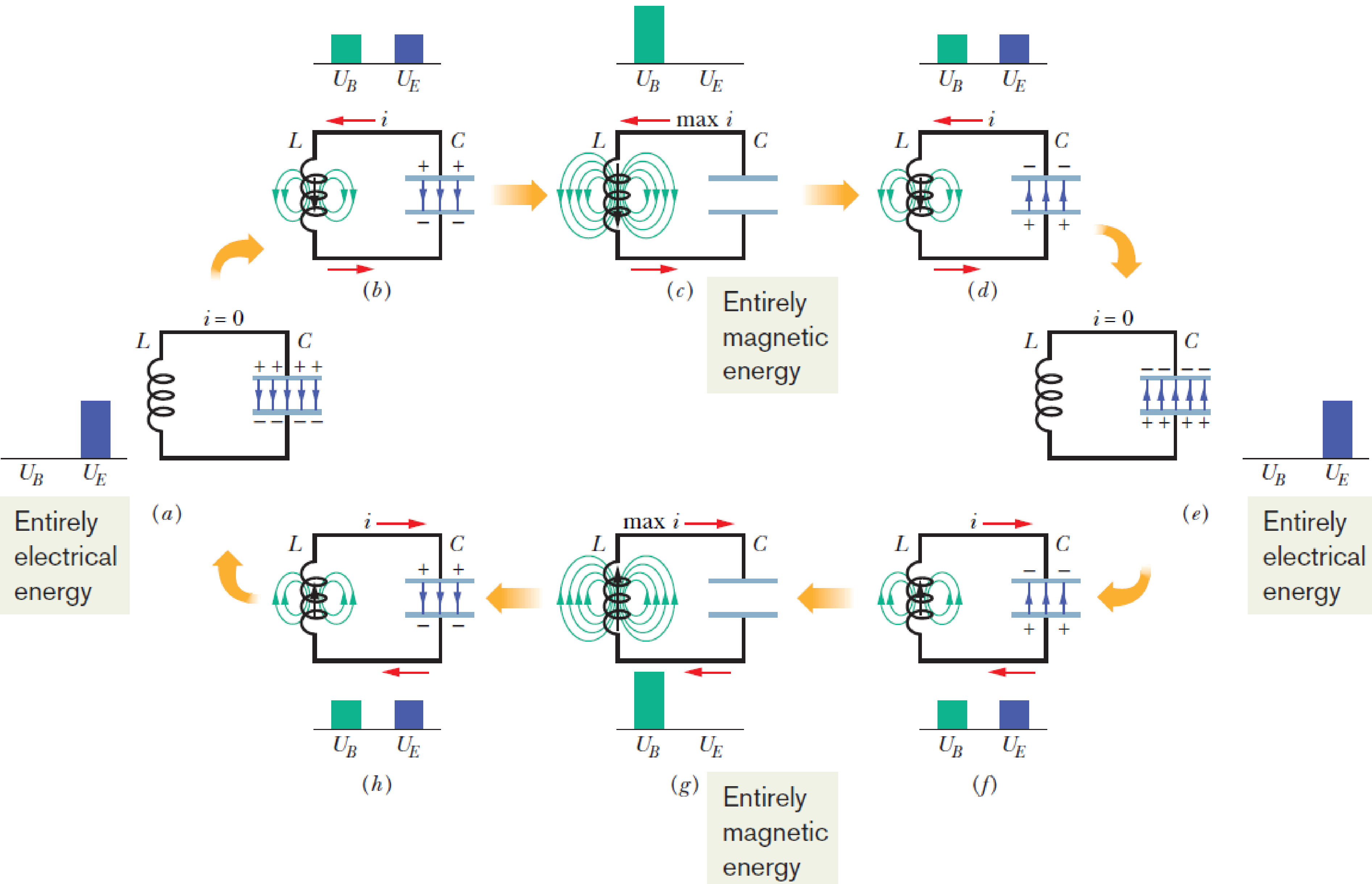
Chapter 6:

Electromagnetic Oscillations and Alternating Current

Homework:

1, 2, 7, 9, 10, 17, 23, 25, 29, 32, 38, 39, 48, 53 (pages 855-858)



U_B U_E
Entirely electrical energy
i = 0
L
C
(a)
U_B U_E
i
L
C
(b)
U_B U_E
max i
L
C
(c)
Entirely magnetic energy
U_B U_E
i
L
C
(d)
i = 0
L
C
U_B U_E
(e)
Entirely electrical energy
i
L
C
U_B U_E
(f)
max i
L
C
U_B U_E
(g)
Entirely magnetic energy
i
L
C
U_B U_E
(h)

1. An oscillating LC circuit consists of a 75.0 mH inductor and a 3.60 μF capacitor. If the maximum charge on the capacitor is 2.90 μC, what are (a) the total energy in the circuit and (b) the maximum current?

$$U = U_E + U_B = \frac{q^2}{2C} + \frac{Li^2}{2}$$

(a) When q is maximum:

$$U = U_{E,\max} = \frac{q_{\max}^2}{2C}$$

(b) i is maximum when q = 0:

$$U_{B,\max} = \frac{Li_{\max}^2}{2} = U_{E,\max}$$

2. The frequency of oscillation of a certain LC circuit is 200 kHz. At time t = 0, plate A of the capacitor has maximum positive charge. At what earliest time t > 0 will (a) plate A again have maximum positive charge, (b) the other plate of the capacitor have maximum positive charge, and (c) the inductor have maximum magnetic field?

$$q = Q\cos(\omega t + \phi)$$

Determine ϕ from the conditions given in the problem, at t = 0:

$q = Q\cos\phi$ is maximum, so $\phi = 0$

(a) So, q is max again as T = (2π/ω) x n

$$T = 2\pi\sqrt{LC} = 1/f = 5\times10^{-6}(s) = 5(\mu s)$$

(b) plate B has maximum positive charge at:

$$t = \frac{1}{2}T + (n-1)T \Rightarrow t = 2.5(\mu s)$$

(c)

$$U_B = \frac{Q^2}{2C}\sin^2(\omega t + \phi) \Rightarrow U_B \text{ max as } t = \frac{T}{4} + \frac{(n-1)}{2}T$$

$$\Rightarrow t = 1.25(\mu s)$$

7. The energy in an oscillating LC circuit containing a 1.25 H inductor is 5.70 μJ. The maximum charge on the capacitor is 175 μC. For a mechanical system with the same period, find the (a) mass, (b) spring constant, (c) maximum displacement, and (d) maximum speed.

q corresponds to x, $1/C$ corresponds to k,
i corresponds to v, and L corresponds to m.

(a) mass m = 1.25 kg

(b) spring constant k = 1/C

$$U = \frac{Q^2}{2C} \Rightarrow C = \frac{Q^2}{2U} = \frac{(175\times10^{-6})^2}{2\times5.7\times10^{-6}} = 2.69\times10^{-3}(F)$$

$$k = \frac{1}{2.69\times10^{-3}} = 372(N/m)$$

(c) x_{max} = 175 μm = 1.75 x 10^{-4} (m)

(d) $$\frac{Li^2}{2} = \frac{Q^2}{2C} \Rightarrow i = \frac{Q}{\sqrt{LC}} = 3.02\times10^{-3}(A)$$

$$v_{max} = 3.02\times10^{-3}(m/s)$$

9. In an oscillating LC circuit with L = 50 mH and C = 4.0 μF, the current is initially a maximum. How long will it take before the capacitor is fully charged for the first time?

$$i = -I\sin(\omega t + \phi)$$

At t = 0, i is max:

$$\phi = \pm\pi/2$$

$$i = -I\sin(\omega t \pm \pi/2)$$

when the capacitor is fully charged, i = 0:

$$\omega t = \pi/2 \Rightarrow 2\pi\frac{t}{T} = \frac{\pi}{2} \Rightarrow t = \frac{T}{4}$$

$$t = \frac{T}{4} = \frac{2\pi\sqrt{LC}}{4} = \frac{2\pi\sqrt{50\times10^{-3}\times4\times10^{-6}}}{4} = 7\times10^{-4}(s)$$

