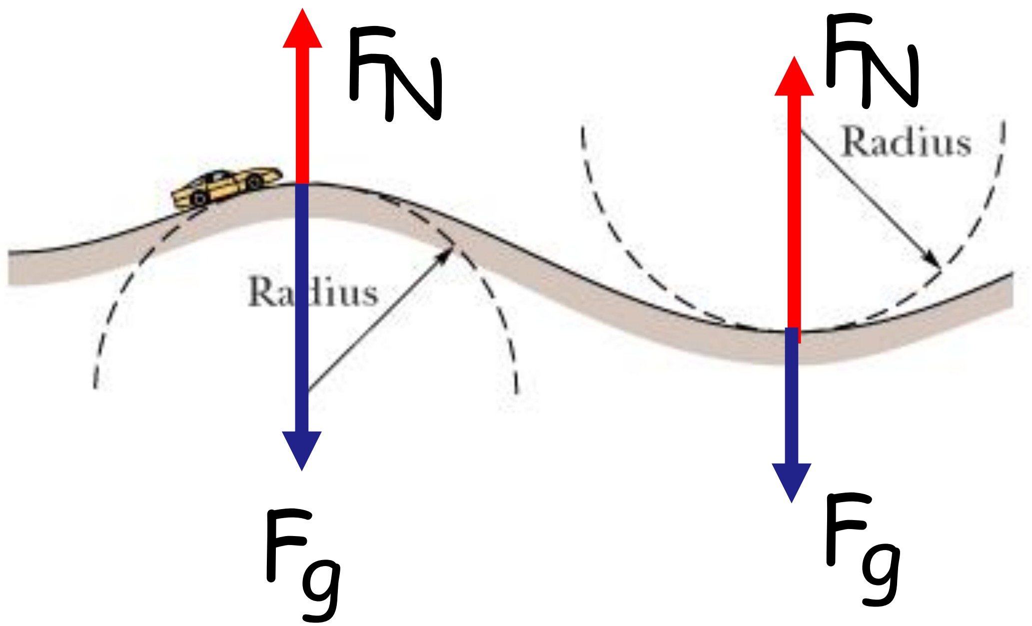
Homework: 49, 51, 70 (p. 134-137)

49. In the figure below, a car is driven at constant speed over a circular hill and then into a circular valley with the same radius. At the top of the hill, the normal force on the driver from the car seat is 0. The driver's mass is 80.0 kg. What is the magnitude of the normal on the driver from the seat when the car passes through the bottom of the valley?



At the top of the hill:

$$F_{centripetal} = F_g - F_N = m\frac{v^2}{R}$$

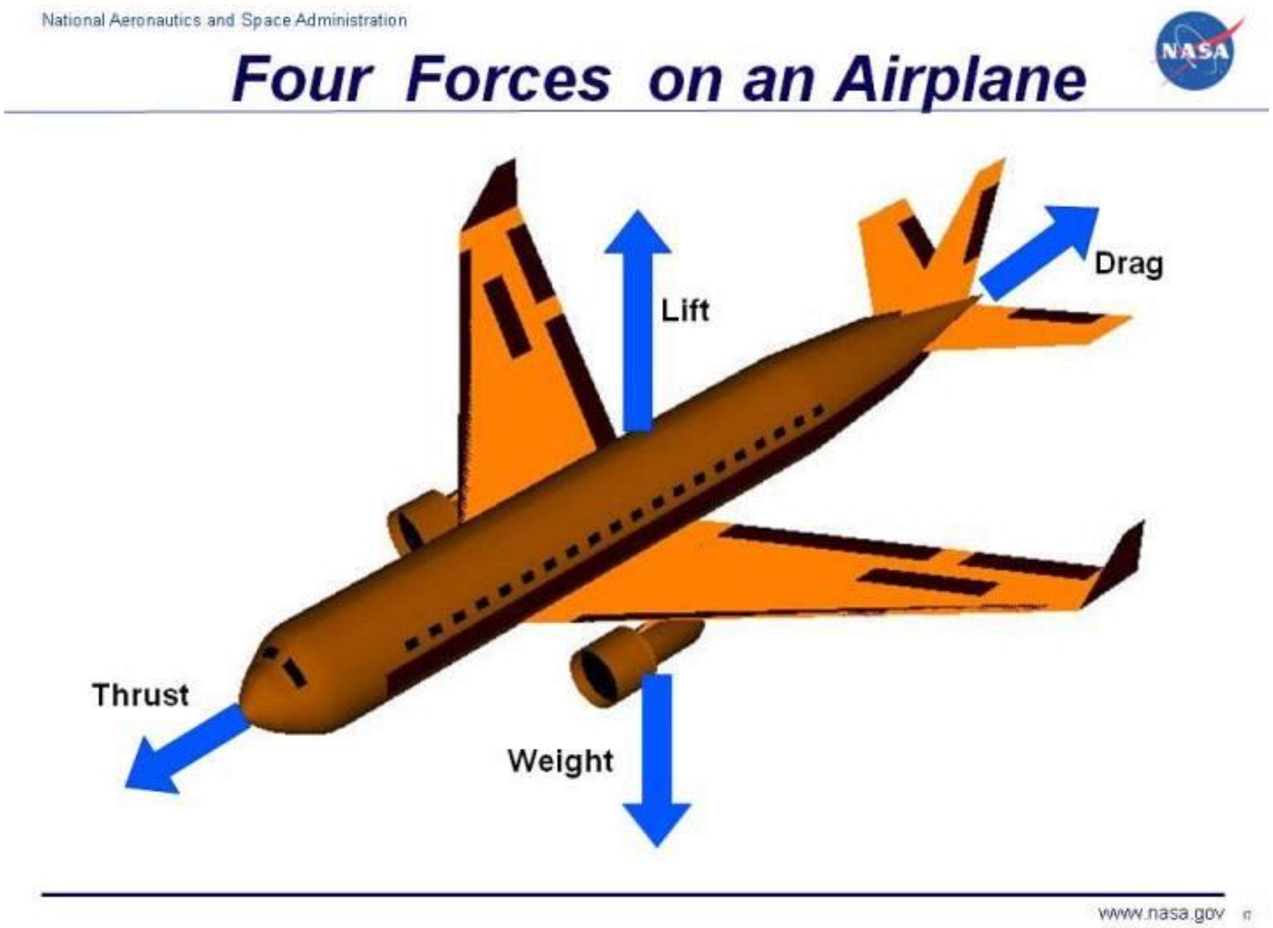
$$F_N = 0 \Rightarrow F_g = m\frac{v^2}{R}$$

