Chapter 2:

Electric Energy and Capacitance

Homework 1:

1, 6, 18, 24, 35, 43, 64 (Page 648-653)

1. A particular 12 V car battery can send a total charge of 84 A.h through a circuit, from one terminal to the other. (a) How many coulombs of charge does this represent? (b) If this entire charge undergoes a change in electric potential of 12 V, how much energy is involved?

(a) In the previous lecture, we mentioned that the coulomb unit is derived from ampere for electric current i:

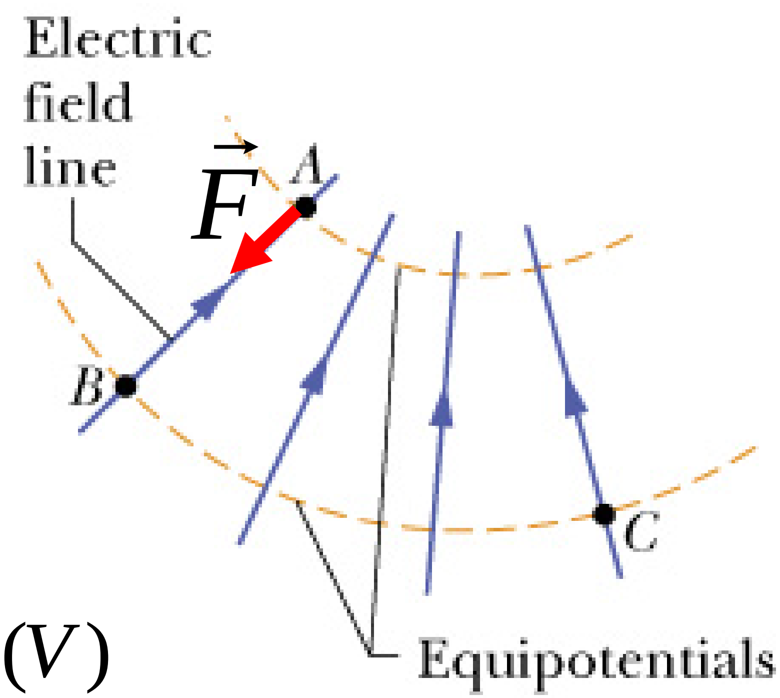
$$i = \frac{dq}{dt} \Rightarrow dq = idt$$

$$Q = 84(C/s) \times 3600(s) = 3 \times 10^5 (C)$$

(b) Energy is computed by:

$$\Delta U = \Delta V \times Q = 12 \times 3 \times 10^5 = 3.6 \times 10^6 (J)$$

6. When an electron moves from A to B along an electric field, see the figure. The electric field does 4.78×10^{-19} J of work on it. What are the electric potential differences (a) $V_B - V_A$, (b) $V_C - V_A$, and (c) $V_C - V_B$?



(a) We have work done by the electric field:

$$W = -q\Delta V$$

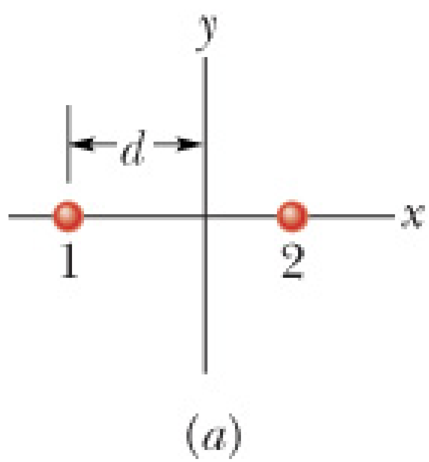
$$W = -(-e)(V_B - V_A)$$

$$V_B - V_A = \frac{W}{e} = \frac{4.78\times10^{-19}}{1.6\times10^{-19}} = 3.0(V)$$

(b) $$V_C - V_A = V_B - V_A = 3.0(V)$$

(c) $V_C - V_B = 0$: on the same equipotential

18. Two charged particles are shown in Figure a. Particle 1, with charge q_1, is fixed in place at distance d. Particle 2, with charge q_2, can be moved along the x axis. Figure b gives the net electric potential V at the origin due to the two particles as a function of the x coordinate of particle 2. The plot has an asymptote of $V = 5.92 \times 10^{-7}$ V as $x \to \infty$. What is q_2 in terms of e?



