Chapter 5:

Electromagnetic Induction

Homework:

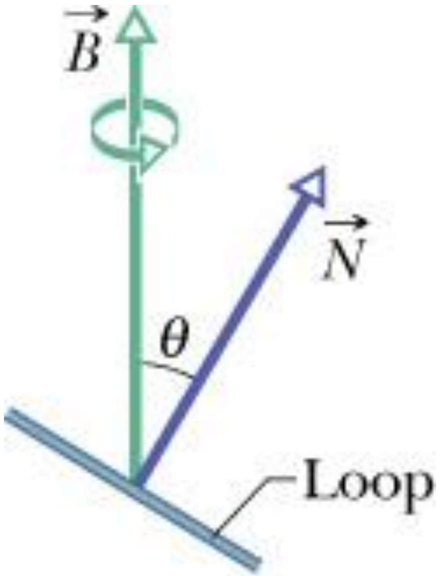
1, 7, 8, 14, 15, 20, 27, 32, 33, 37, 40, 47, 55, 56, 68, 73, 79 (pages 818-824)

1. In the figure below, a circular loop of wire 10 cm in diameter (seen edge-on) is placed with its normal N at an angle θ = 30° with the direction of a uniform magnetic field B of magnitude 0.50 T. The loop is then rotated such that N rotates in a cone about the field direction at the rate 100 rev/min; angle θ remains unchanged during the process. What is the emf induced in the loop?

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

$$\Phi_B = B.A.\cos\theta = \text{constant}$$

Therefore: $$\varepsilon = 0$$

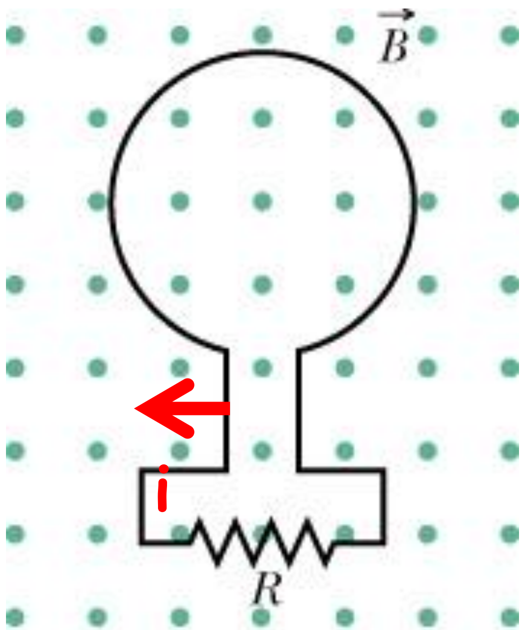


7. In the figure below, the magnetic flux through the loop increases according to the relation $\Phi_B = 6.0t^2 + 7.0t$, where Φ_B is in milliwebers and t is in seconds. (a) What is the magnitude of the emf induced in the loop when t = 2.0 s? (b) Is the direction of the current through R to the right or left?

(a) Using Faraday's law: $\varepsilon = -\dfrac{d\Phi_B}{dt}$

$$|\varepsilon| = (12t + 7) = 31(mV)$$

(b) B increases, using Lenz's law, $B_{induced}$ should be point into the page, so the current through R is to the left



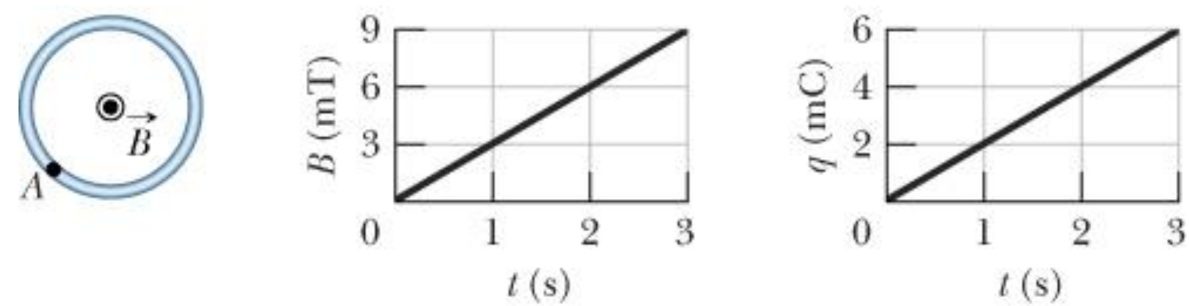
8. A uniform magnetic field B is perpendicular to the plane of a circular loop of diameter 10 cm formed from wire of diameter 2.5 mm and resistivity 1.69 × 10^{-8} Ω.m. At what rate must the magnitude of B change to induce a 10 A current in the loop?

