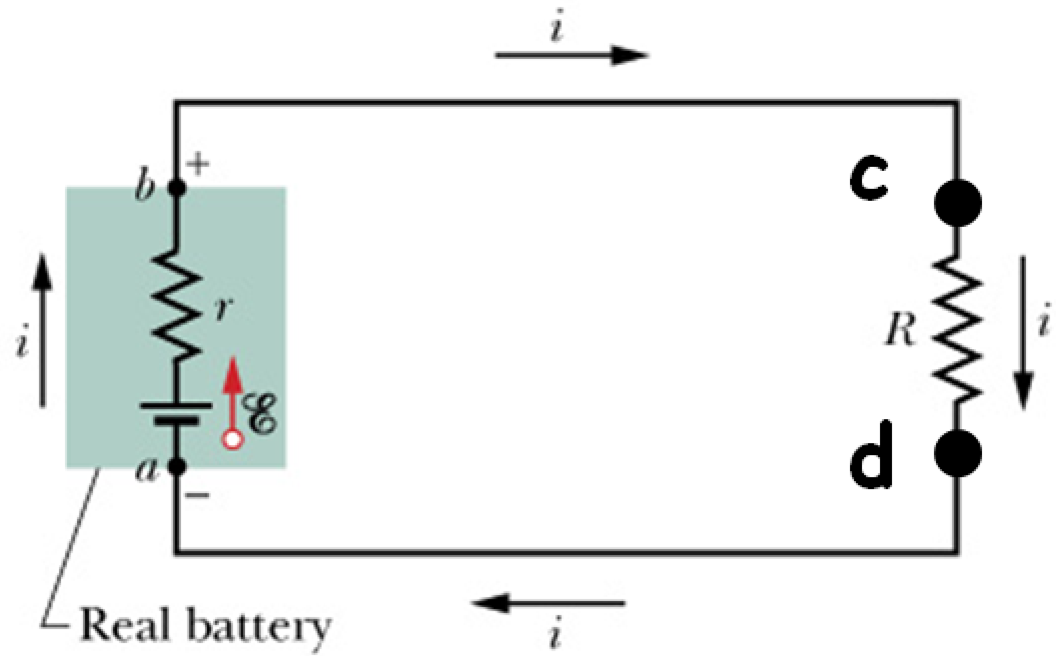
In this section, we study resistances in series and in parallel using Kirchhoff's rules. First, we apply the rules for a single-loop circuit

3.7.1. A single-loop circuit:

Internal Resistance:

- A real battery has internal resistance r

Using the loop rule clockwise beginning at point a:



$$V_b - V_a + V_d - V_c = 0$$

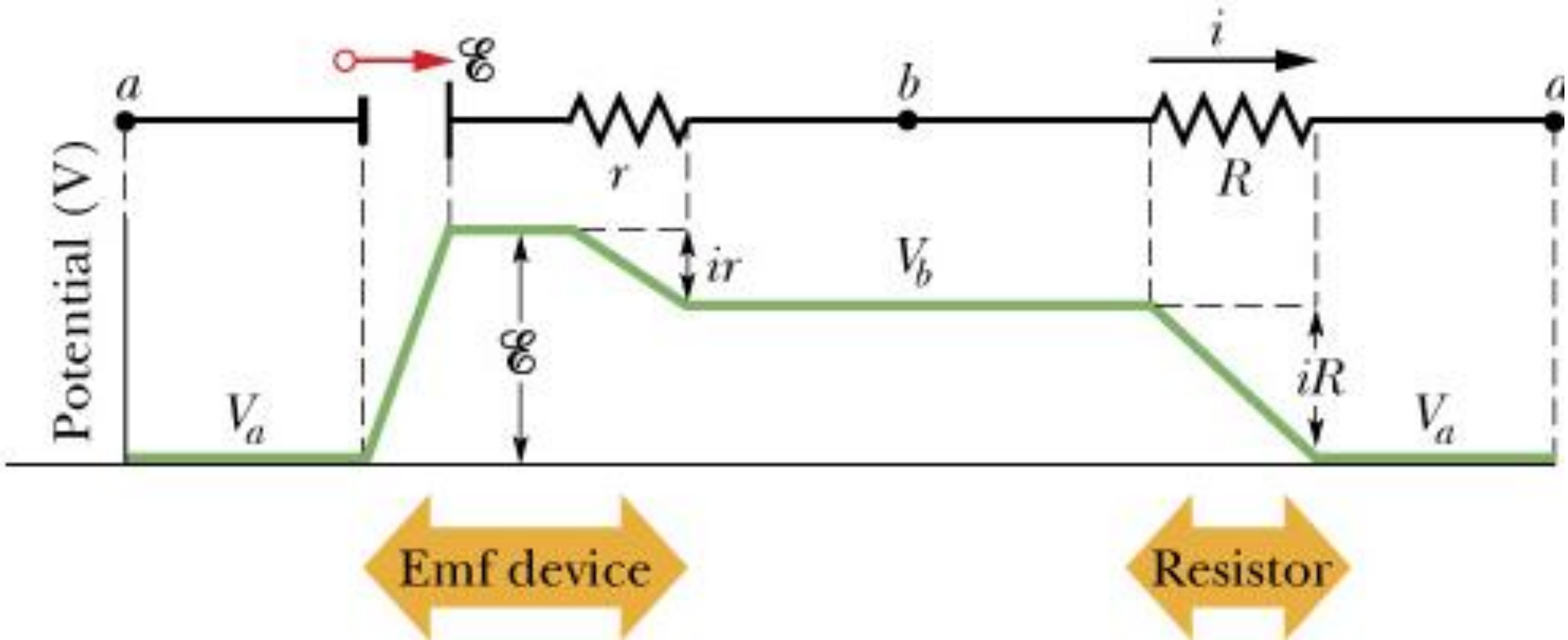
As the battery has a resistance r: $V_b - V_a = \varepsilon - ir$

$$(\varepsilon - ir) + (-iR) = 0$$

So, we can calculate the current i:

$$i = \frac{\varepsilon}{r + R}$$

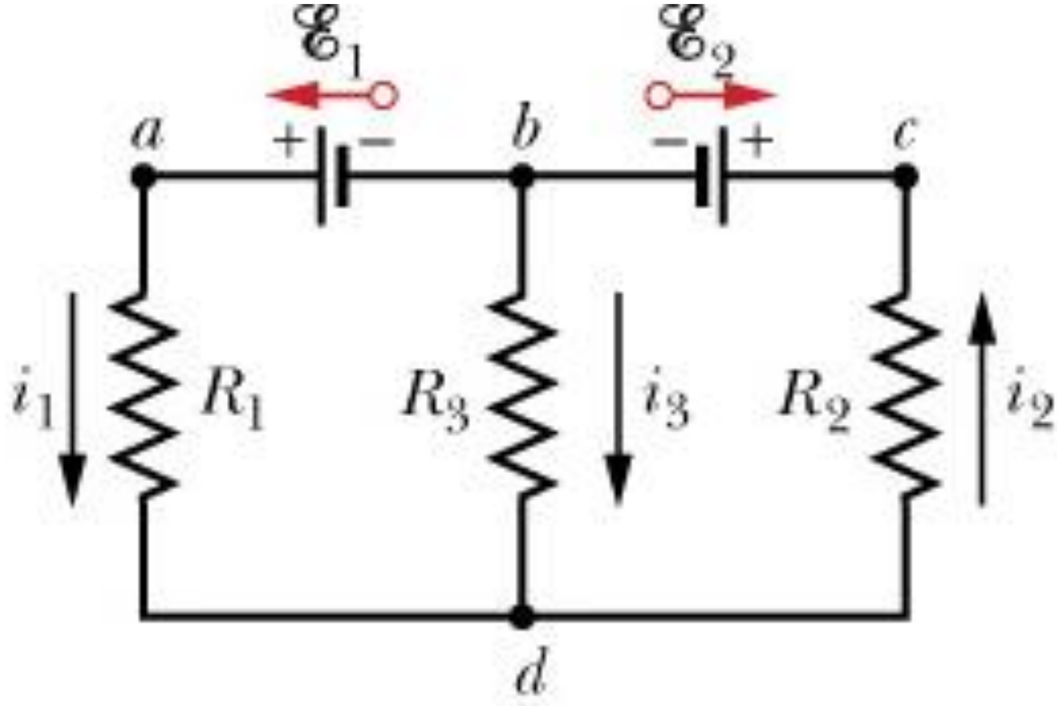
The changes in electric potential around the circuit



Note: in general, if a battery is not described as real or if no internal resistance is indicated, you can assume that it is ideal

3.7.2. Multiloop circuits:

Example: The figure shows a multiloop circuit consisting of three branches



Junction rule for point b:

$$i_1 + i_3 = i_2 \quad (1)$$

Applying the loop rule for left-hand loop badb in a counter-clockwise direction:

$$\varepsilon_1 - i_1 R_1 + i_3 R_3 = 0 \quad (2)$$

For right-hand loop bdcb: $-i_3 R_3 - i_2 R_2 - \varepsilon_2 = 0 \quad (3)$

For big loop badcb: $\varepsilon_1 - i_1 R_1 - i_2 R_2 - \varepsilon_2 = 0 \quad (4)$

Note: Equation (4) can be derived from the sum of (2) and (3), so we can use only two of the three equations above for such a multiloop circuit

3.7.3. Resistors in Series:

Problem: Calculate the resistance of resistors connected in series

We apply the Kirchhoff's rules in the clockwise direction:

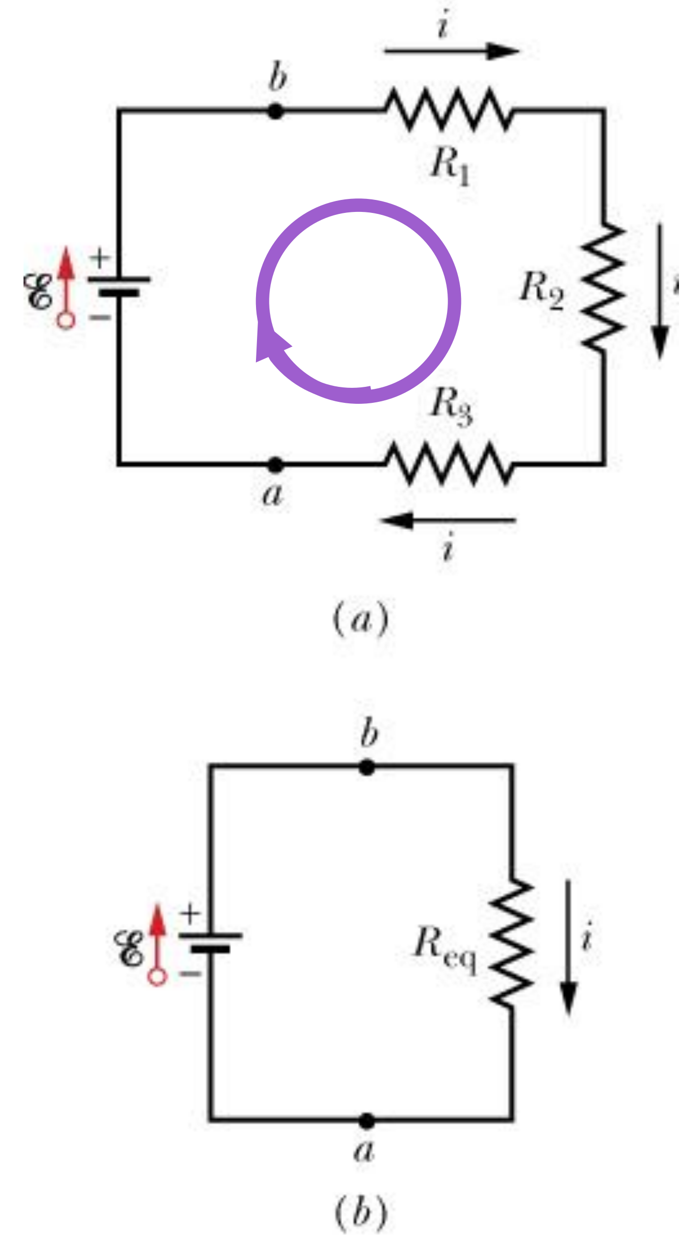
- **Junction Rule**: When a potential difference V is applied across resistances connected in series, the resistances have identical currents i:

$$i = i_1 = i_2 = i_3$$

- **Loop Rule**: The sum of the potential differences across resistances is equal to the applied potential difference V:

$$\varepsilon - iR_1 - iR_2 - iR_3 = 0$$

$$i = \frac{\varepsilon}{R_1 + R_2 + R_3}$$



(a) (b)