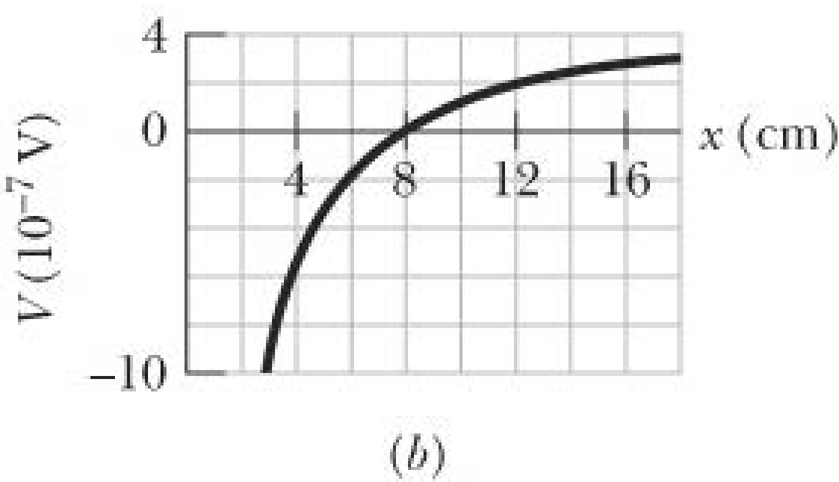
(a)

Potential due to a point charge: $V = k\dfrac{q}{r}$

Potential at the origin (O) due to q_1 and q_2:

$$V_O = k\frac{q_1}{d} + k\frac{q_2}{x}$$

$$V_{O,x=\infty} = k\frac{q_1}{d} = 5.92\times10^{-7}\,(V)$$



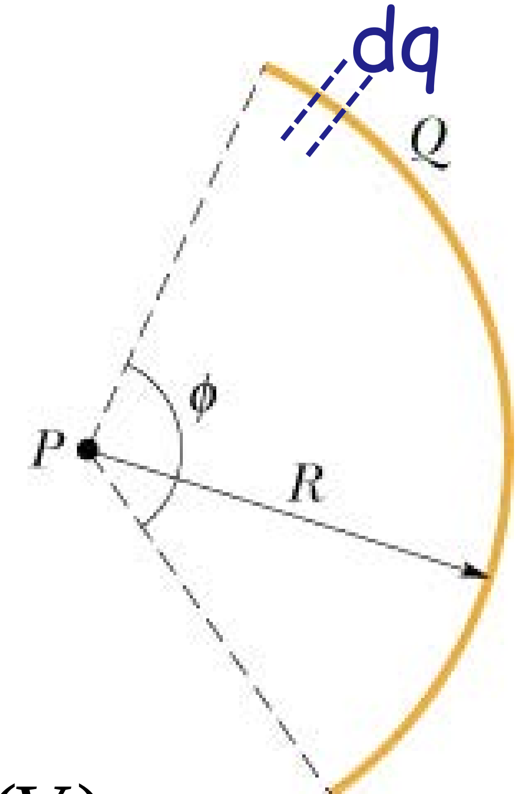
(b)

At x = 8 cm, $V_O = 0$:

$$V_{O,x=8} = V_{O,x=\infty} + k\frac{q_2}{x}$$

$$q_2 = -\frac{V_{O,x=\infty}\,x}{k} = -\frac{5.92\times10^{-7}\times0.08}{8.99\times10^{9}} = -5.27\times10^{-18}\,(C) \text{ or } -33e$$

24. The figure shows a plastic rod having a uniformly distributed charge Q = -28.9 pC has been bent into a circular arc of radius R = 3.71 cm and central angle $\Phi = 120^0$. With V=0 at infinity, what is the electric potential at P, the center of curvature of the rod?