At the bottom of the valley:

$$F_{centripetal} = F_N - F_g = m\frac{v^2}{R} \Rightarrow F_N = m\frac{v^2}{R} + F_g$$

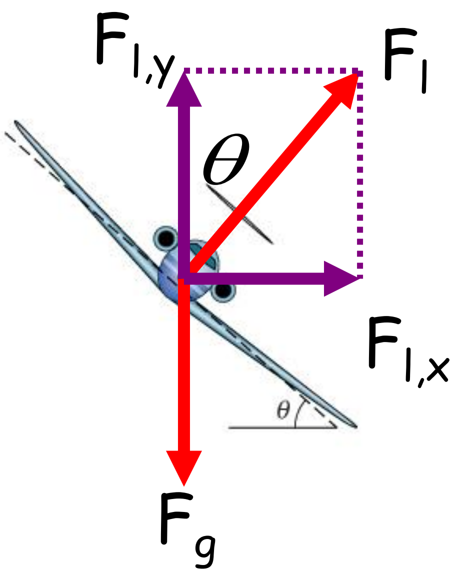
$$F_N = 2F_g = 2mg = 2 \times 80 \times 9.8 = 1568\,(\text{N})$$

51. An airplane is flying in a horizontal circle at a speed of 600 km/h. If its wings are tilted at angle $\theta=40^0$ to the horizontal, what is the radius of the circle in which the plane is flying? Assume that the required force is provided entirely by an "aerodynamic lift" that is perpendicular to the wing surface.



Animation

• According to the Bernoulli's principle, the aerodynamic lift appears due to the air-stream velocity over the top of the airplane greater than that at the bottom.



$$F_{l,y} = F_g$$

$$F_l \cos\theta = mg \quad (1)$$

$F_{l,x}$ is the centripetal force: $F_{l,x} = m\frac{v^2}{R}$

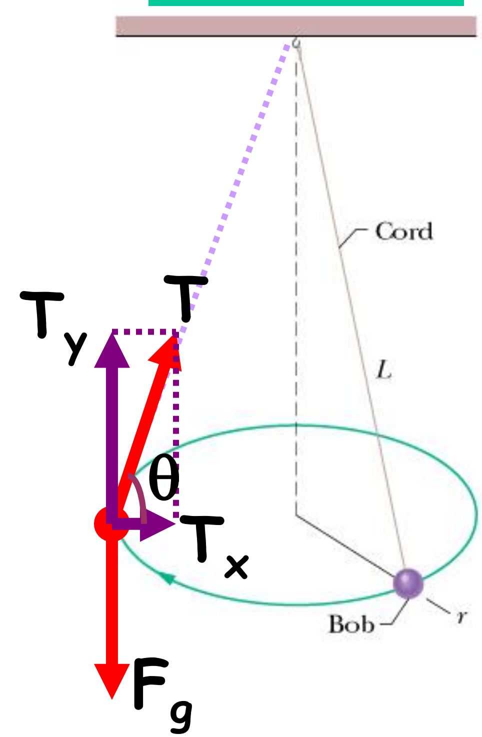
$$F_l \sin\theta = m\frac{v^2}{R} \quad (2)$$