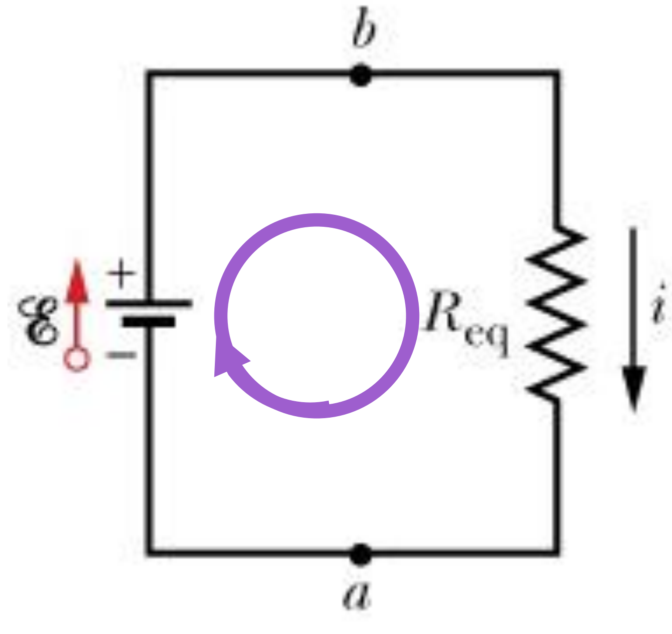
- If we replace three resistors by an equivalent resistor, so its resistance is:

$$\varepsilon - iR_{eq} = 0$$

$$R_{eq} = R_1 + R_2 + R_3$$

- For n resistors connected in series:

$$R_{eq} = \sum_{j=1}^{n} R_j$$



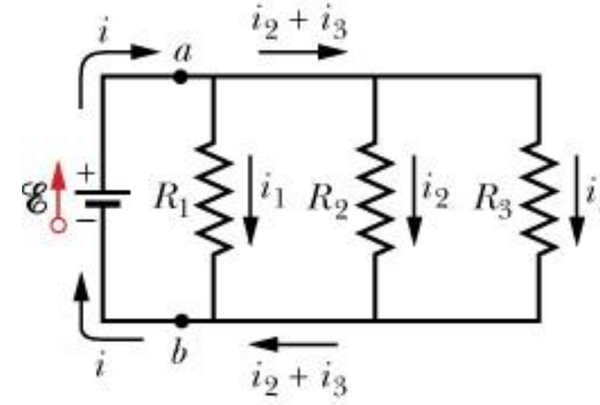
(b)

3.7.4. Resistors in Parallel:

Problem: Calculate the resistance of resistors connected in parallel

For resistors in parallel:

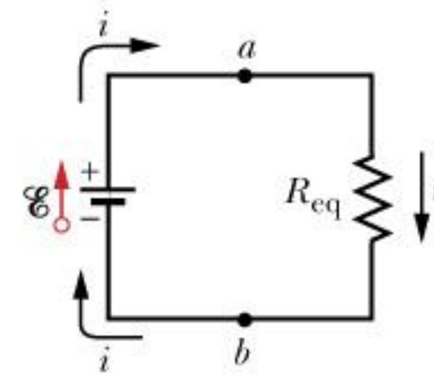
When a potential difference V is applied across resistances connected in parallel, the resistances all have that same potential difference V.



(a)

- **Junction Rule**: $$i = i_1 + i_2 + i_3$$

$$i = V\left(\frac{1}{R_1} + \frac{1}{R2} + \frac{1}{R3}\right)$$



(b)

- **Loop Rule**: If we replace three resistors by an equivalent resistor:

$$\varepsilon - iR_{eq} = 0$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R2} + \frac{1}{R3}$$

if n resistors in parallel:

$$\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}$$

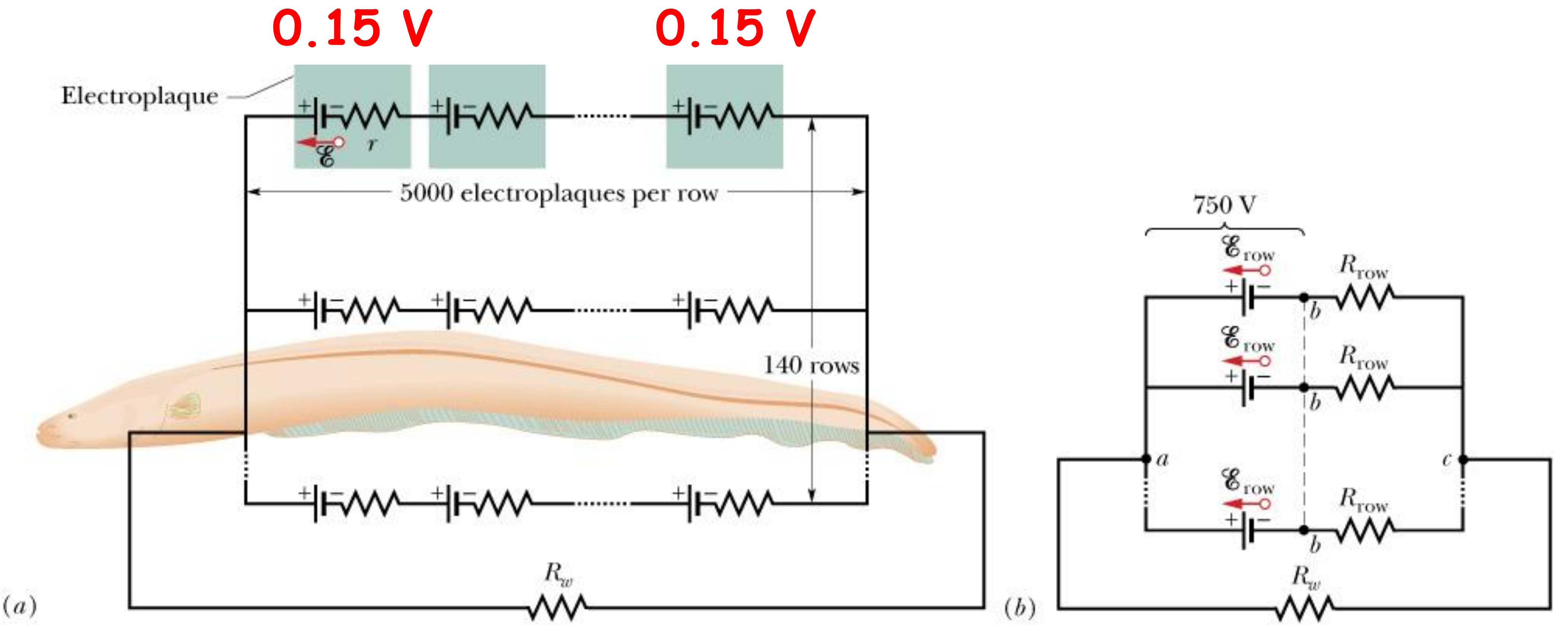
Series and Parallel Resistors and Capacitors

