

## Chapter 1:

# Electric Charges, Electric Fields, Gauss' Law

## Homework 1:

3, 6, 9, 10, 16, 23, 33 (page 575-577)

3. What must be the distance between point charge  $q_1 = 26.0 \mu\text{C}$  and point charge  $q_2 = -47 \mu\text{C}$  for the electrostatic force between them to have a magnitude of  $5.70 \text{ N}$ ?

$$F = k \frac{q_1 q_2}{r^2}$$

$$r = \sqrt{k \frac{|q_1| |q_2|}{F}}$$

$$1 \mu\text{C} = 10^{-6} \text{ C}$$

$$r = \sqrt{\frac{8.99 \times 10^9 (26 \times 10^{-6})(47 \times 10^{-6})}{5.7}} = 1.39 \text{ (m)}$$

6. Two equally charged particles are held  $3.2 \times 10^{-3}$  m apart and then released from rest. The initial acceleration of the first particle is observed to be  $6.0 \text{ m/s}^2$  and that of the second to be  $9.0 \text{ m/s}^2$ . If the mass of the first particle is  $6.3 \times 10^{-7}$  kg, what are (a) the mass of the second particle and (b) the magnitude of the charge of each particle.

$$F_{21} = m_1 a_1 \qquad F_{12} = m_2 a_2$$

$$F_{12} = F_{21} : \qquad m_2 = \frac{a_1}{a_2} m_1$$

$$F_{12} = F_{21} = k \frac{q_1 q_2}{r^2} = m_1 a_1$$

$$q = \sqrt{\frac{m_1 a_1 r^2}{k}} = \sqrt{\frac{6.3 \times 10^{-7} \times 6 \times (3.2 \times 10^{-3})^2}{8.99 \times 10^9}} = 6.6 \times 10^{-11} \text{ (C)}$$

$$q = 66 \text{ (pC)}$$

9. Two identical conducting spheres, fixed in place, attract each other with an electrostatic force of 0.108 N when their center-to-center separation is 50.0 cm. The spheres are then connected by a thin conducting wire. When the wire is removed, the spheres repel each other with an electrostatic force of 0.036 N of the initial charges, with **a positive net charge**, what was (a) the negative charge on one of them and (b) the positive charge on the other?

Using the shell theorem:

Before: 
$$F_1 = -k \frac{q_1 q_2}{r^2} \Rightarrow q_1 q_2 = -3.0 \times 10^{-12} \quad (C)$$

After: the net charge is positive

$$F_2 = k \frac{q^2}{r^2} = k \frac{\left( \frac{q_1 + q_2}{2} \right)^2}{r^2} \Rightarrow q_1 + q_2 = 2.0 \times 10^{-6} \quad (C)$$

$$q_1^2 - 2.0 \times 10^{-6} q_1 - 3.0 \times 10^{-12} = 0$$

The solutions:

1<sup>st</sup>:  $q_1 = 3.0 \times 10^{-6} (C) \Rightarrow q_2 = -1.0 \times 10^{-6} (C)$

2<sup>nd</sup>:  $q_1 = -1.0 \times 10^{-6} (C) \Rightarrow q_2 = 3.0 \times 10^{-6} (C)$

Note: if the net charge is negative

$$q_1 + q_2 = -2.0 \times 10^{-6} (C)$$

the solutions should be:

$$q_1(\text{or } q_2) = 1.0 \times 10^{-6} (C) \Rightarrow q_2(\text{or } q_1) = -3.0 \times 10^{-6} (C)$$