$$R = \rho\frac{l}{A} = 1.69\times10^{-8}\times\frac{2\pi0.05}{\pi\frac{0.0025^2}{4}} = 1.1\times10^{-3}(\Omega)$$

$$i = \frac{\varepsilon}{R} = -\frac{1}{R}\frac{d\Phi_B}{dt} = -\frac{\pi r^2}{R}\frac{dB}{dt}$$

$$\left|\frac{dB}{dt}\right| = \frac{iR}{\pi r^2} = \frac{10\times1.1\times10^{-3}}{\pi\times(0.05)^2} = 1.4(T/s)$$

14. In Fig. 30-40a, a uniform magnetic field B increases in magnitude with time t as given by Fig. 30-41b. A circular conducting loop of area 8.0×10^{-4} m^2 lies in the field, in the plane of the page. The amount of charge q passing point A on the loop is given in Fig. 30-41c as a function of t. What is the loop's resistance?



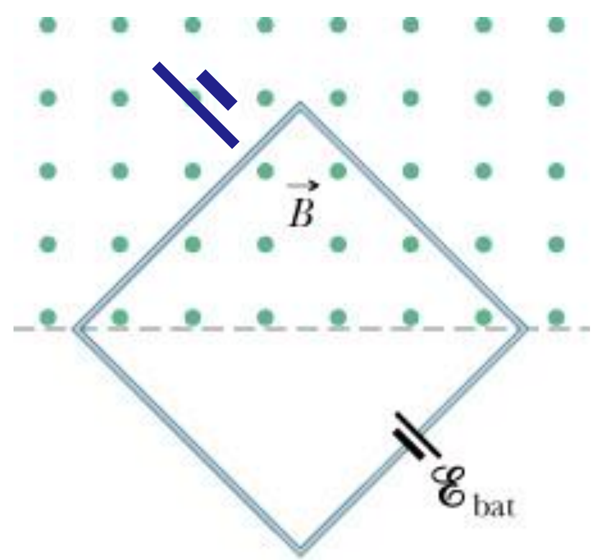
(a) (b) (c)

$$\varepsilon = -\frac{d\Phi_B}{dt} = -A\frac{dB}{dt} = -0.003A \quad (V)$$

$$i = \frac{dq}{dt} = 0.002 \quad (A)$$

$$\Rightarrow R = \frac{|\varepsilon|}{i} = \frac{0.003 \times 8 \times 10^{-4}}{0.002} = 0.0012(\Omega)$$

15. A square wire loop with 2.00 m sides is perpendicular to a uniform magnetic field, with half the area of the loop in the field as shown in the figure below. The loop contains an ideal battery with emf ε = 20.0 V. If the magnitude of the field varies with time according to B = 0.0420 - 0.870t, with B in tesla and t in seconds, what are (a) the net emf in the circuit and (b) the direction of the (net) current around the loop?



(a) As time goes on, B decreases

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{L^2}{2}(-0.87) = 1.74(V)$$

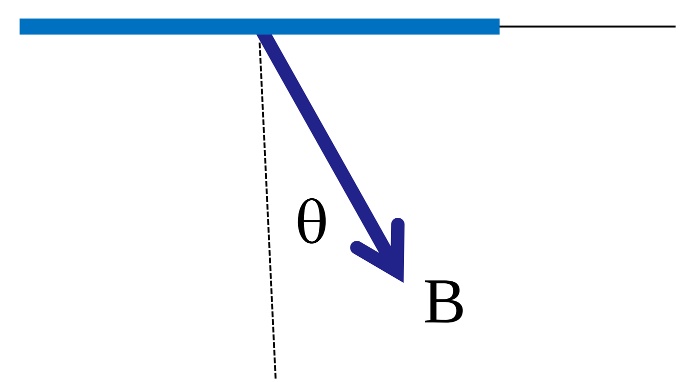
The induced B points out of the page, so the induced current as well as the induced emf is counterclockwise.