Consider potential at P due to an element dq:

$$dV = k\frac{dq}{R}$$

$$V = \int k\frac{dq}{R} = k\frac{Q}{R}$$

$$V = \frac{8.99\times10^{9}\times(-28.9\times10^{-12})}{3.71\times10^{-2}} = -7.0(V)$$

35. The electric potential at points in an xy plane is given by $V = (2\ \text{V/m}^2)x^2 - (3\ \text{V/m}^2)y^2$. In unit vector notation, what is the electric field at the point (3.0 m, 2.0 m)?

We have:

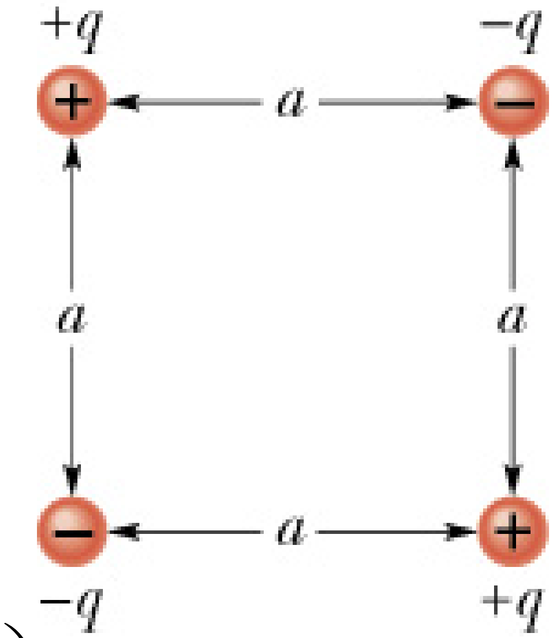
$$\vec{E} = -\nabla V$$

$$E_x = -\frac{\partial V}{\partial x}; E_y = -\frac{\partial V}{\partial y}$$

$$E_x = -4x = -12(V/m); E_y = 6y = 12(V/m)$$

$$\vec{E} = -12(V/m)\hat{i} + 12(V/m)\hat{j}$$

43. How much work is required to set up the arrangement of the figure below if q = 2.3 pC, a = 64 cm, and the particles are initially infinitely far apart and at rest?



We have 4 charges, so we have N = 6 pairs:

$$N = \frac{n(n-1)}{2}$$

$$W_{\text{applied}} = U_{\text{system}}$$

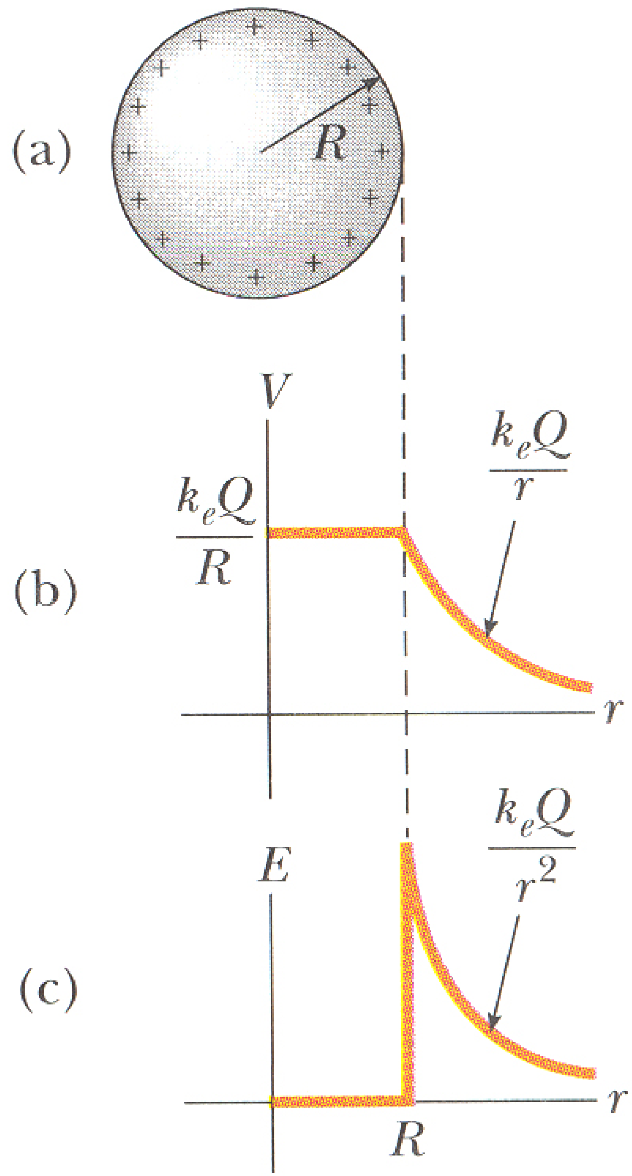
$$U_{\text{system}} = kq^2\left(-\frac{1}{a} - \frac{1}{a} + \frac{1}{a\sqrt{2}} - \frac{1}{a} + \frac{1}{a\sqrt{2}} - \frac{1}{a}\right)$$