$$v = 600 \text{ km/h} = 166.7 \text{ m/s}$$

$$(1) \text{ and } (2) \Rightarrow R = \frac{v^2}{g\tan\theta} = \frac{166.7^2}{9.8\tan(40)} \approx 3379 \text{ (m) or } 3.38 \text{ (km)}$$

70. The figure below shows a *conical pendulum*, in which the bob (the small object at the lower end of the cord) moves in a horizontal circle at constant speed. The cord sweeps out a cone as the bob rotates. The bob has a mass of 0.050 kg, the string has length L=0.90 m and negligible mass, and the bob follows a circular path of circumference 0.94 m. What are (a) the tension in the string and (b) the period of the motion?

Animation



$$T_y - F_g = 0 \Rightarrow T\sin\theta = mg$$

T_x: the centripetal force

θ: the angle between the cord and the horizontal circle.

$$T_x = m\frac{v^2}{R} \Rightarrow T\cos\theta = m\frac{v^2}{R}$$

$$\cos\theta = \frac{R}{L}; R = \frac{C}{2\pi} = \frac{0.94\,(m)}{2\times 3.14} = 0.15\,m$$

$$\theta = \arccos\left(\frac{R}{L}\right) = 80.4^0$$

(a) $T = \dfrac{mg}{\sin\theta} \approx 0.5\,(N)$ **(b)** $v = \sqrt{\dfrac{T\times R\cos\theta}{m}} = 0.5\,(m/s);\ P = \dfrac{C}{v} = 1.88\,(s)$

Part B Laws of Conservation

Chapter 3 Work and Mechanical Energy

3.1. Kinetic Energy and Work. Power

3.2. Work-Kinetic Energy Theorem

3.3. Work and Potential Energy

3.4. Conservative and Non-conservative Forces. Conservative Forces and Potential Energy

3.5. Conservation of Mechanical Energy

3.6. Work Done on a System by an External Force. Conservation of Energy

What is energy?

Energy is the capacity of a system to do work.

There are many forms of energy:

- Mechanical energy:
  - Potential energy, stored in a system
  - Kinetic energy, from the movement of matter
- Radiant energy (solar energy)
- Thermal energy
- Chemical energy (chemical bonds of molecules)
- Electrical energy (movement of electrons)
- Electromagnetic energy (from X-rays to radio waves)
- Nuclear energy

Energy can be transformed from one type to another and transferred from one object to another, but the total amount is always the same (the principle of energy conservation).

3.1. Kinetic Energy and Work. Power

3.1.1. Kinetic Energy and Work

Kinetic energy is energy associated with the state of motion of an object.

$$K = \frac{1}{2}mv^2$$

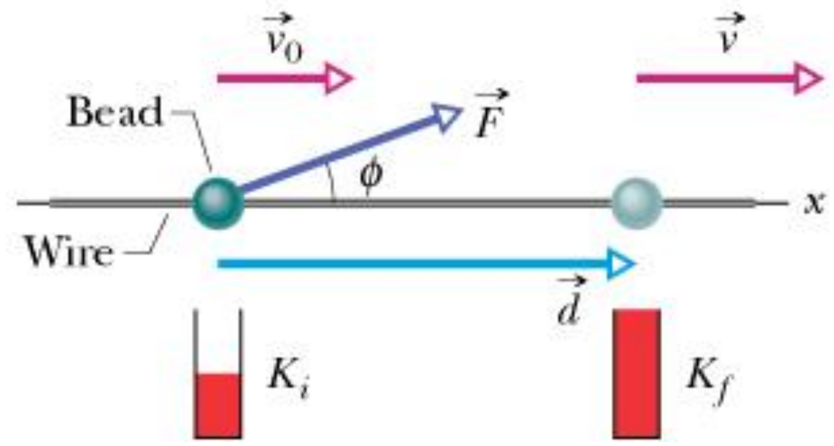
1 joule = 1 J = 1 kg. m^2/s^2

Work is energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work and energy transferred from the object is negative work.