(b) The net current is counterclockwise:

$$\varepsilon_{net} = \varepsilon + \varepsilon_i$$

20. At a certain place, Earth's magnetic field has magnitude B = 0.590 gauss and is inclined downward at an angle of 70.0° to the horizontal. A flat horizontal circular coil of wire with a radius of 10.0 cm has 1000 turns and a total resistance of 85.0 Ω. It is connected in series to a meter with 140 Ω resistance. The coil is flipped through a half-revolution about a diameter, so that it is again horizontal. How much charge flows through the meter during the flip?

$$\varepsilon = -\frac{d\Phi_B}{dt}; i = \frac{dq}{dt}$$



$$i = \frac{|\varepsilon|}{R} \Rightarrow \frac{dq}{dt} = \frac{\left|\frac{d\Phi_B}{dt}\right|}{R}$$

$$|dq| = \frac{|d\Phi_B|}{R} = \frac{N}{R}\left(BA\cos\theta - (-BA\cos\theta)\right)$$

$$|dq| = \frac{2NBA\cos\theta}{R}$$ with $\theta = 20^0$; $1\text{gauss} = 10^{-4}\,\text{T}$

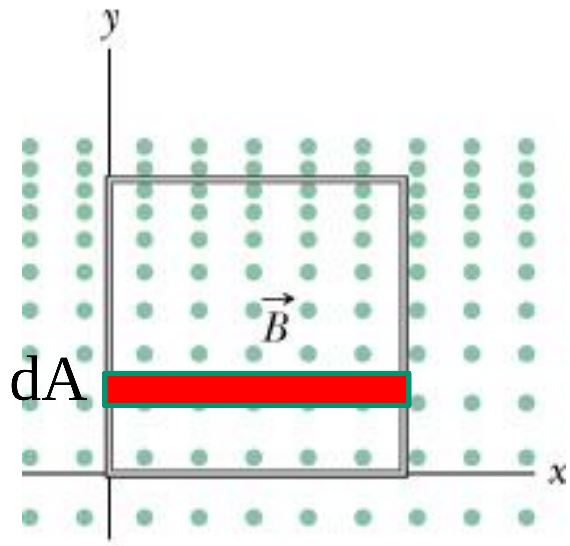
27. As seen in Fig. 30-48, a square loop of wire has sides of length 2.0 cm. A magnetic field is directed out of the page; its magnitude is given by $B = 4.0t^2y$, where B is in teslas, t is in seconds, and y is in meters. At t = 2.5 s, what are the (a) magnitude and (b) direction of the emf induced in the loop?

(a)

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

$$\Phi_B = \int \vec{B}d\vec{A} = \int 4t^2 y(ldy) = 4lt^2 \int_0^l ydy = 2l^3t^2$$

$$\varepsilon = -\frac{d\Phi_B}{dt} = -4l^3t$$



At t = 2.5 s:

$$|\varepsilon| = 4l^3t = 8\times10^{-5}(V)$$

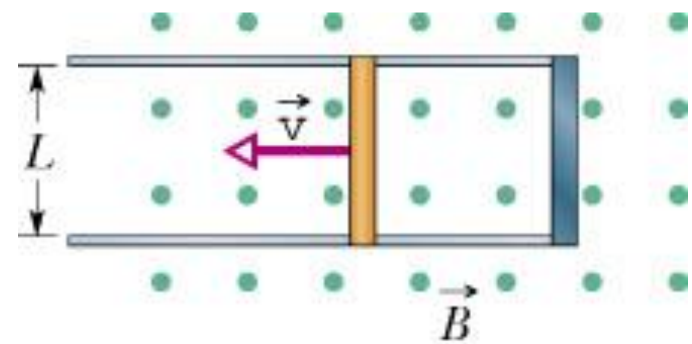
(b) B increases, the current direction is clockwise, so the induced emf direction is clockwise

32. A loop antenna of area 2.00 cm^2 and resistance 5.21 $\mu\Omega$ is perpendicular to a uniform magnetic field of magnitude *17.0 μT.* The field magnitude drops to zero in 2.96 ms. How much thermal energy is produced in the loop by the change in field?