$$U_{\text{system}} = \frac{2kq^2}{a}\left(\frac{1}{\sqrt{2}} - 2\right)$$

Note: q = 2.3 pC = 2.3×10^{-12} C; a = 64 cm = 0.64 m

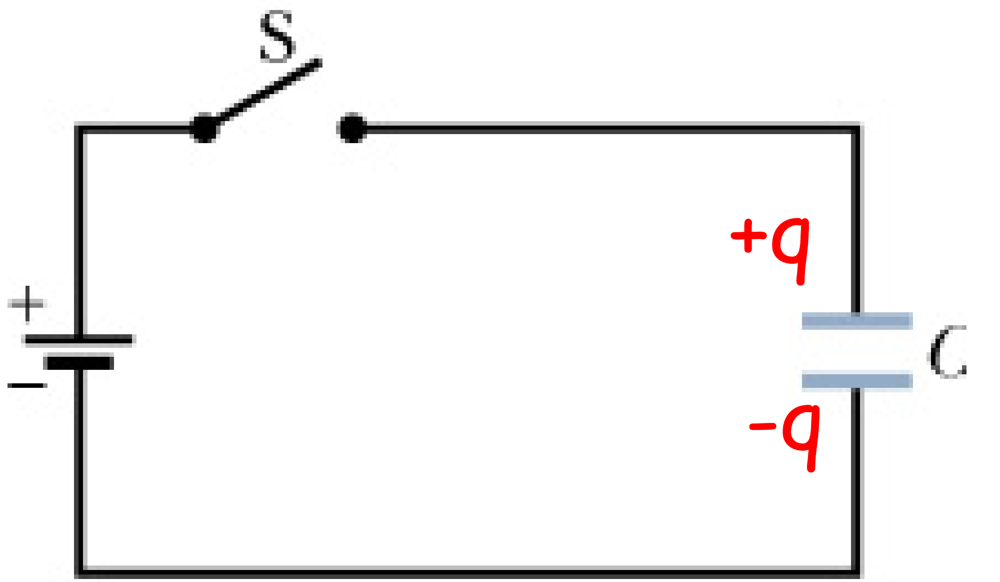
64. A hollow metal sphere has a potential of +300 V with respect to ground (defined to be at V = 0) and a charge of 5.0×10^{-9} C. Find the electric potential at the center of the sphere.

V = constant = +300 V throughout the entire conductor, this is valid for solid and hollow metal spheres.



Homework 2:
2, 6, 11, 16, 26, 31, 33, 42, 48, 51 (Page 676-680)

2. The capacitor in the figure below has a capacitance of 30 μF and is initially uncharged. The battery provides a potential difference of 120 V. After switch S is closed, how much charge will pass through it?