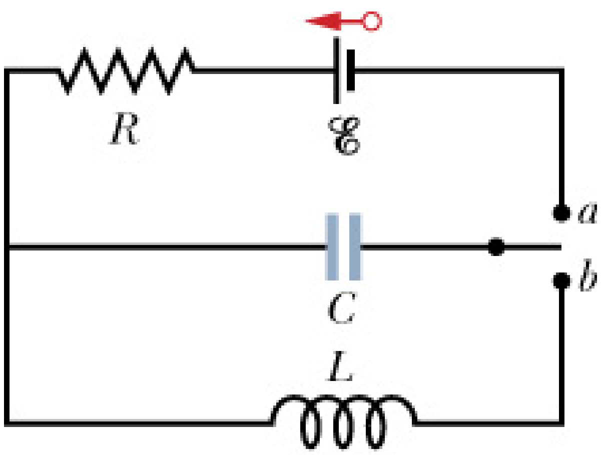
10. LC oscillators have been used in circuits connected to loudspeakers to create some of the sounds of electronic music. What inductance must be used with a 6.7 μF capacitor to produce a frequency of 10 kHz, which is near the middle of the audible range of frequencies?

$$T = 2\pi\sqrt{LC} = 1/f \Rightarrow L = \frac{1}{4\pi^2 f^2 C}$$

17. In Figure below, R = 14.0 Ω, C = 6.20 μF, and L = 54.0 mH, and the ideal battery has emf ε = 34.0 V. The switch is kept at a for a long time and then thrown to position b. What are the (a) frequency and (b) current amplitude of the resulting oscillations?

(a)

$$T = 2\pi\sqrt{LC} = 1/f \Rightarrow f = \frac{1}{2\pi\sqrt{LC}}$$



(b) When the capacitor is fully charge (the switch is on a):

$$Q = C\varepsilon$$

The maximum current when the switch is on b:

$$I = \omega Q = 2\pi f Q$$

23. In an oscillating LC circuit, $L = 25.0$ mH and $C = 7.80$ μF. At time $t = 0$ the current is 9.20 mA, the charge on the capacitor is 3.80 μC, and the capacitor is charging. What are (a) the total energy in the circuit, (b) the maximum charge on the capacitor, and (c) the maximum current? (d) If the charge on the capacitor is given by $q = Q\cos(\omega t + \phi)$, what is the phase angle ϕ? (e) Suppose the data are the same, except that the capacitor is discharging at $t = 0$. What then is ϕ?

(a) t = 0:

$$U = U_E + U_B = \frac{q^2}{2C} + \frac{Li^2}{2}$$

(b) the maximum charge:

$$U = U_{E,\max} = \frac{Q^2}{2C} \Rightarrow Q = \sqrt{2CU}$$

(c) the maximum current:

$$U = U_{B,\max} = \frac{LI^2}{2} \Rightarrow I = \sqrt{\frac{2U}{L}}$$

(d) the charge is given:

$$q = Q\cos(\omega t + \phi)$$

At t = 0: q = 3.8 μC

$$\cos\phi = \frac{q}{Q} \Rightarrow \phi = \pm 47^0$$

The capacitor is charging at t = 0:

$$\frac{dq}{dt} = -\omega Q \sin(\omega t + \phi) = -\omega Q \sin\phi > 0$$

$$\phi = -47^0$$

(d) if the capacitor is discharging:

$$\frac{dq}{dt} = -\omega Q \sin(\omega t + \phi) = -\omega Q \sin\phi < 0$$

$$\phi = +47^0$$

25. What resistance R should be connected in series with an inductance L = 220 mH and capacitance C = 12.0 μF for the maximum charge on the capacitor to decay to 99.0% of its initial value in 50.0 cycles? (Assume ω' ≈ ω).