$$|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = \frac{\Delta B \times A}{\Delta t} = \frac{17\times10^{-6}\times2\times10^{-4}}{2.96\times10^{-3}} = 1.2\times10^{-6}(V)$$

$$E_{\text{thermal}} = i^2 R\Delta t = \frac{\varepsilon^2}{R}\Delta t = \frac{1.2^2\times10^{-12}}{5.21\times10^{-6}}2.96\times10^{-3} = 8\times10^{-10}(J)$$

33. In Fig. 30-52, a metal rod is forced to move with constant velocity v along two parallel metal rails, connected with a strip of metal at one end. A magnetic field of magnitude B = 0.350 T points out of the page. (a) If the rails are separated by L = 25.0 cm and the speed of the rod is 55.0 cm/s, what emf is generated? (b) If the rod has a resistance of 18.0 Ω and the rails and connector have negligible resistance, what is the current in the rod? (c) At what rate is energy being transferred to thermal energy?



(a) $$\varepsilon = \left| -\frac{d\Phi_B}{dt} \right| = \frac{d(BLvt)}{dt} = BvL$$

$$\varepsilon = 0.35 \times 0.55 \times 0.25 = 0.048(V)$$

(b) $$i = \frac{\varepsilon}{R} = \frac{0.048}{18} = 2.7 \times 10^{-3}(A)$$

using Lenz's law: the current direction is clockwise

(c)

$$P = i^2 R = 0.13(mW)$$

37. A long solenoid has a diameter of 12.0 cm. When a current i exists in its windings, a uniform magnetic field of magnitude B = 30.0 mT is produced in its interior. By decreasing i, the field is caused to decrease at the rate of 6.50 mT/s. Calculate the magnitude of the induced electric field (a) 2.20 cm and (b) 8.20 cm from the axis of the solenoid (see also Sample Problem 30-4, page 804)

(a) $$\oint \vec{E}.d\vec{s} = -\frac{d\Phi_B}{dt}; \quad \oint \vec{E}.d\vec{s} = E2\pi r; \Phi_B = BA = B\pi r^2$$

$$\Rightarrow E = \frac{r}{2}\frac{dB}{dt} = \frac{2.2\times10^{-2}}{2}6.5\times10^{-3} = 7.15\times10^{-5}(V/m)$$

(b) $$\oint \vec{E}.d\vec{s} = E2\pi r; \Phi_B = BA = B\pi R^2$$

$$\Rightarrow E = \frac{R^2}{2r}\frac{dB}{dt} = \frac{6.0^2\times10^{-4}}{2\times8.2\times10^{-2}}6.5\times10^{-3}$$

$$= 1.43\times10^{-4}(V/m)$$

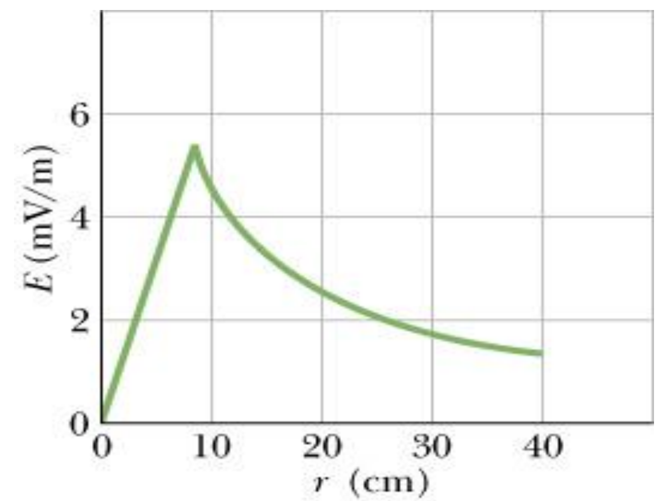


Figure 30-15 of Sample Problem 30-4

40. The inductance of a closely packed coil of 400 turns is 8.0 mH. Calculate the magnetic flux through the coil when the current is 5.0 mA.