Series	Parallel	Series	Parallel
Resistors		**Capacitors**	
$R_{eq} = \sum_{j=1}^{n} R_j$	$\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}$	$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$	$C_{eq} = \sum_{j=1}^{n} C_j$
Same current through all resistors	Same potential difference across all resistors	Same charge on all capacitors	Same potential difference across all capacitors

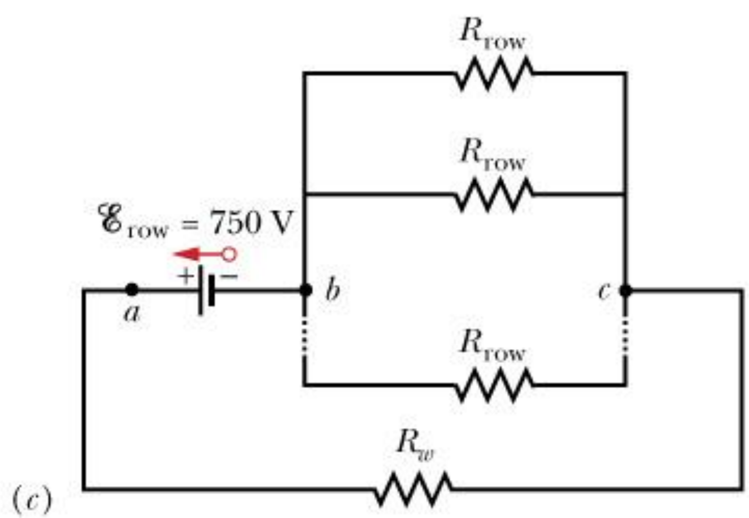
Sample Problem (p.718): Electric fish are able to generate current with biological cells called *electroplaques*, which are physiological emf devices. The electroplaques in the type of electric fish known as a South American eel are arranged in 140 rows, each row stretching horizontally along the body and each containing 5000 electroplaques. The arrangement is suggested in Figure a; each electroplaque has an emf ε of 0.15 V and an internal resistance r of 0.25 Ω. The water surrounding the eel completes a circuit between the two ends of the electroplaque array, one end at the animal's head and the other near its tail.

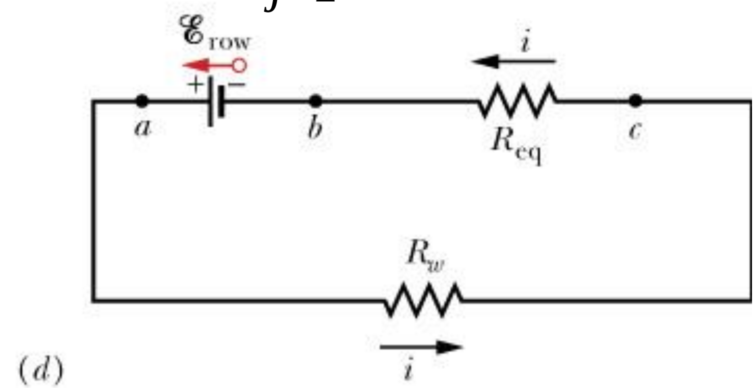
(a) If the water surrounding the eel has resistance R_w = 800 Ω, how much current can the eel produce in the water?

To solve this problem, we can simply the circuit of Figure a by replacing combinations of emfs and internal resistances with equivalent emfs and resistances as shown in the following figures b, c, and d:

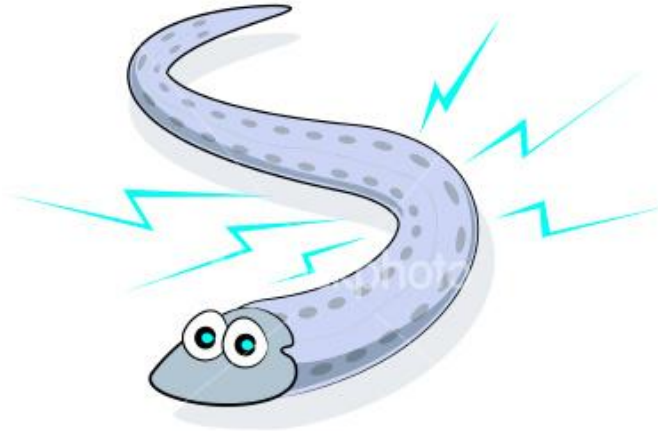


$$\frac{1}{R_{eq}} = \sum_{j=1}^{140} \frac{1}{R_j} = 140\frac{1}{R_{row}}$$





The equivalent resistance:



$$R_{eq} = \frac{R_{row}}{140} = 8.93(\Omega)$$

Using the loop rule:

$$\varepsilon_{row} - iR_w - iR_{eq} = 0$$

$$i = \frac{\varepsilon_{row}}{R_w + R_{eq}} = \frac{750}{800 + 8.93} = 0.93(A)$$

Therefore, if the head or tail of the eel is near a fish, some of this current could pass along a narrow path through the fish, stunning or killing the fish

(b) How much current i_{row} travels through each row of Figure a?