Unit: joule

3.1.2. Work done by a force

A. Work done by a constant force:



•To establish an expression for work, we consider a constant force F that accelerates a bead along a wire:

$$F_x = ma_x$$

$$v^2 = v_0^2 + 2a_x d$$

$$\Rightarrow \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = F_x d$$

Therefore, the work W done on the bead by F is:

$$W = F_x d = Fd\cos\phi$$

$$\boxed{W = \vec{F}\,\vec{d} \text{ (work done by a constant force)}}$$

B. Work done by a general variable force:

One-dimensional analysis:

- Choose Δx small enough, work done by the force in the j*th* interval:

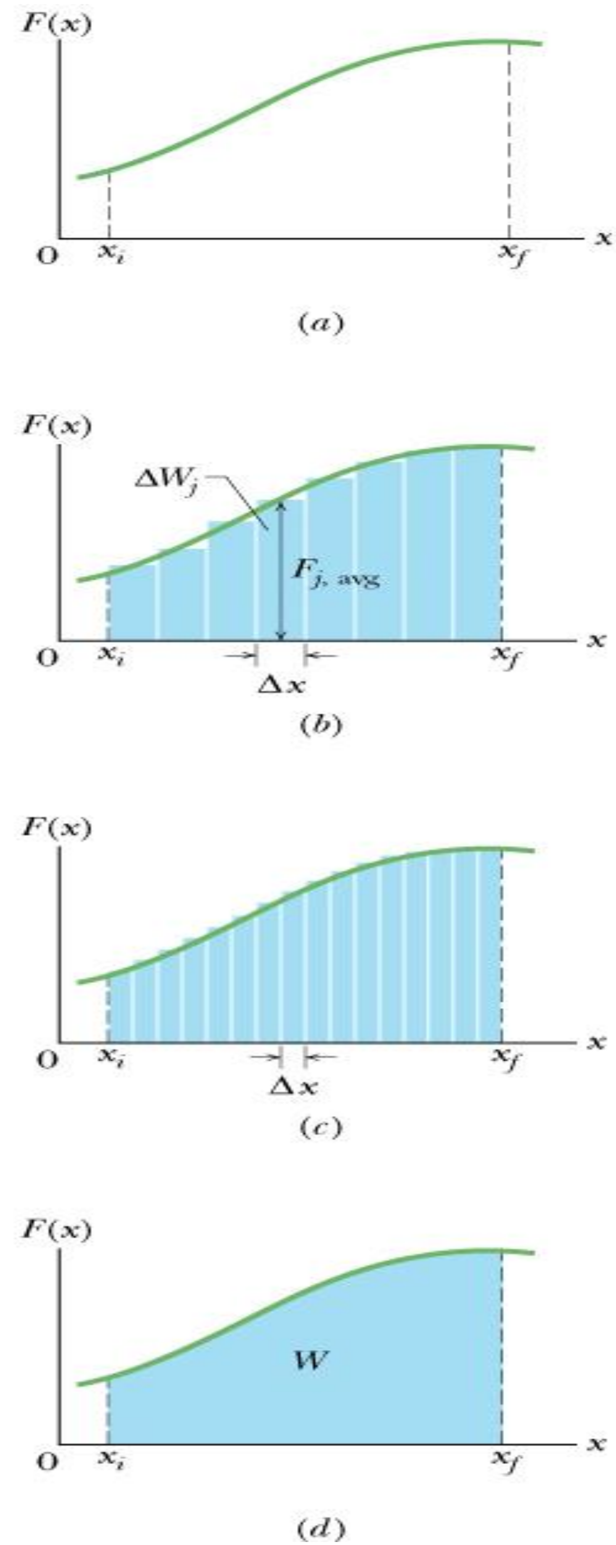
$$\Delta W_j = F_{j,avg}\Delta x$$

- The total work:

$$W = \sum \Delta W_j = \sum F_{j,avg}\Delta x$$

$$W = \lim_{\Delta x \to 0} \sum F_{j,avg}\Delta x$$

$$W = \int_{x_i}^{x_f} F(x)dx \text{ (work done by a variable force)}$$



Three-dimensional analysis:

$$dW = \vec{F}d\vec{r} = F_x dx + F_y dy + F_z dz$$

$$W = \int_{r_i}^{r_f} dW = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$$

3.1.3. Power

Power is the rate at which work is done.