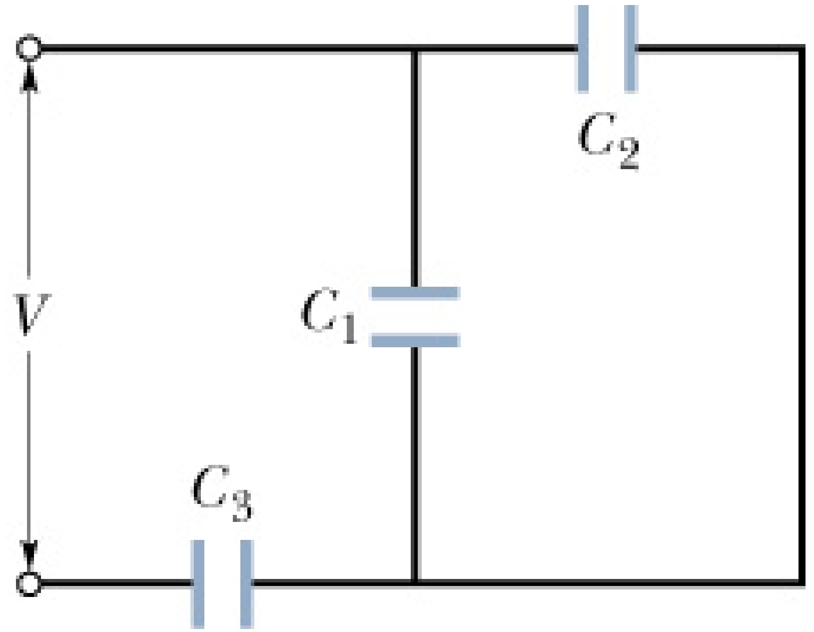


S is closed, the charge on the capacitor plates is:

$$q = CV$$

$$q = 30\times10^{-6}\times120 = 3.6\times10^{-3}(C)$$

11. In the figure below, find the equivalent capacitance of the combination. Assume that C_1 = 10.0 μF, C_2 = 5.0 μF, and C_3 = 4.0 μF



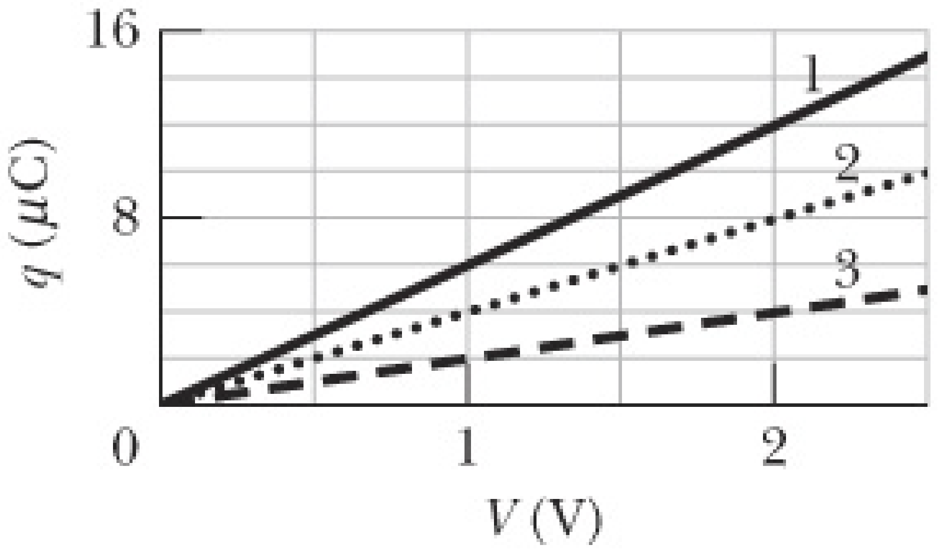
C_1 and C_2 are in parallel, the equivalent capacitance :

$$C_{12} = C_1 + C_2 = 15(\mu F)$$

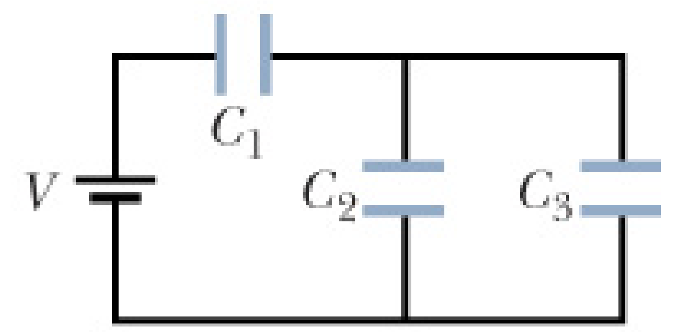
C_{12} and C_3 in series:

$$C_{123} = \frac{C_{12}C_3}{C_{12} + C_3} = \frac{15 \times 4}{15 + 4} = 3.16(\mu F)$$

16. Plot 1 in Figure a gives the charge q that can be stored on capacitor 1 versus the electric potential V set up across it. Plots 2 and 3 are similar plots for capacitors 2 and 3, respectively. Figure b shows a circuit with those three capacitors and a 10.0 V battery. What is the charge stored on capacitor 2 in that circuit?