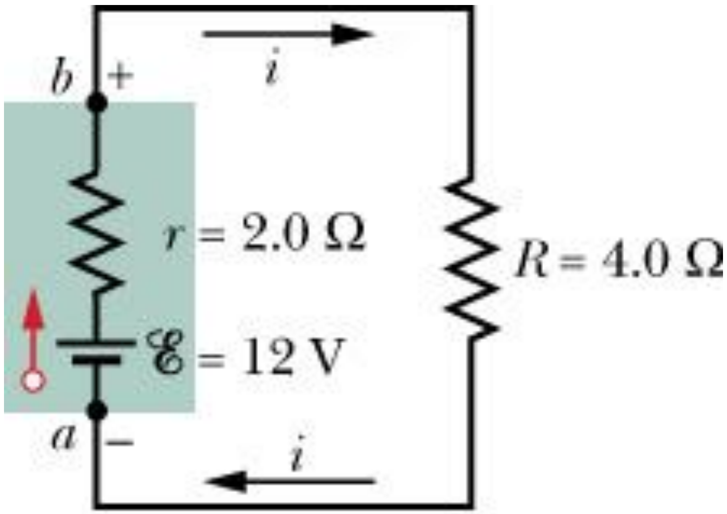
$$i_{row} = \frac{i}{140} = \frac{0.927}{140} = 6.6 \times 10^{-3}(A)$$

3.7.5. Calculating Potential Difference Between Two Points:

Usually we need to calculate potential difference between two points in a circuit. This section will show you how to do this in some common cases and other issues related to potential difference

- Calculate $V_b - V_a$ in the figure:

We start at point a with potential V_a, when we pass through the battery's emf, the potential increases by ε, when we pass through the battery's internal resistance r the potential decreases by ir. We are then at point b with potential V_b:



$$V_a + \varepsilon - ir = V_b$$

So:

$$V_b - V_a = \varepsilon - ir; \quad i = \frac{\varepsilon}{r+R}$$

$$V_b - V_a = \frac{\varepsilon}{r+R} R = \frac{12}{2+4} 4 = 8(V)$$

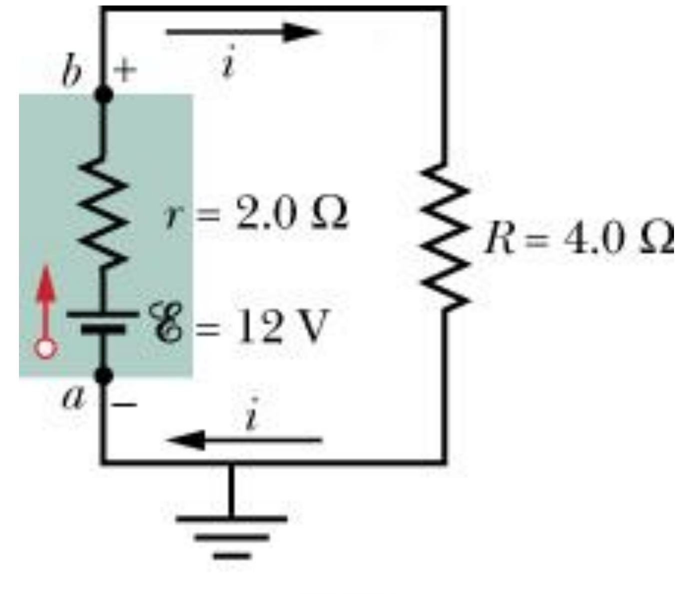
V_b-V_a is the terminal-to-terminal potential difference V:

$$V_b - V_a = \varepsilon - ir$$

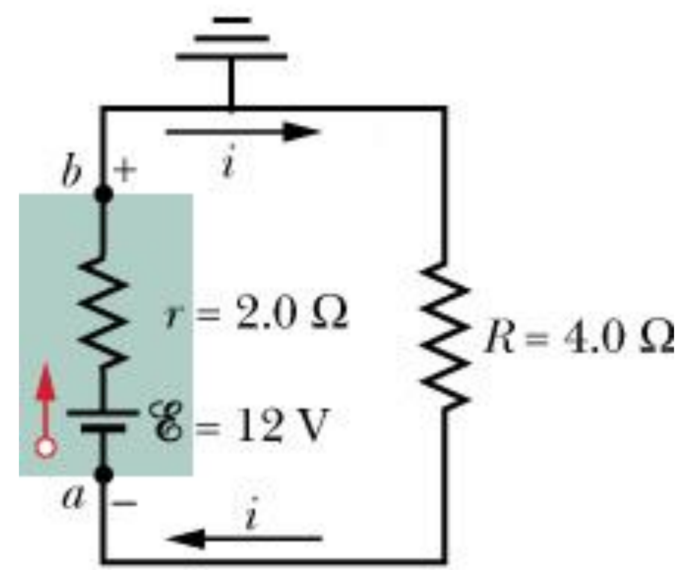
So, for a real battery V is less ε

For an ideal battery: $V = \varepsilon$

Grounding a circuit: The figures show point a or b directly connected to ground (symbol: ). In this case, the potential is defined to be zero at the grounding point in the circuit



(a)



(b)

So, we have:

- In Figure a, V_b-V_a= 8 V, V_a = 0 → V_b = 8 V
- In Figure b, V_b = 0 → V_a = -8 V

The relationship between Power and Potential:

We will calculate work done by an emf device (e.g., a battery) on the charges to establish a current i; the dissipation rate of energy due to the internal resistance r of the emf device and the power of the emf device

