

Chapter 3 The Kinetic Theory of Gases

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Overview

In this chapter, we consider the physics of gases at the microscopic level:

- A gas consists of atoms that fill their container's volume and the volume is a result of the freedom of the atoms spread throughout the container.
- The temperature is a measure of the kinetic energy of the atoms.
- The pressure exerted by a gas is produced by the collisions of the atoms with the container's wall.

The kinetic theory of gases relates the motion of the atoms to the volume, pressure, and temperature of the gas.

Ideal gases

- A truly ideal gas does not exist in nature.
- But all real gases approach the ideal state at low enough densities:
 - + Molecules are far enough apart, so they do not interact with one another.
 - + An ideal gas obeys the ideal gas law (see next slides).

3.1. Ideal Gases

Avogadro's Number

Avogadro's law

Equal volumes of gases, at the same temperature and pressure, contain the same number of molecules.

- Useful unit to measure the sizes of our samples is mole.
- One mole is the number of atoms in a 12 g sample of carbon-12

In chapter 2, section 2.3, one mole contains 6.02×10^{23} elementary units.

$$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$$

N_A : Avogadro's number

- Number of moles contained in a sample of any substance:

$$n = \frac{N}{N_A}$$

where N is the number of molecules in the sample

$$n = \frac{M_{\text{sample}}}{M} = \frac{M_{\text{sample}}}{mN_A}$$

where M_{sample} is the mass of the sample

M is the *molar mass* (the mass of 1 mol)

m is the mass of one molecule

Problem 1 (p. 530)

Find the mass in kilograms of 7.5×10^{24} atoms of arsenic (As), which has molar mass of 74.9 g/mol.

The number of moles n : $n = \frac{M_{\text{arsenic}}}{M}$

$$M_{\text{arsenic}} = nM = \frac{N}{N_A} M = \frac{7.5 \times 10^{24}}{6.02 \times 10^{23}} \times 74.9 = 933 \text{ (g)}$$

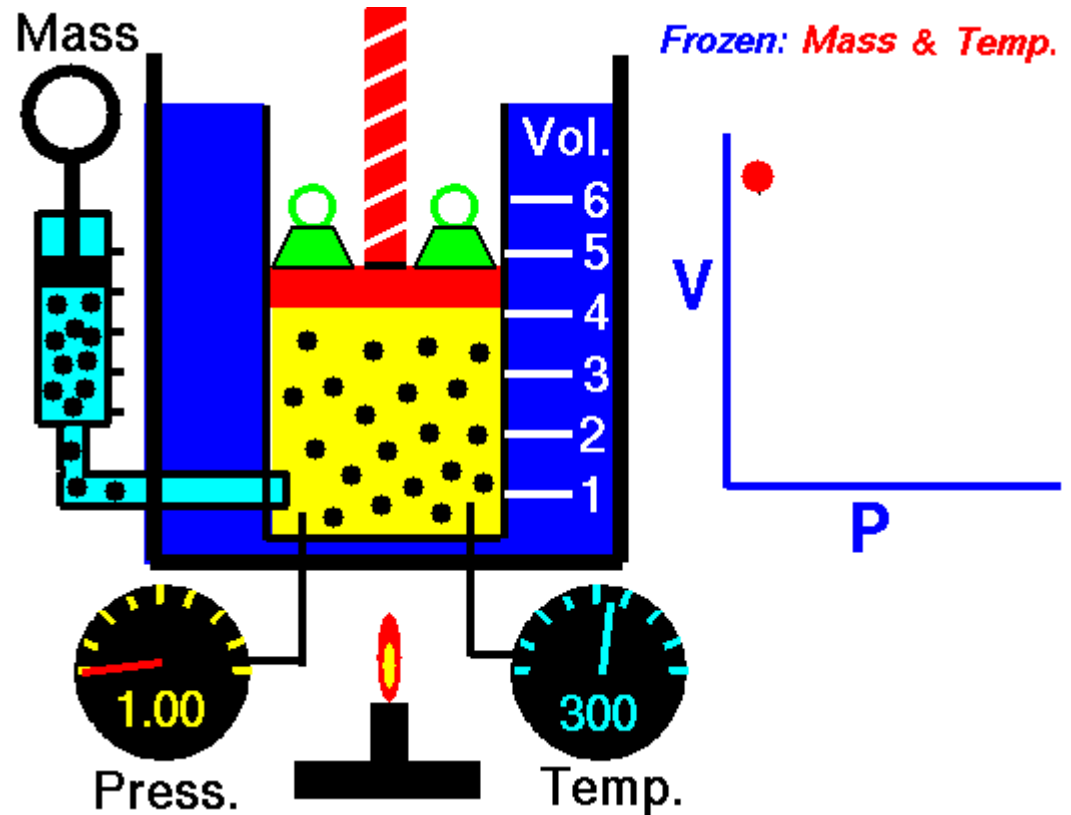
3.1.1. Experimental Laws and the Equation of State