$$q = Qe^{-Rt/2L}\cos(\omega' t + \phi)$$

$$\omega' = \sqrt{\omega^2 - (R/2L)^2} \qquad \omega = 1/\sqrt{LC}$$

We have:

$$\frac{q_{\max}}{Q} = e^{-Rt/2L} = 0.99$$

$$t = 50T = 50\frac{2\pi}{\omega} = 100\pi\sqrt{LC}$$

$$-Rt/2L = \ln\left(\frac{q_{\max}}{Q}\right) \Rightarrow R = -\frac{2L}{t}\ln\left(\frac{q_{\max}}{Q}\right)$$

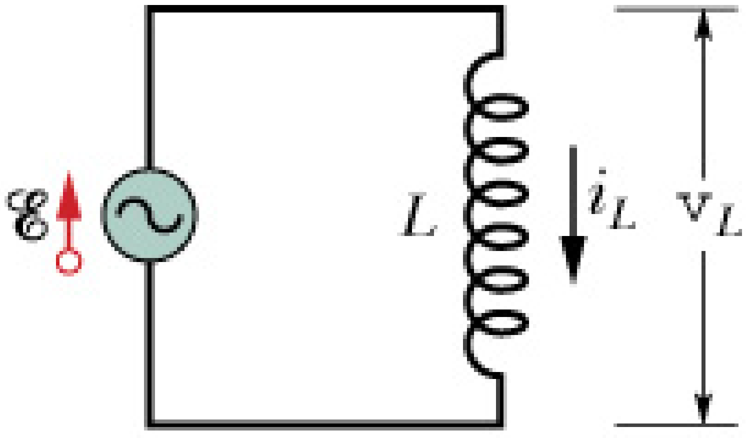
29. A 50.0 mH inductor is connected (as Figure) to an ac generator with ε_m = 30.0 V. What is the amplitude of the resulting alternating current if the frequency of the emf is (a) 1.00 kHz and (b) 8.00 kHz?

$$i_L = I_L \sin(\omega_d t - \phi)$$

$$V_L = I_L X_L = \varepsilon_m$$

$$X_L = \omega_d L = 2\pi f L$$

$$I_L = \frac{\varepsilon_m}{X_L} = \frac{\varepsilon_m}{2\pi f_d L}$$



(a)

32. An ac generator has emf $\varepsilon = \varepsilon_m \sin\omega_d t$, with ε_m= 25.0 V and ω_d= 377 rad/s. It is connected to a 12.7 H inductor. (a) What is the maximum value of the current? (b) When the current is a maximum, what is the emf of the generator? (c) When the emf of the generator is -12.5 V and increasing in magnitude, what is the current?

(a)
$$I_L = \frac{\varepsilon_m}{X_L} = \frac{\varepsilon_m}{\omega_d L} = \frac{25}{377 \times 12.7} = 5.22 \times 10^{-3} (A)$$

$$i_L = I_L \sin(\omega_d t - \frac{\pi}{2})$$

(b) when i_L is maximum, $\sin(\omega_d t - \frac{\pi}{2}) = 1 \Rightarrow \omega_d t = (2n+1)\pi$

$$\varepsilon = \varepsilon_m \sin(\omega_d t) = 0$$

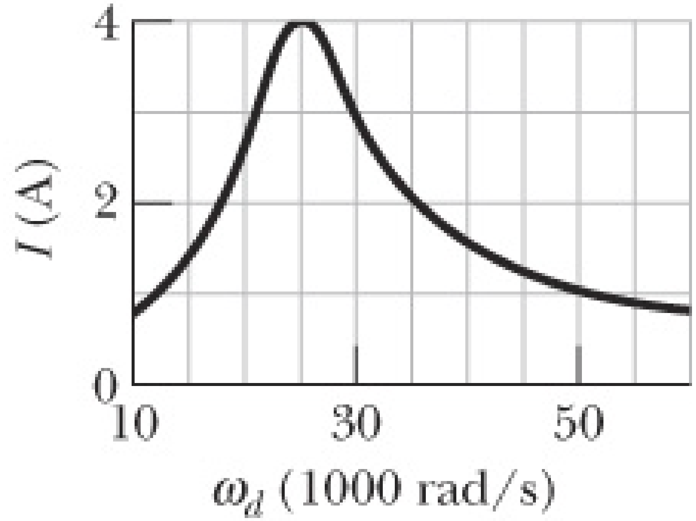
(c) $\sin(\omega_d t) = -\frac{1}{2} \Rightarrow \omega_d t = 2n\pi + \frac{7}{6}\pi$ or $\omega_d t = 2n\pi + \frac{11}{6}\pi$