- **The net rate P of energy transfer from the emf device to the charge carriers:**

$$P = iV$$

- **We also have:**

$$V = \varepsilon - ir$$

$$P = i(\varepsilon - ir) = i\varepsilon - i^2 r$$

- **The term i^2r is the rate of energy transfer to thermal energy within the emf device:**

$$P_r = i^2 r$$

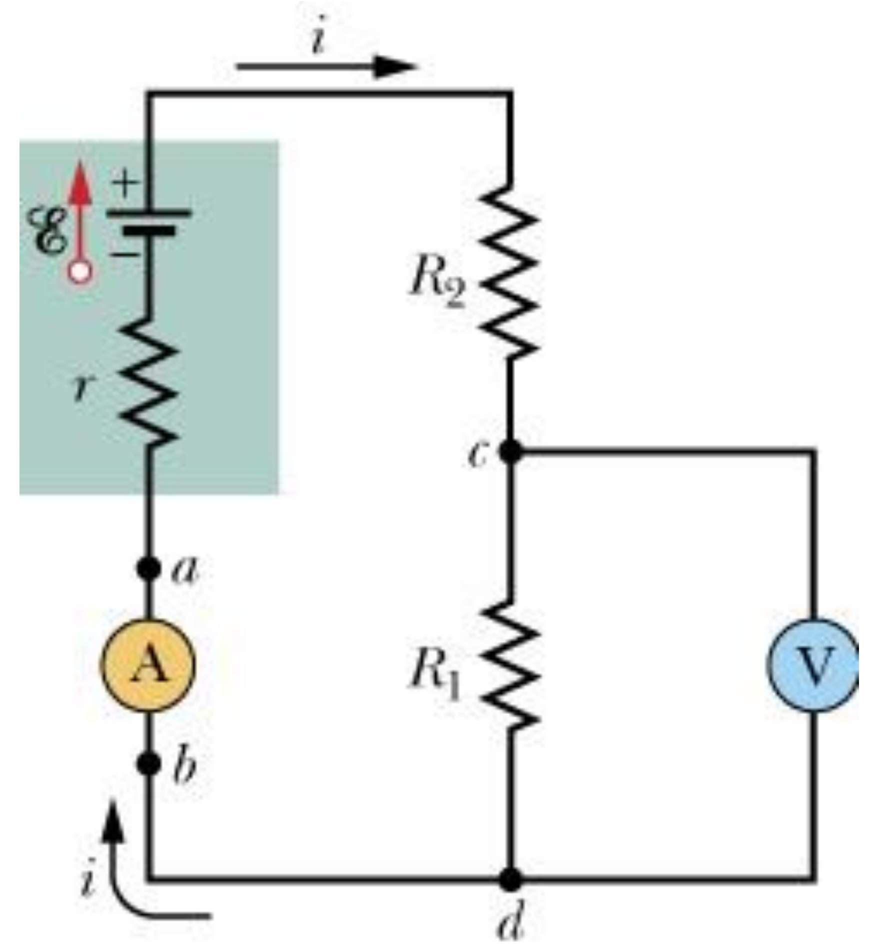
- **The rate of energy transfer from the emf device both to the charges and to the thermal energy:**

$$P_{emf} = i\varepsilon$$

3.7.6. The Ammeter and the Voltmeter:

- An instrument that is used to measure currents is called an ammeter
- A meter to measure potential difference is called a voltmeter

- The figure shows how to set up an ammeter and a voltmeter in a circuit

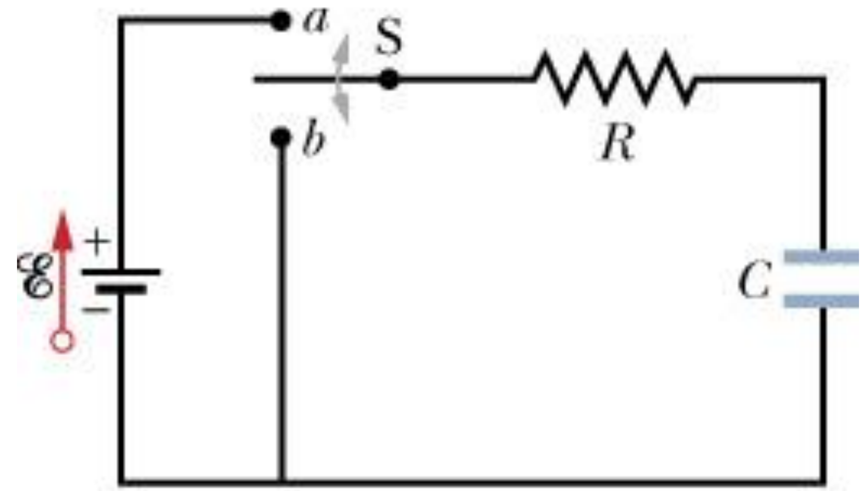


- A meter can measure currents (ammeter), potential difference (voltmeter) and resistance (ohmmeter), by means of a switch, is called a multimeter

3.8. RC Circuits:

In this section we begin to study time-varying currents

3.8.1. Charging a Capacitor:



- **An RC series circuit:** a circuit consists a capacitor C, an ideal battery and a resistor R
- When S is closed on a, C is charged: the charge begins to flow between the capacitor plates and the battery terminals, establishing a current i.

When $V_C = V_{battery} = \varepsilon$, the current is zero

- Now, we examine the charging process:
  - charge q(t), potential difference V_C(t), and current i(t).

  Using the loop rule:

$$\varepsilon - iR - V_C = 0 \Rightarrow \varepsilon - iR - \frac{q}{C} = 0$$

$$i = \frac{dq}{dt} \Rightarrow \boxed{R\frac{dq}{dt} + \frac{q}{C} = \varepsilon \text{ (charging equation)}}$$

The solution for the differential equation above is:

$$q = C\varepsilon(1 - e^{-t/RC})$$

(see text for solving the equation)