Boyle's Law (Boyle-Mariotte): For a given mass, at constant temperature (isothermal), the pressure times the volume is a constant for an ideal gas.

$$pV = \text{constant}$$



Robert Boyle (1627-1691)



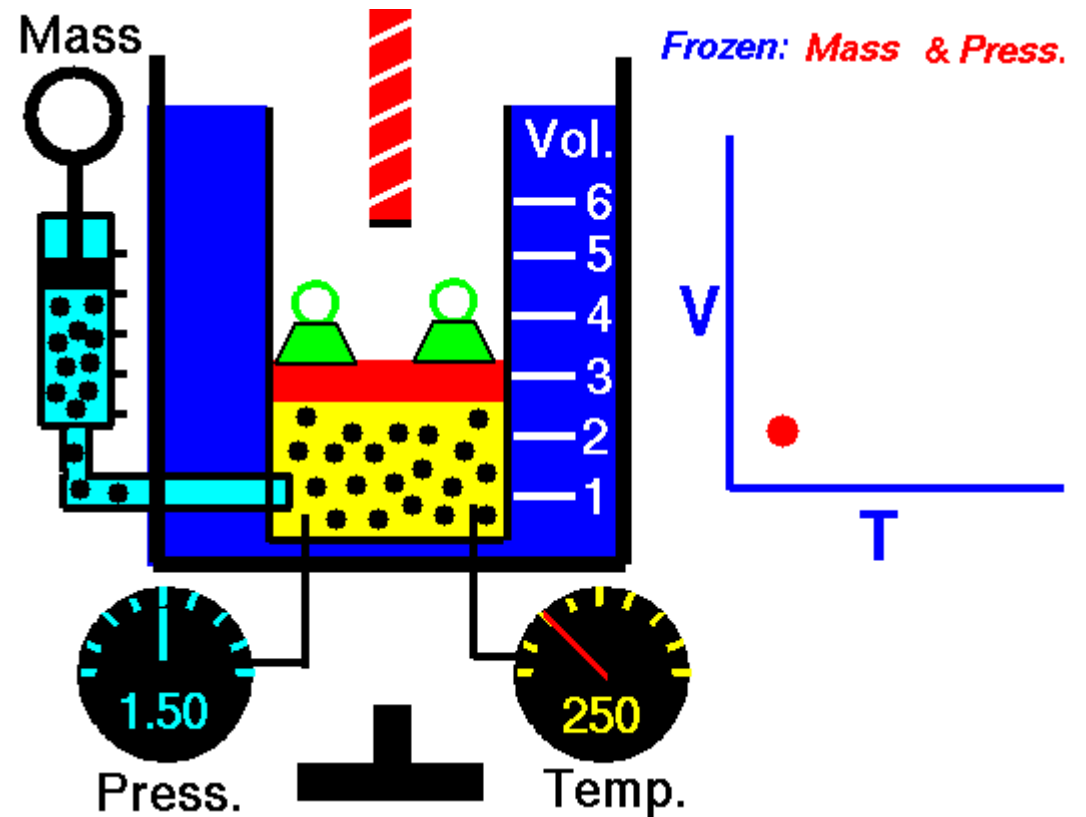
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Charles's Law: For a given mass, at constant pressure (isobaric), the volume is directly proportional to the temperature.