$$\frac{d\varepsilon}{dt} = \varepsilon_m \omega_d \cos(\omega_d t) < 0 \Rightarrow \omega_d t = 2n\pi + \frac{7}{6}\pi$$

$$i_L = I_L \sin(2n\pi + \frac{2}{3}\pi) = 5.22 \times 10^{-3} \times 0.866 = 4.52 \times 10^{-3} (A)$$

38. The current amplitude I versus driving angular frequency ω_d for a driven RLC circuit is given (as in Figure). The inductance is 200 μH, and the emf amplitude is 8.0 V. What are (a) C and (b) R?

(a) The current I is maximum when:



$$\omega_d = 25\times10^3 (rad/s)$$

$$\omega_d = \omega = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{\omega_d^2 L}$$

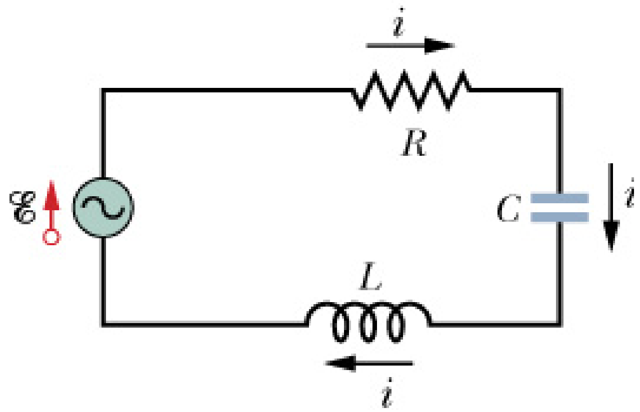
(b) At resonance:

$$Z = R$$

$$R = Z = \frac{\varepsilon_m}{I} = \frac{8}{4} = 2(\Omega)$$

39. In Fig. 31-7, set R = 200 Ω, C = 70.0 μF, L = 230 mH, f_d = 60.0 Hz, and ε_m = 36.0 V. What are (a) Z, (b) ϕ, and (c) I? (d) Draw a phasor diagram.

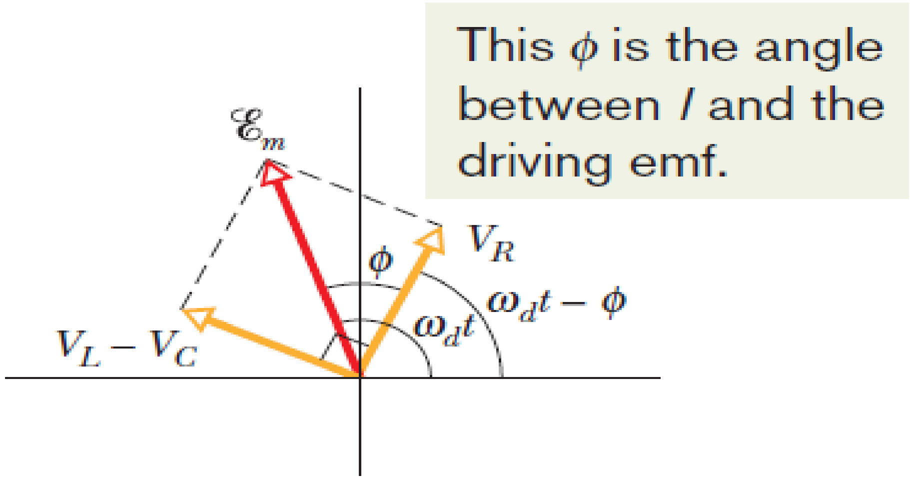
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$