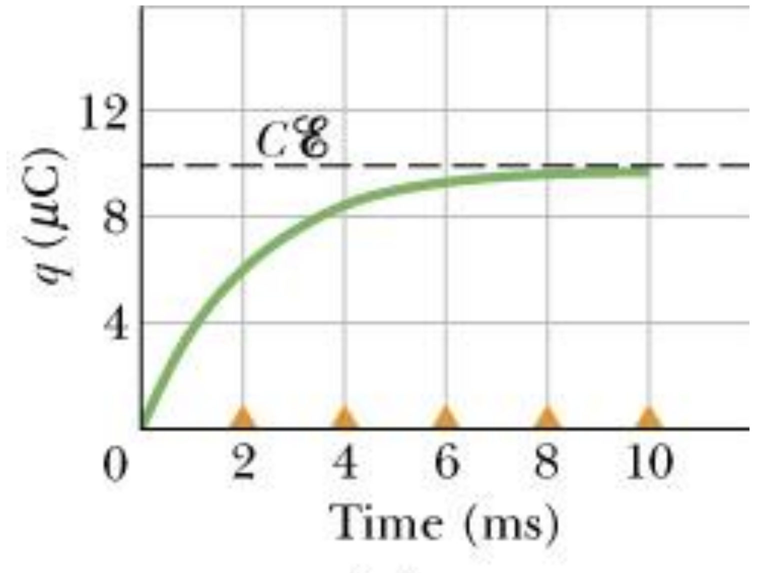
So, the current i(t):

$$i = \frac{dq}{dt} = \left(\frac{\varepsilon}{R}\right)e^{-t/RC}$$

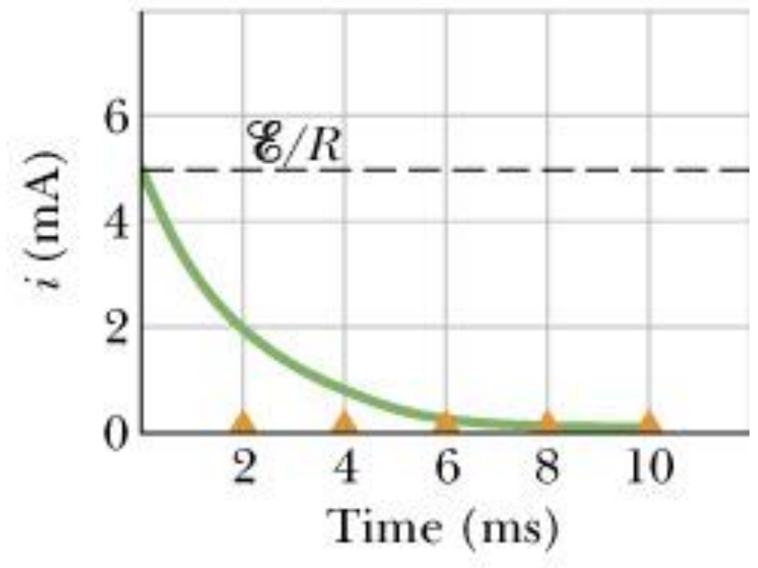
And the potential difference V(t) across the capacitor:

$$V_C = \frac{q}{C} = \varepsilon(1 - e^{-t/RC})$$

Note: When the capacitor becomes fully charged as $t \rightarrow \infty$: $q = C\varepsilon$, $i = 0$, and $V_C = \varepsilon$



(a)



(b)

The time constant:

$$\tau = RC$$

(Unit: s)

At $t = \tau = RC$:

$$q = C\varepsilon(1 - e^{-1}) = 0.63C\varepsilon$$

During the first time τ, the charge increases from zero to 63% of its final value $C\varepsilon$

The greater τ is the greater the charging time

3.8.2. Discharging a Capacitor:

After being fully charged, the capacitor has a potential $V_0 = \varepsilon$, we switch S from a to b, so the capacitor discharges through resistor R. Using the loop rule:

$$R\frac{dq}{dt} + \frac{q}{C} = 0 \text{ (discharging equation)}$$

The solution for this equation is:

$$q = q_0 e^{-t/RC}$$

Where:

$$q_0 = CV_0$$

At **t = τ = RC**, the charge is reduced to $q_0 e^{-1}$ or about 37% of the initial value

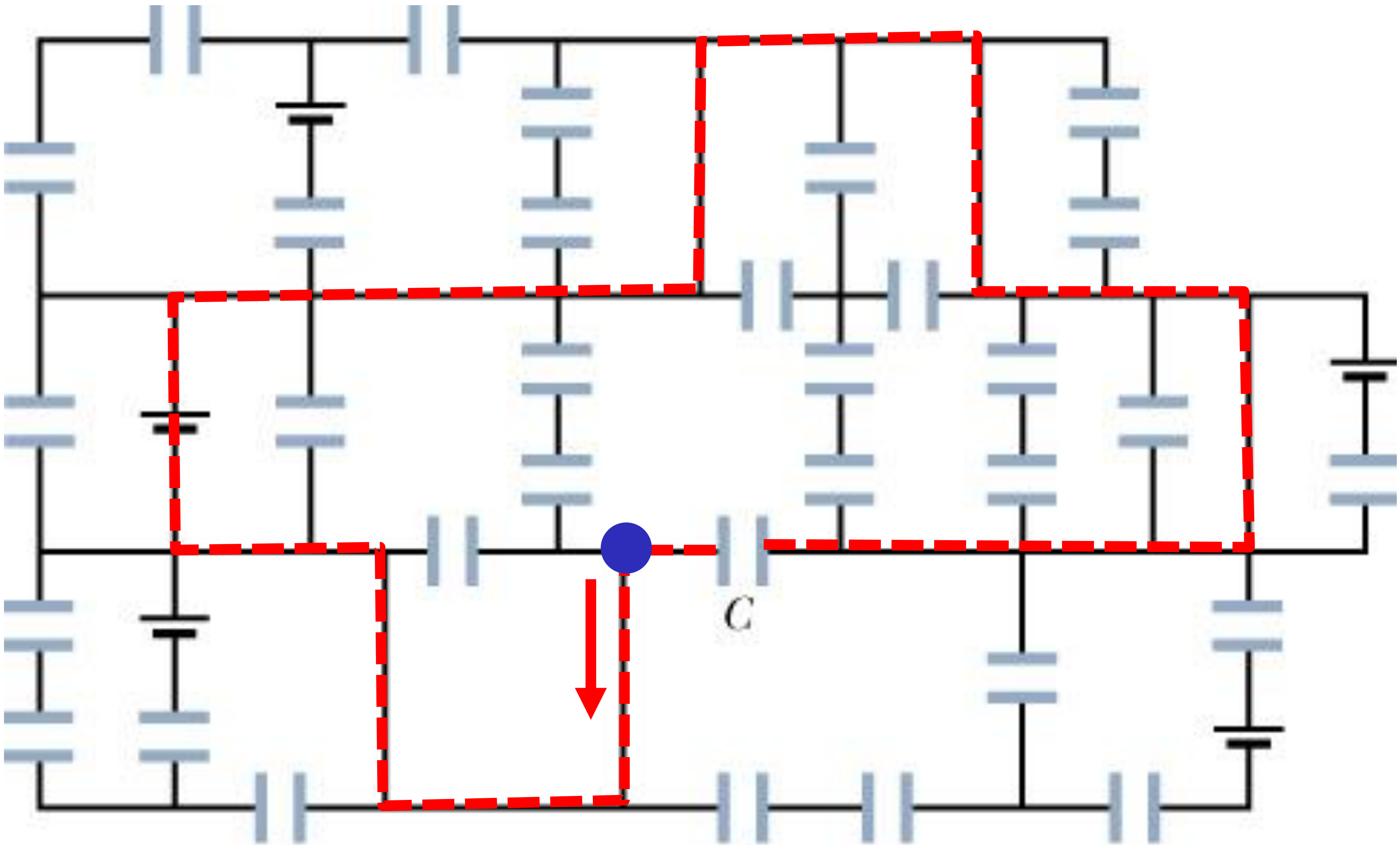
The current i(t):