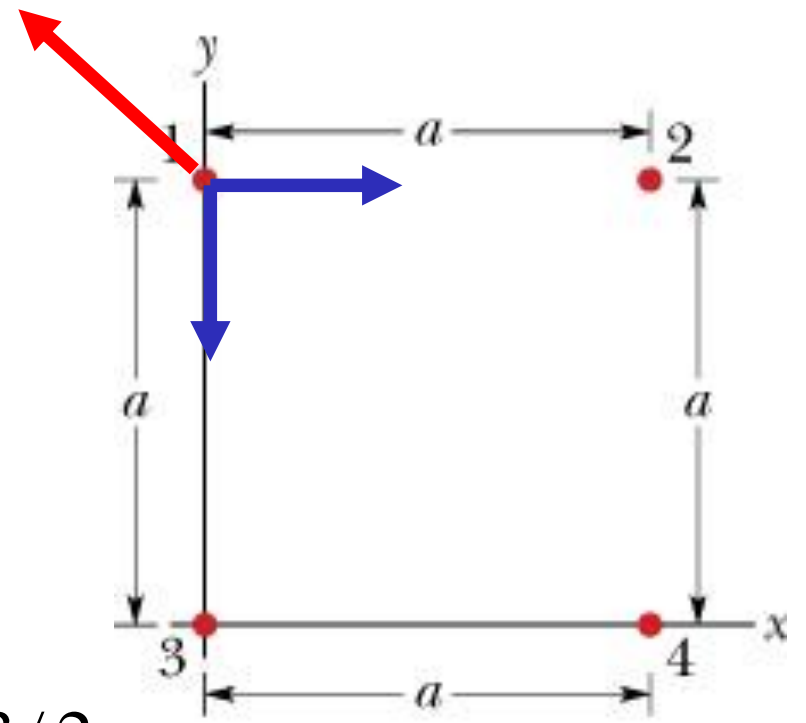
**10.** In the figure as shown, four particles form a square. The charges are  $q_1 = q_4 = Q$  and  $q_2 = q_3 = q$ . (a) What is  $Q/q$  if the net electrostatic force on particles 1 and 4 is zero? (b) Is there any value of  $q$  that makes the net electrostatic force on each of the four particles zero? explain.

(a)  $q_1$  &  $q_4$  have the same sign, all three forces act on  $q_1$  as shown

$$\vec{F}_{41} = \vec{F}_{21} + \vec{F}_{31}$$

$$F_{41} = \sqrt{2}F_{21}$$

$$k \frac{Q^2}{(a\sqrt{2})^2} = \sqrt{2}k \frac{|qQ|}{(a)^2} \Rightarrow \left| \frac{Q}{q} \right| = 2^{3/2} = 2.83$$



(b) if the net force acting on particle 3 is also zero:  $\left| \frac{q}{Q} \right| = 2.83$   
 → this is inconsistent with (a), so the answer is NO

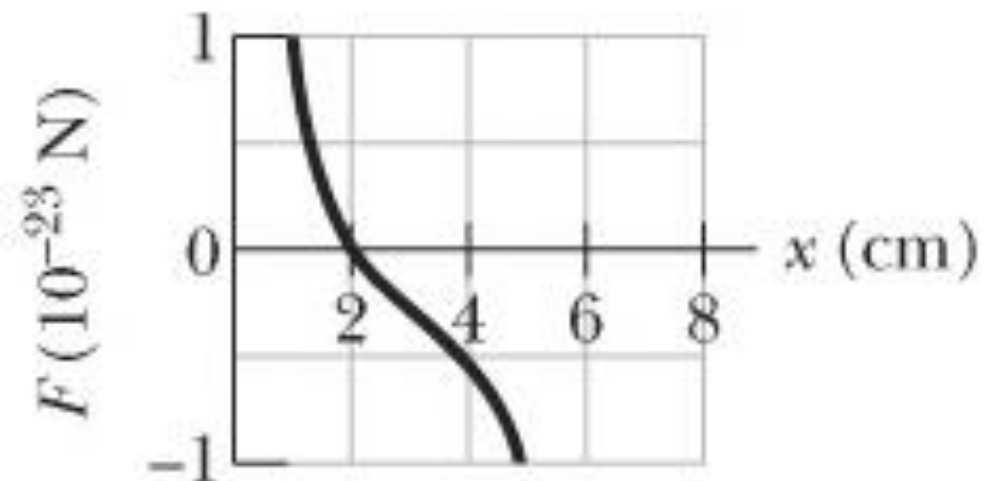
**16.** See the figure as shown, particle 1 (of charge  $q_1$ ) and particle 2 (of charge  $q_2$ ) are fixed in place on an  $x$  axis, 8.0 cm apart. Particle 3 (of charge  $q_3 = +6.0 \times 10^{-19}$  C) is to be placed on the line between particles 1 and 2 so that they produce a net electrostatic force  $F_{3,\text{net}}$  on it. The diagram gives the  $x$  component of that force versus the coordinate  $x$  at which particle 3 is placed. What are (a) the sign of charge  $q_1$  and (b) the ratio  $q_2/q_1$ ?