- Φ_B: magnetic flux
- $N\Phi_B$: magnetic flux linkage

$$\Phi_B = \frac{Li}{N} = \frac{8.0\times10^{-3}\times5.0\times10^{-3}}{400} = 1.0\times10^{-7}\,(Wb)$$

47. *Inductors in series.* Two inductors L_1 and L_2 are connected in series and are separated by a large distance so that the magnetic field of one cannot affect the other. (a) Show that the equivalent inductance is given by $L_{eq} = L_1 + L_2$. (*Hint:* Review the derivations for resistors in series and capacitors in series. Which is similar here?) (b) What is the generalization of (a) for N inductors in series?

$$\varepsilon_L = -L\frac{di}{dt}$$

Consider two inductors in series:

$$\varepsilon_{L_1} = -L_1\frac{di}{dt}; \varepsilon_{L_2} = -L_2\frac{di}{dt}$$

$$\varepsilon_{L_{eq}} = -L_{eq}\frac{di}{dt}; \ \varepsilon_{L_{eq}} = \varepsilon_{L_1} + \varepsilon_{L_2}$$

$$\Rightarrow L_{eq} = L_1 + L_2$$

55. A battery is connected to a series RL circuit at time t = 0. At what multiple of τ_L will the current be 0.100% less than its equilibrium value?

$$i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right)$$

The equilibrium value:

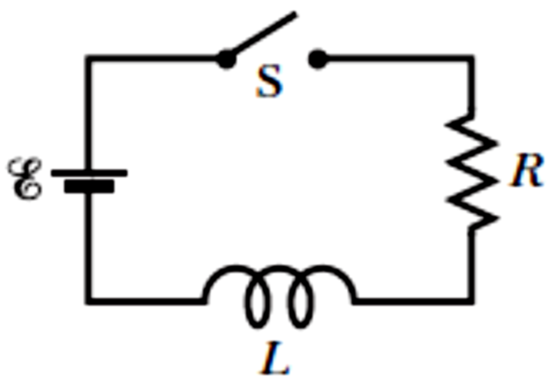
$$i_{equil} = \frac{\varepsilon}{R}$$

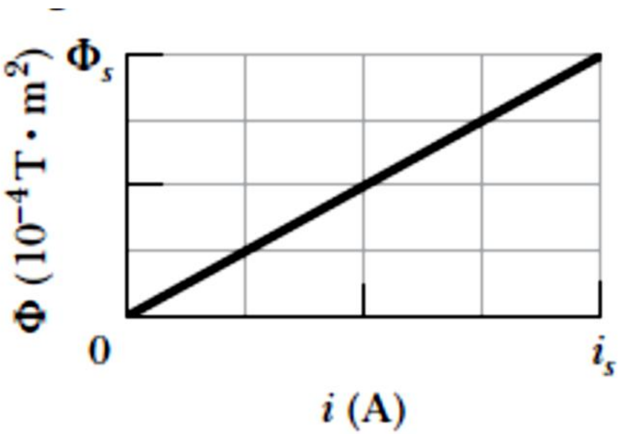
Time at which:

$$i = 0.999 i_{equil} \Rightarrow 0.999\frac{\varepsilon}{R} = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right)$$

$$t = 6.91\tau_L$$

56. The inductor has 25 turns and the ideal battery has an emf of 16 V. Figure b gives the magnetic flux Φ through each turn versus the current i through the inductor. The vertical axis scale is set by $\Phi s = 4.0 \times 10^{-4}$ T.m², and the horizontal axis scale is set by i_s = 2.00 A. If switch S is closed at time t = 0, at what rate *di/dt* will the current be changing at t = 1.5 τ_L?





68. A toroidal inductor with an inductance of 90.0 mH encloses a volume of 0.0200 m^3. If the average energy density in the toroid is 70.0 J/m^3, what is the current through the inductor?

73. Two coils are at fixed locations. When coil 1 has no current and the current in coil 2 increases at the rate 15.0 A/s, the emf in coil 1 is 25.0 mV.
(a) What is their mutual inductance?
(b)When coil 2 has no current and coil 1 has a current of 3.60 A, what is the flux linkage in coil 2?

79. The battery is ideal and ε=10 V, R_1= 5.0 Ω, R_2 = 10 Ω, and L= 5.0 H (as Figure). Switch S is closed at time t = 0. Just afterwards, what are (a) i_1, (b) i_2, (c) the current i_S through the switch, (d) the potential difference V_2 across resistor 2, (e) the potential difference V_L across the inductor, and (f) the rate of change di_2/dt? A long time later, what are (g) i_1, (h) i_2, (i) i_S ,(j) V_2, (k) V_L, and (l) di_2/dt?