(a)



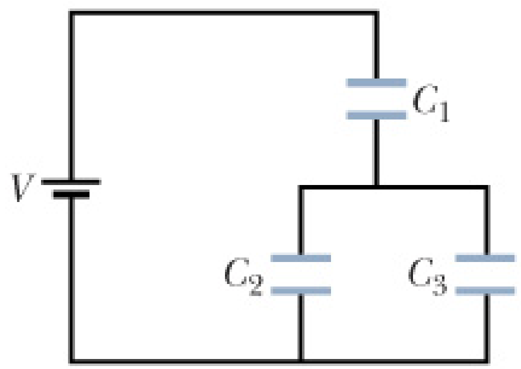
(b)

$$C_1 = \frac{q_1}{V_1} = \frac{12(\mu C)}{2(V)} = 6\mu F; C_2 = \frac{q_2}{V_2} = \frac{8(\mu C)}{2(V)} = 4\mu F; C_3 = \frac{q_3}{V_3} = \frac{4(\mu C)}{2(V)} = 2\mu F$$

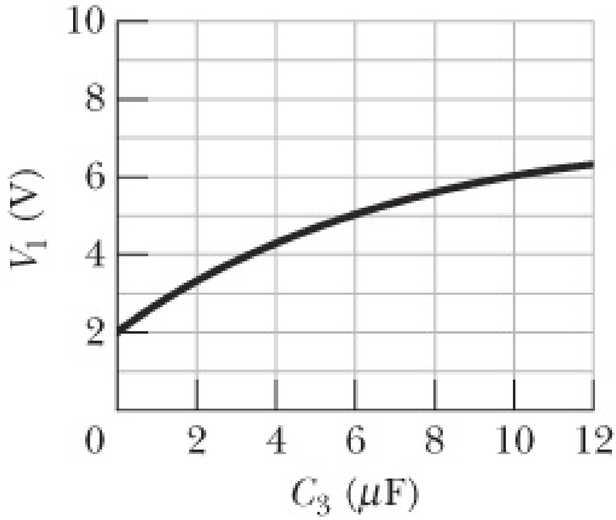
$$C_{123} = 3(\mu F)$$

$$V_1 = \frac{q}{C_1} = \frac{C_{123}V}{C_1} = \frac{1}{2}10 = 5(\text{V}) \Rightarrow q_2 = C_2V_2 = 4\mu\text{F} \times 5\text{V} = 20\mu\text{C}$$

26. Capacitor 3 in Figure a is a variable capacitor (its capacitance C_3 can be varied). Figure b gives the electric potential V_1 across capacitor 1 versus C_3. Electric potential V_1 approaches an asymptote of 8 V as $C_3 \rightarrow \infty$. What are (a) the electric potential V across the battery, (b) C_1, and (c) C_2?



(a)



(b)

(a) When $C_3 \rightarrow \infty$, $C_{123} = C_1$; so, $V = V_1 = 8$ V

(b)

$$C_{123} = \frac{C_1 C_{23}}{C_1 + C_{23}} = \frac{C_1(C_2 + C_3)}{C_1 + C_2 + C_3};$$

$$V_1 = \frac{q}{C_1} = \frac{C_{123}V}{C_1} = \frac{C_2 + C_3}{C_1 + C_2 + C_3}V$$

- **At C_3 = 0, V_1 = 2 V:**

$$C_1 = 3C_2$$

- **At C_3 = 6 μF, V_1 = 5 V:**

$$V_1 = \frac{C_2 + 6}{3C_2 + C_2 + 6}8 = 5$$

$$C_2 = 1.5\mu F; C_1 = 4.5\mu F$$

33. A charged isolated metal sphere of diameter 10 cm has a potential of 8000 V relative to V = 0 at infinity. Calculate the energy density in the electric field near the surface of the sphere.