Average power:

$$P_{avg} = \frac{W}{\Delta t}$$

Instantaneous power:

$$P = \frac{dW}{dt}$$

Unit: watt (W)

$$1\,\text{watt} = 1\,\text{W} = 1\,\text{J/s}$$

$$1\,\text{horsepower} = 1\,\text{hp} = 550\,\text{ft.lb/s} = 746\,\text{W}$$

$$1\,\text{kilowatt-hour} = 1\text{kW.h} = (10^3\,\text{W})(3600\text{s}) = 3.6\times10^6\,\text{J} = 3.6\,\text{MJ}$$

F=constant:

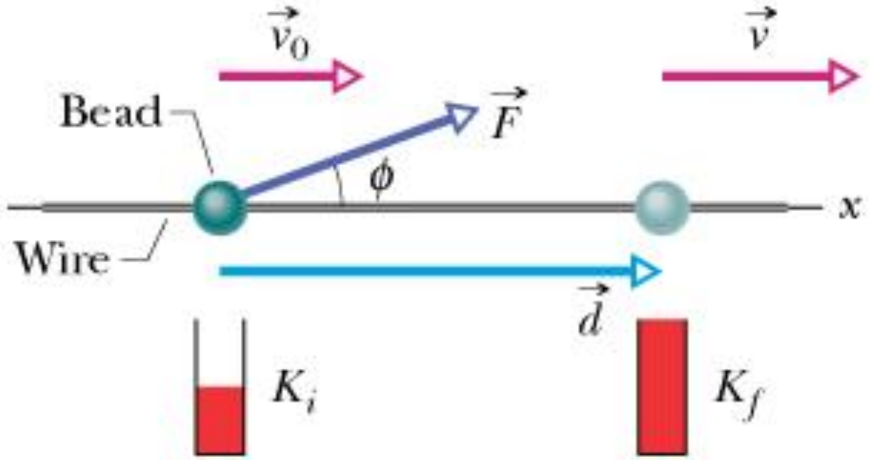
$$P = \frac{dW}{dt} = \frac{F\cos\phi\, dx}{dt} = F\cos\phi\left(\frac{dx}{dt}\right)$$

$$P = Fv\cos\phi$$

Instantaneous power:

$$P = \vec{F}\vec{v}$$

3.2. Work-Kinetic Energy Theorem



Let ΔK be the change in the kinetic energy of the bead.

$$\Delta K = K_f - K_i = W$$

This can be read as follows:

$$\begin{pmatrix}\text{change in the kinetic} \\ \text{energy of an object}\end{pmatrix} = \begin{pmatrix}\text{net work done on} \\ \text{the object}\end{pmatrix}$$

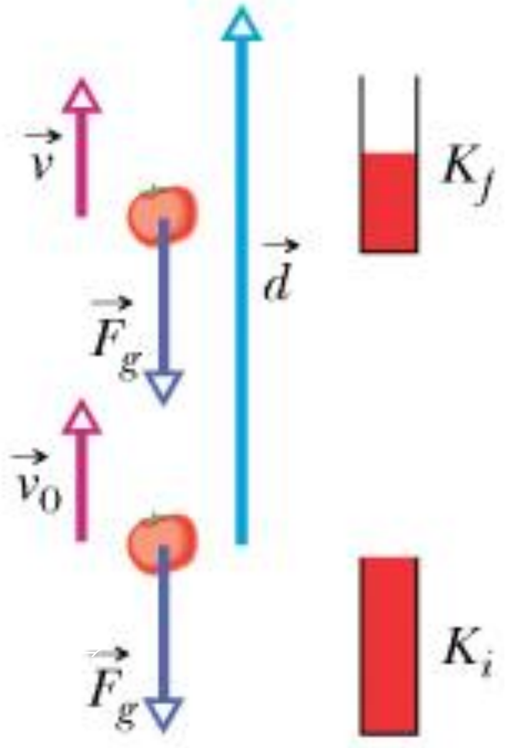
or

$$K_f = K_i + W$$

$$\begin{pmatrix}\text{kinetic energy after} \\ \text{the net work is done}\end{pmatrix} = \begin{pmatrix}\text{kinetic energy} \\ \text{before the net work}\end{pmatrix} + \begin{pmatrix}\text{the net} \\ \text{work done}\end{pmatrix}$$

Examples:

E1. Work done by the gravitational force:



$$W = F_x d = Fd\cos\phi$$

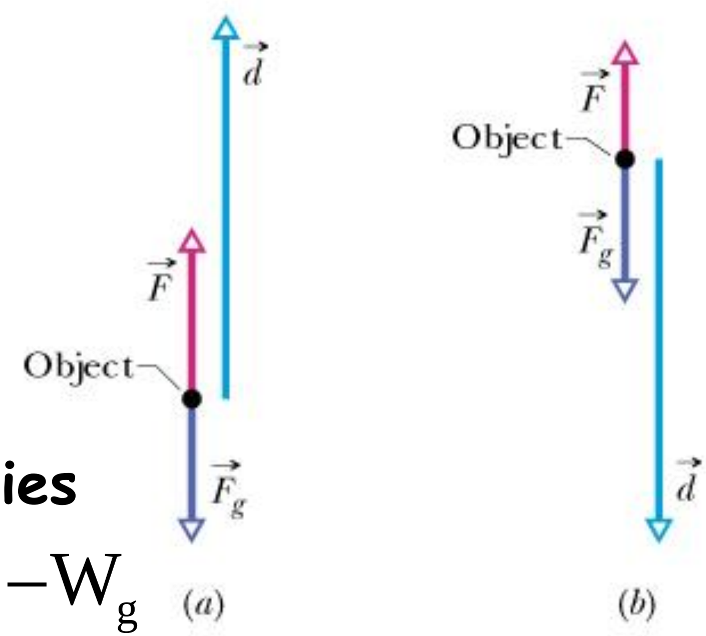
For a rising object, Φ=180⁰:

$$W = -mgd$$

For a falling object, Φ=0⁰:

$$W = +mgd$$

Work done in lifting and lowering an object



Gravity and an applied force acting on the object:

$$\Delta K = K_f - K_i = W_a + W_g$$

where W_a is the work done by the applied force; W_g is the work done by the gravitational force. If initial and final velocities are zero:

$$K_f = K_i = 0 \Rightarrow \quad \Delta K = 0 \Rightarrow W_a = -W_g$$

$$W_a = -mgd\cos\phi$$

E2. Work done by a spring force:

The spring force is computed by:

$$\vec{F}_s = -k\vec{d} \text{ (Hooke's law)}$$

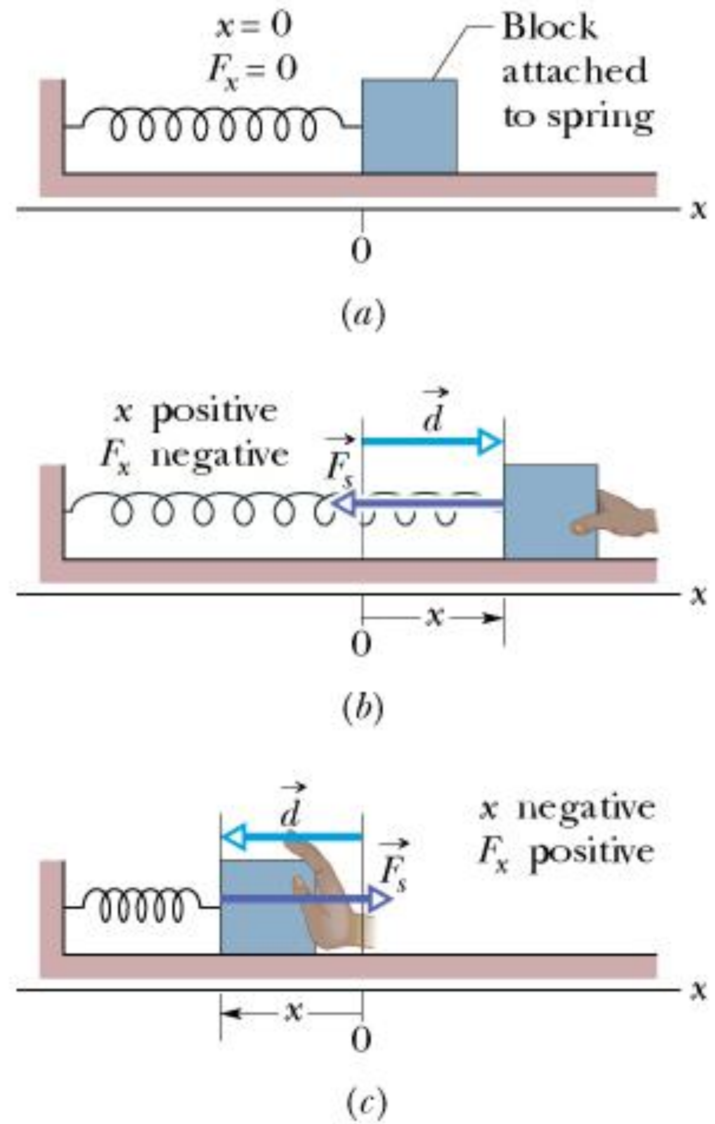
k: the spring constant (or force constant)

Is an x axis is parallel to the length of the spring:

$$F_x = -kx \text{ (Hooke's law)}$$

➔A spring force is a variable force F=F(x)

To find the work done by the spring force, We have to make assumptions: (1) the spring is massless; (2) It is an ideal spring (it exactly obeys Hooke's law).