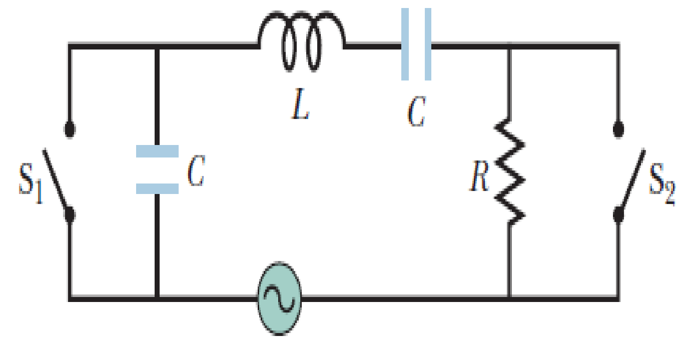
$$\tan\phi = \frac{X_L - X_C}{R}$$

If ϕ > 0: ε_m leads the current
If ϕ < 0: the current leads ε_m

$$I = \frac{\mathcal{E}_m}{Z}$$



48. Figure 31-32 shows a driven RLC circuit that contains two identical capacitors and two switches. The emf amplitude is set at 12.0 V, and the driving frequency is set at 60.0 Hz. With both switches open, the current leads the emf by 30.9°. With switch S_1 closed and switch S_2 still open, the emf leads the current by 15.0°. With both switches closed, the current amplitude is 447 mA. What are (a) R, (b) C, and (c) L?



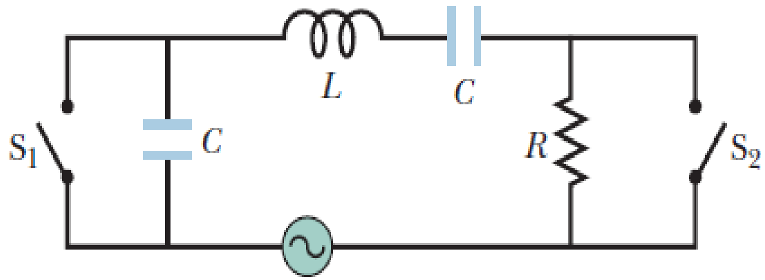
When S_1 and S_2 closed, we have a simple LC circuit; $Z = X_L - X_C$

$$I = \frac{\varepsilon_m}{Z} = \frac{\varepsilon_m}{X_L - X_C} \Rightarrow X_L - X_C = \varepsilon_m I \quad (1)$$

a) When S_1 closed and S_2 opened, we have RLC circuit

$$\tan\phi = \frac{X_L - X_C}{R} \quad (2)$$

From (1) + (2) $\Rightarrow$ $$R = \frac{\varepsilon_m}{I\tan(\phi)} = \frac{12}{0.447\tan(15^0)} = 100\Omega$$



b) When both switches (S_1 and S_2) opened, we have RLC circuit with R, L and C_{eq} = C/2 => X_{Ceq}=2X_c

$$\tan\phi' = \frac{X_L - 2X_C}{R} \qquad (3)$$

From (1) + (3) $$X_C = \frac{\varepsilon_m}{I} - R\tan\phi' = \frac{12}{0.447} - 100\tan(-31.9) = 89(\Omega)$$

$$X_C = \frac{1}{\omega C} \Rightarrow C = \frac{1}{2\pi f Xc} = \frac{1}{2\pi(60)(89)} = 3\times 10^{-5}(F)$$

c) $$X_L = \frac{\varepsilon_m}{I} + X_C = \frac{12}{0.447} + 89 = 116(\Omega)$$

$$X_L = L\omega \Rightarrow L = X_L 2\pi f = 116x2\pi x60 = 0.307(H)$$

53. An air conditioner connected to a 120 V rms ac line is equivalent to a 12.0 Ω resistance and a 1.30 Ω inductive reactance in series. Calculate (a) the impedance of the air conditioner and (b) the average rate at which energy is supplied to the appliance.

(a) the impedance of the air conditioner

$$Z = \sqrt{R^2 + {X_L}^2} = \sqrt{12^2 + 1.3^2} = 12.1(\Omega)$$

(b) the average rate

$$P_{avg} = \varepsilon_{rms} I_{rms} = V_{rms} \frac{V_{rms}}{Z} = 1.19kW$$