$$V = \text{constant} \times T$$



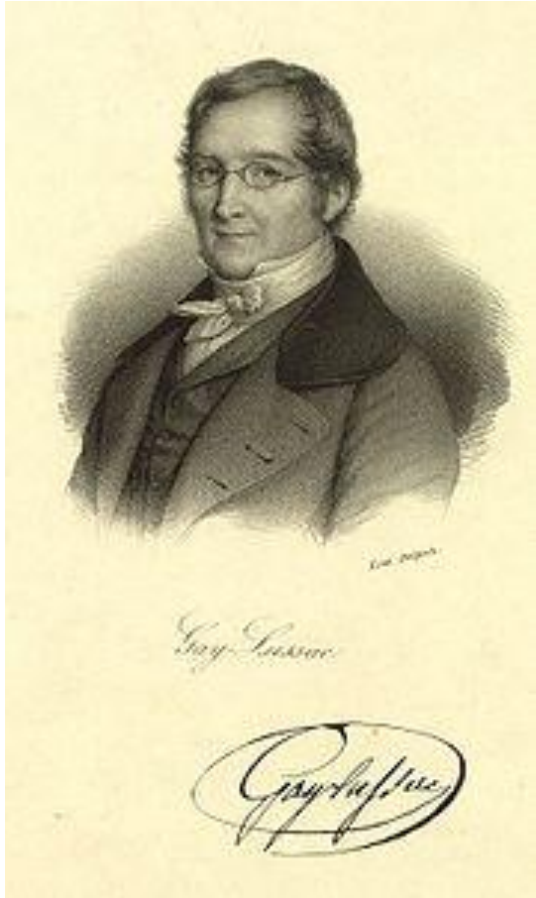
Jacques Charles (1746-1823)



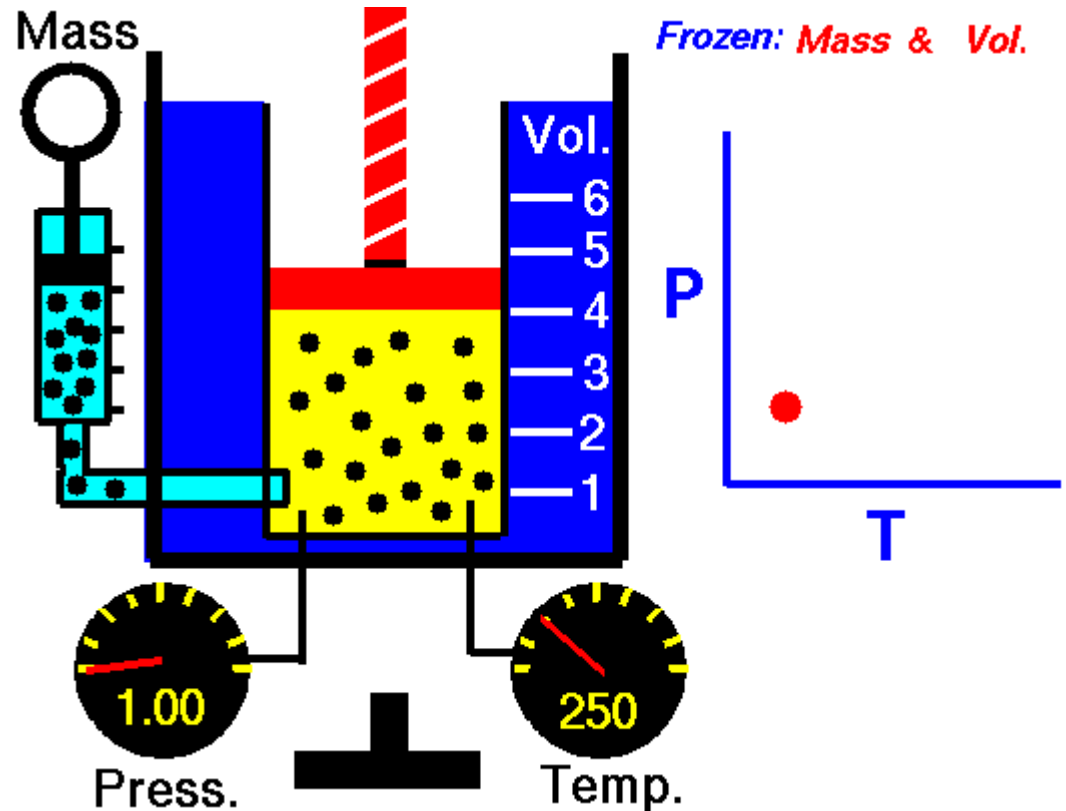
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Gay-Lussac's Law: For a given mass, at constant volume (isochoric), the pressure is directly proportional to the temperature.