In a general case, the energy density is computed by:

$$u = \frac{1}{2}\varepsilon_0 E^2$$

For a charged isolated metal sphere:

$$u = \frac{1}{2}\varepsilon_0 \left(\frac{V}{R}\right)^2 = \frac{1}{2} 8.85 \times 10^{-12} \left(\frac{8000}{0.05}\right)^2 = 0.113 (\mathrm{J/m^3})$$

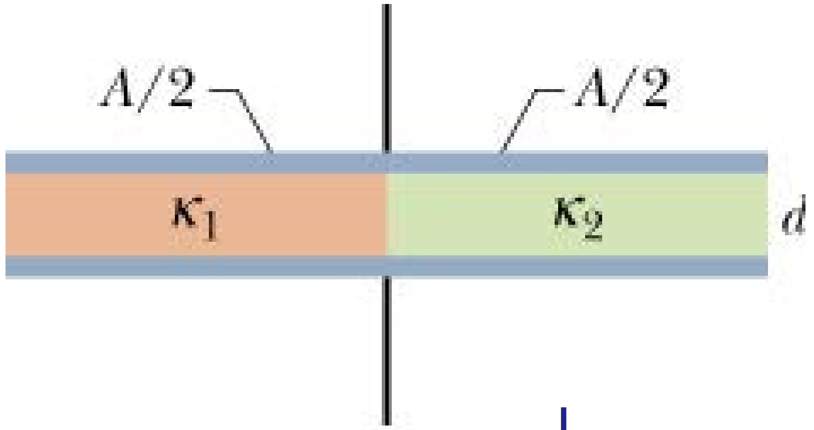
42. A parallel-plate air-filled capacitor has a capacitance of 50 pF: (a) If each of its plates has an area of 0.30 m^2, what is the separation? (b) If the region between the plates is now filled with material having κ = 5.6, what is the capacitance?

(a) For parallel-plate capacitors: $C = \dfrac{\varepsilon_0 A}{d}$

$$d = \frac{\varepsilon_0 A}{C} = \frac{8.85\times 10^{-12}\times 0.30}{50\times 10^{-12}} = 5.3\times 10^{-2}(m) = 5.3(cm)$$

(b) With a dielectric: $C' = \kappa C = 5.6\times 50 = 280(pF)$

48. The figure below shows a parallel-plate capacitor with a plate area A = 5.56 cm^2 and separation d = 5.56 mm. The left half of the gap is filled with material of dielectric constant κ_1 = 7.00; the right half is filled with material of dielectric constant κ_2 = 12.0. What is the capacitance?

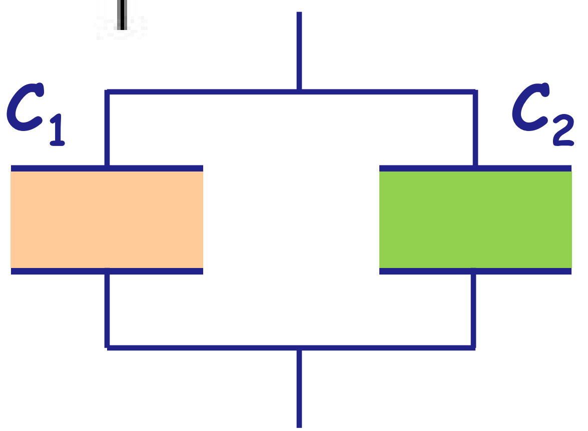


Their configuration is equivalent to a combination of **two capacitors in parallel** with dielectrics κ_1 and κ_2, respectively

$$C_0 = \frac{\varepsilon_0 (A/2)}{d} =$$

$$= \frac{8.85\times10^{-12}\times5.56\times10^{-4}}{2\times5.56\times10^{-3}} = 4.43\times10^{-13}(F)$$

$$C_0 = 0.443 \text{ pF}$$



equivalent capacitor

$$C_{equivalent} = C_1 + C_2 = (\kappa_1 + \kappa_2)C_0 = 8.42(\text{pF})$$