$$W_s = \lim_{\Delta x \to 0} \sum \vec{F}_{xj} \Delta\vec{x} = \lim_{\Delta x \to 0} \sum F_{xj} \Delta x \cos\theta = \lim_{\Delta x \to 0} \sum -F_{xj} \Delta x$$

Note: $\theta = 180^0$ and F_{xj} is the magnitude of the spring force

$$W_s = \int_{x_i}^{x_f} -|F_x|dx = \int_{x_i}^{x_f} -kxdx$$

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

$$\text{If } x_i = 0, x_f = x : W_s = -\frac{1}{2}kx^2$$

Work done by an applied force:

$$\Delta K = K_f - K_i = W_a + W_s$$

If the block is stationary before and after the displacement, ΔK=0:

$$W_a = -W_s$$

Homework: 1, 2, 8, 15, 24, 26, 29, 36, 43 (p. 159-163)

Review

(All sections of Chapter 1, 2)

Chapter 1:

Motion in one dimension:

To describe motion, we need to measure:

+ Displacement: $\Delta x = x_f - x_0$ (measured in m or cm)

+ Time interval: $\Delta t = t - t_0$ (measured in s)

Average velocity: $$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

Average speed: $$s_{avg} = \frac{\text{total distance}}{\Delta t}$$

Instantaneous velocity: $$v(t) = \lim_{\Delta t \to 0} \frac{\Delta x(t)}{\Delta t} = \frac{dx(t)}{dt}$$

Average acceleration: $$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

Instantaneous acceleration: $$a(t) = \frac{dv(t)}{dt} = \frac{d^2x}{dt^2}$$

Two basic equations for constant acceleration:

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$v^2 - v_0^2 = 2a(x - x_0)$$

For freely falling objects: $a = g = 9.8\,(m/s^2)$

Motion in two dimensions:

$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} = \Delta x\hat{i} + \Delta y\hat{j}$$

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

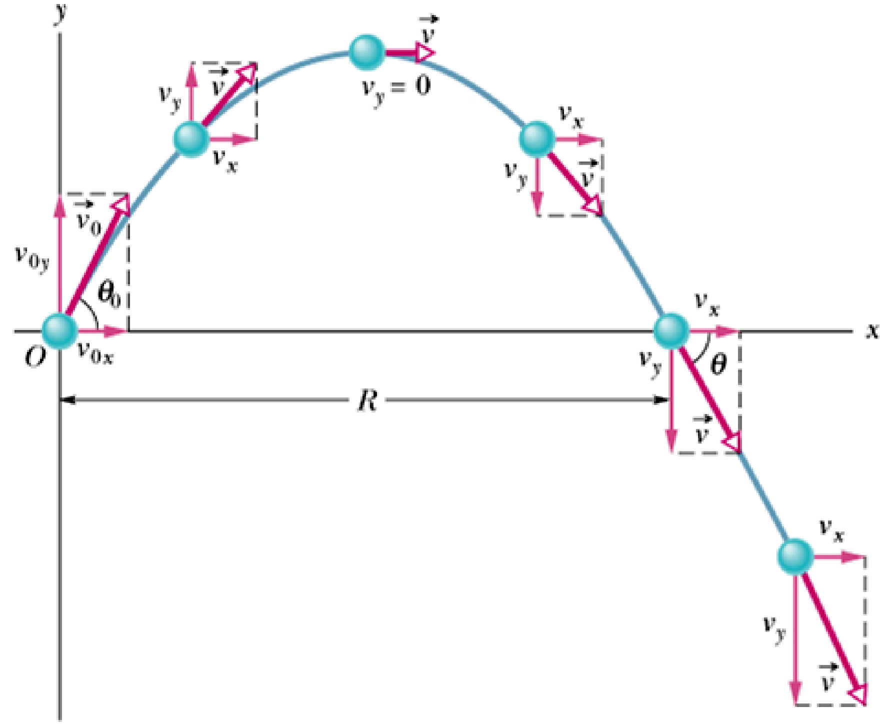
$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

Projectile motion:

- Ox: Horizontal motion (no acceleration):

$$v_x = v_0 \cos\theta_0 = \text{constant}$$

$$x = x_0 + v_0 \cos\theta_0 t$$



- Oy: Vertical motion (free fall)