$$p = \text{constant} \times T$$



J. L. Gay-Lussac (1778-1850)



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Equation of State:

Boyle's Law: $pV = \text{constant}$

Charles's Law: $V = \text{constant} \times T$

Gay-Lussac's Law: $p = \text{constant} \times T$

The gas laws of Boyle, Charles and Gay-Lussac can be combined into a single equation of state:

$$pV = nRT \text{ (ideal gas law)}$$

where p is the absolute pressure

n is the number of moles of gas

T is the temperature (in K)

R is a constant, called the gas constant

$$R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$$

The Boltzmann constant k is:

$$k = \frac{R}{N_A} = \frac{8.31 \text{ J mol}^{-1} \text{ K}^{-1}}{6.02 \times 10^{23} \text{ mol}^{-1}} = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

$$nR = \frac{N}{N_A} k N_A = kN \quad (\mathbf{N \text{ is the number of molecules}})$$

$$\mathbf{pV = NkT \quad (\text{ideal gas law})}$$

Sample Problem (p. 510)

A cylinder contains 12 L of oxygen at 20°C and 15 atm. The temperature is raised to 35°C, and the volume is reduced to 8.5 L. What is the final pressure of the gas in atmospheres? Assume that the gas is ideal.

Key equation:

$$pV = nRT$$

At state i:
$$p_i V_i = nRT_i$$

At the final state f:
$$p_f V_f = nRT_f$$

$$\Rightarrow \frac{p_f V_f}{p_i V_i} = \frac{T_f}{T_i} \Rightarrow p_f = \frac{p_i V_i T_f}{T_i V_f}$$

We must convert temperatures in °C to that in K:

$$T_i = (273 + 20) \text{ K} = 293 \text{ K}; \quad T_f = (273 + 35) \text{ K} = 308 \text{ K}$$

$$p_f = \frac{(15 \text{ atm})(308 \text{ K})(12 \text{ L})}{(293 \text{ K})(8.5 \text{ L})} \approx 22.3 \text{ atm}$$

Problem 4 (p. 530)

A quantity of ideal gas at 10°C and 100 kPa occupies a volume of 3.0 m^3 . (a) How many moles of the gas are present? (b) If the pressure is now raised to 300 kPa and the temperature is raised to 30°C , how much volume does the gas occupy? Assume no leaks.