$$i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right) e^{-t/RC}$$

The potential difference V_C(t):

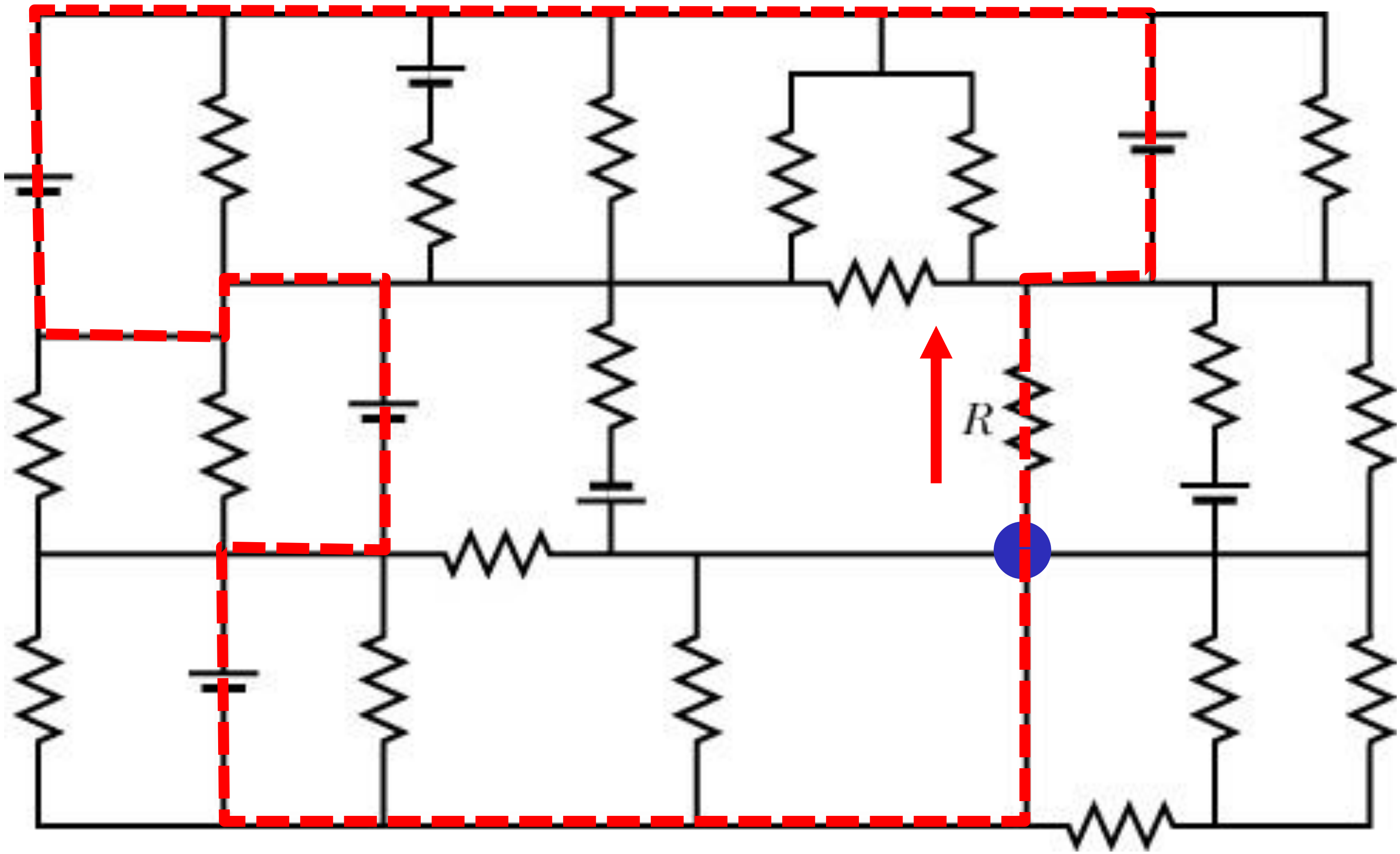
$$V_C = V_0 e^{-t/RC}$$

Checkpoint: (Cap-monster maze) All the capacitors have a capacitance of 6 μF, and all the batteries have an emf of 10 V. What is the charge on capacitor *C*?



$$\varepsilon - \frac{q}{C} = 0 \Rightarrow q = C\varepsilon = 6\times10^{-6}\times10 = 60\times10^{-6}(C) = 60(\mu C)$$

Checkpoint: (Res-monster maze) **All the resistors have a resistance of 4 Ω, and all the batteries have an emf of 4 V. What is the current through resistor R?**



$$-iR+\varepsilon-\varepsilon-\varepsilon-\varepsilon=0 \Rightarrow i=-2\varepsilon / R=-2(A)$$

Homework: 2, 5, 7, 10, 17, 22, 24, 30, 34, 44, 45, 54, 57, 60, 65 (pages 726-731)