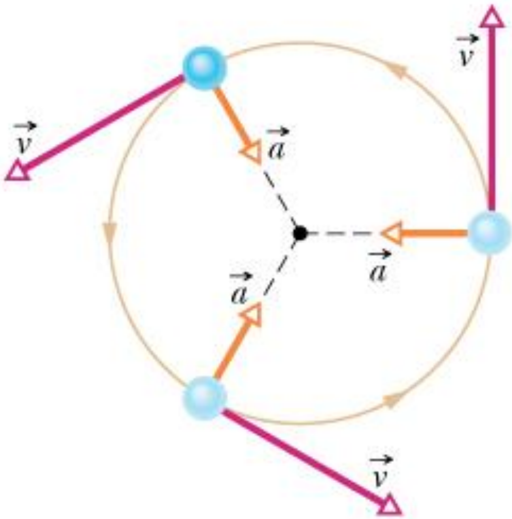
$$v_y = v_0 \sin\theta_0 - gt$$

$$y = y_0 + v_0 \sin\theta_0 t - \frac{1}{2} g t^2$$

- Horizontal range:

$$R = \frac{v_0^2}{g} \sin 2\theta_0$$

Uniform Circular Motion:



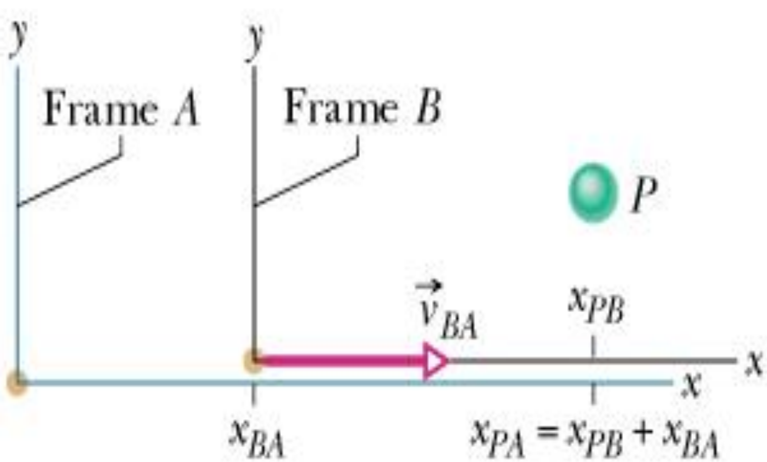
The particle is accelerating with a centripetal acceleration:

$$a = \frac{v^2}{r}$$

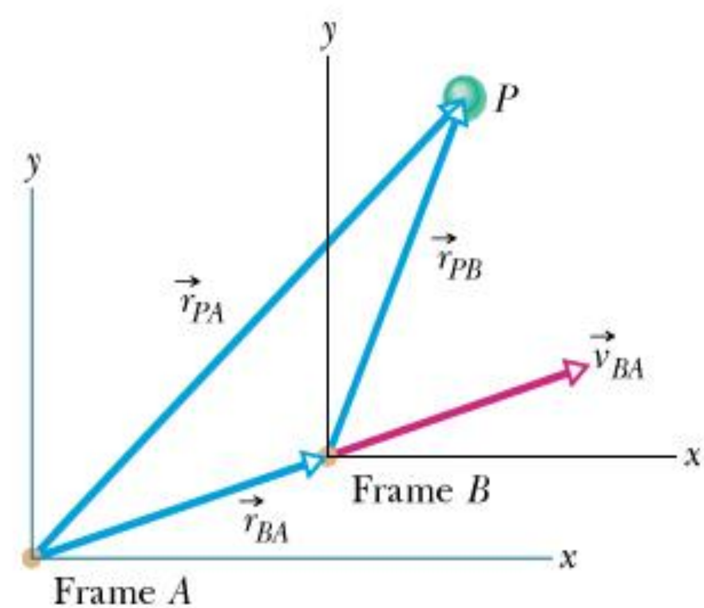
Where r is the radius of the circle
v the speed of the particle

$$T = \frac{2\pi r}{v}$$ (T: period)

Relative Velocity and Relative Acceleration:



$$v_{PA} = v_{PB} + v_{BA}$$



$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$$

Chapter 2:

Newton's Laws

$$\vec{F} = 0 \quad \text{or} \quad \sum_{i=1}^{n} \vec{F}_i = 0$$

$$\vec{F}_{net} = m\vec{a}$$

$$\vec{F}_{BC} = -\vec{F}_{CB}$$

Some particular forces: gravitational, normal, tension and frictional forces

Friction and Properties of Friction:

$$f_{s,max} = \mu_s F_N$$ μ_s is the coefficient of static friction

$$f_k = \mu_k F_N$$ μ_k is the coefficient of kinetic friction

Uniform Circular Motion and Non-uniform Circular Motion:

- Uniform circular motion: $a = \frac{v^2}{R}$ $F = ma = m\frac{v^2}{R}$
- Non-uniform circular motion: $F_r = m\frac{v^2}{R}; F_t = m\frac{dv}{dt}$

$$\vec{a} = \vec{a}_r + \vec{a}_t$$