$$pV = nRT; \quad R = 8.31\text{ J mol}^{-1}\text{K}^{-1}$$

$$(a) \quad n = \frac{pV}{RT}$$

$$p = 100\text{ kPa} = 10^5\text{ Pa}; \quad V = 3.0\text{ m}^3; \quad T = 273 + 10 = 283\text{ K}$$
$$\Rightarrow n = \frac{10^5 \times 3.0}{8.31 \times 283} \approx 127.6\text{ (moles)}$$

$$(b) \text{ At any state } i \text{ (} p, V, T \text{):} \quad p_i V_i = nRT_i$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} = nR \Rightarrow V_2 = \frac{p_1 V_1 T_2}{p_2 T_1}$$

$$T_2 = 273 + 30 = 303\text{ K}: \quad V_2 = \frac{p_1 V_1 T_2}{p_2 T_1} = \frac{100 \times 303}{300 \times 283} \times 3.0 \approx 1.1\text{ (m}^3\text{)}$$

Problem 5 (p. 530)

The best laboratory vacuum has a pressure of about 10^{-18} atm, or 1.01×10^{-13} Pa. How many gas molecules are there per cubic centimeter in such a vacuum at 293 K?

$$pV = nRT$$

The number of moles: $n = \frac{pV}{RT}$; $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$

$$p = 1.01 \times 10^{-13} \text{ Pa}; V = 1 \text{ cm}^3 = 10^{-6} \text{ m}^3; T = 293 \text{ K}$$

The number of molecules: $N = n \times N_A$

$$\Rightarrow N = \frac{1.01 \times 10^{-13} \times 10^{-6}}{8.31 \times 293} \times 6.02 \times 10^{23} \approx 25 \text{ (molecules)}$$

Review

Chapter 1:

density:

$$\rho = \frac{m}{V}$$

pressure:

$$p = \frac{F}{A}$$

fluids at rest:

$$p = p_0 + \rho gh$$

absolute pressure

atmosphere pressure

gauge pressure

Pascal's law:

$$\frac{F_1}{A_1} = \frac{F_0}{A_0}$$

Archimede's principal:

$$F_b = \rho_{\text{fluid}} gV : \text{buoyant force}$$

Equation of continuity: $A_1 v_1 = A_2 v_2 = \text{constant}$

Bernoulli's equation:

$$p_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2 = \text{constant}$$

Chapter 2:

$$T(\text{K}) = T(\text{C}^0) + 273.15$$

thermal expansion:

- Linear expansion: (solids) $\Delta L = L \alpha \Delta T$
- Area expansion: (solids) $\Delta A = A \alpha_A \Delta T$; $\alpha_A = 2\alpha$
- Volume expansion: (solids and liquids)

$$\Delta V = V \beta \Delta T; \beta = 3\alpha$$

Heat capacity:

$$Q = C \Delta T = C (T_f - T_i)$$

specific capacity:

$$Q = cm \Delta T = cm (T_f - T_i)$$

• Latent heat:

$$Q = Lm$$

Heat Transfer Mechanisms:

• Conduction:

$$P_{\text{cond}} = \frac{Q}{t} = kA \frac{T_H - T_C}{L} \quad (\text{Unit: } W = J/s)$$

steady-state:

$$P_{\text{cond}} = \frac{k_2 A (T_H - T_X)}{L_2} = \frac{k_1 A (T_X - T_C)}{L_1}$$

If the slab consists of n materials:

$$P_{\text{cond}} = \frac{A(T_H - T_C)}{\sum_{i=1}^n (L_i/k_i)}$$

Chapter 3:

$$\Delta E_{\text{int}} = E_{\text{int},f} - E_{\text{int},i} = Q - W$$

Three special cases:

1. **Adiabatic processes:** $Q = 0 \Rightarrow \Delta E_{\text{int}} = -W$

2. **Constant-volume (isochoric) processes:** $W = 0 \Rightarrow \Delta E_{\text{int}} = Q$

3. **Cyclical processes:** $\Delta E_{\text{int}} = 0 \Rightarrow Q = W$

• **Work done by the gas:**

-Expansion: $W > 0$

-Compression: $W < 0$

• **Energy transferred as heat Q :**

-Heat transferred to the gas
(receiving energy as heat):

$$Q > 0$$

-Heat transferred from the gas
(releasing energy as heat):

$$Q < 0$$