- at 2 cm:  $F_{\text{net}} = 0$  so 1 and 2 must have the same sign.

- when  $q_3$  approaches  $q_1$ ,  $F_{13}$  increases in magnitude.  $F_{\text{net}}$  increases in the positive  $x$  direction, so  $F_{13}$  is a repulsive force,  $q_1 > 0$

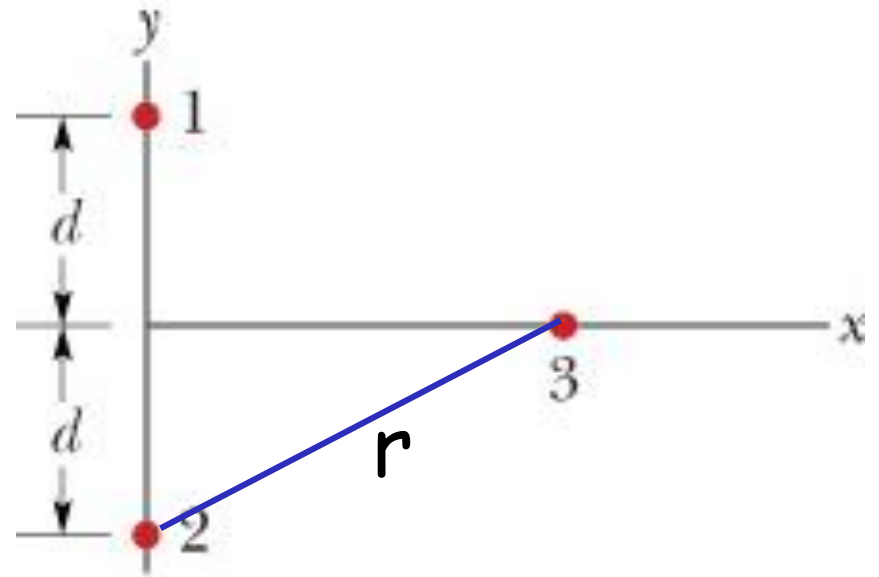
$$k \frac{q_1 q_3}{r_{13}^2} = k \frac{q_2 q_3}{r_{23}^2}$$

$$\frac{q_2}{q_1} = \left( \frac{r_{23}}{r_{13}} \right)^2 = \left( \frac{6}{2} \right)^2 = 9$$



**23.** See the figure, particles 1 and 2 of charge  $q_1 = q_2 = +3.2 \times 10^{-19} \text{ C}$  are on a y axis at distance  $d = 17.0 \text{ cm}$  from the origin. Particle 3 of charge  $q_3 = +6.4 \times 10^{-19} \text{ C}$  is moved gradually along the x axis from  $x = 0$  to  $x = +5.0 \text{ m}$ . At what values of  $x$  will the magnitude of the electrostatic force on the third particle from the other two particles be (a) minimum and (b) maximum? what are the (c) minimum and (d) maximum magnitudes?

The net force acting on particle 3:



$$F_{net} = 2 \times k \frac{qq_3}{r^2} \times \cos \theta$$

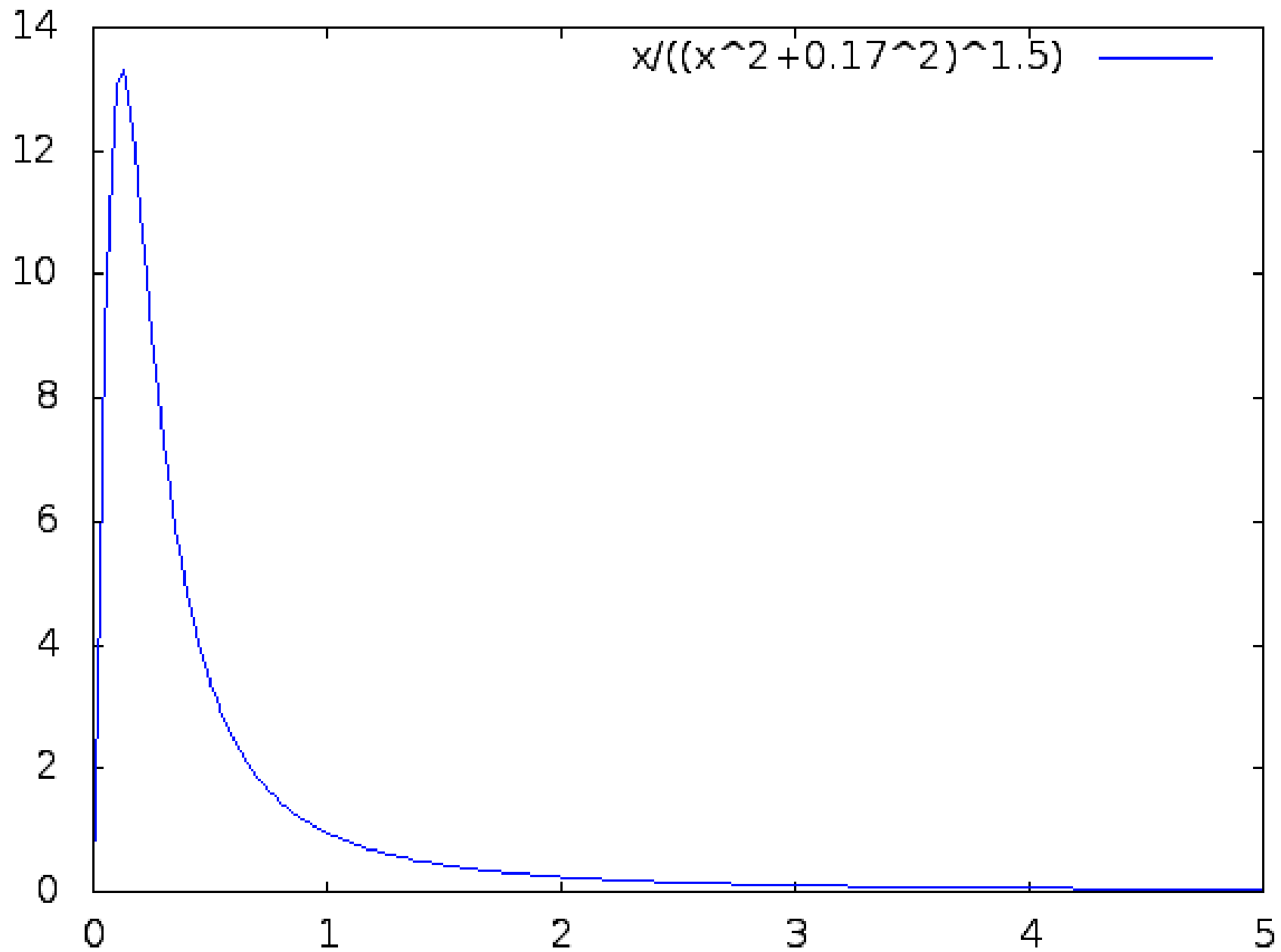
$$r = \sqrt{x^2 + d^2}; \cos \theta = \frac{x}{r}$$

$$F_{net} = 2kqq_3 \frac{x}{(x^2 + d^2)^{3/2}} \Rightarrow x = 0 : \text{minimum}$$

$$F'_{net} = 0 \Rightarrow d^2 - 2x^2 = 0 \Rightarrow x = d / \sqrt{2} : \text{maximum}$$



<http://www.function-grapher.com/index.php>



33. Calculate the number of coulombs of positive charge in 250 cm<sup>3</sup> of (neutral) water. (Hint: A hydrogen atom contains one proton; an oxygen atom contains eight protons)

The mass of the sample:

$$m = \rho V = 1 \times 250 = 250(\text{g})$$

The number of moles:

$$n = \frac{m}{M_{\text{molar}}} = \frac{250}{18} = 13.9$$

The positive charge:

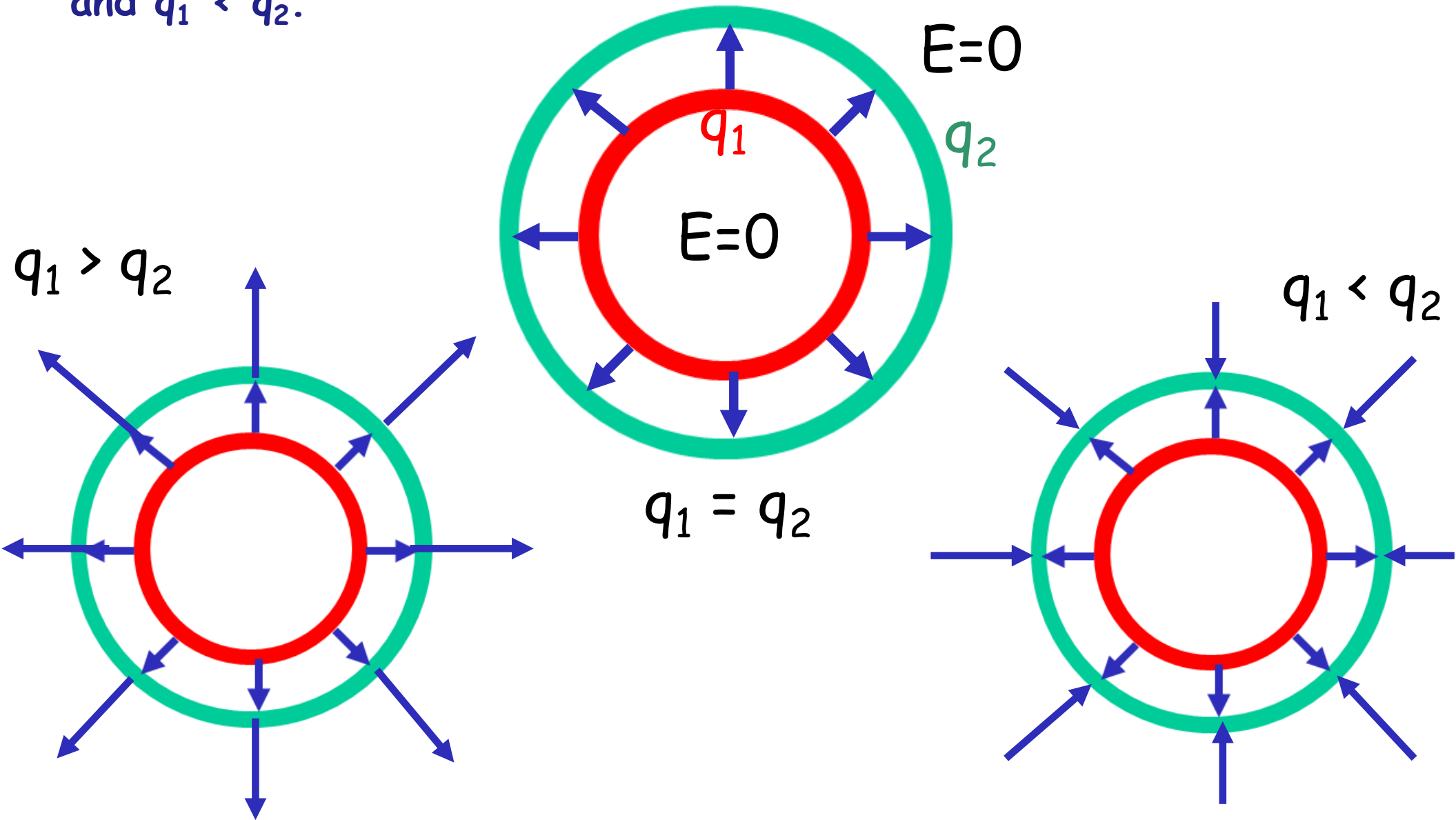
$$Q = nN_A q$$

$$= 13.9 \times 6.023 \times 10^{23} \times 10 \times 1.6 \times 10^{-19} = 1.34 \times 10^7 (\text{C})$$

## Homework 2:

1, 5, 14, 15, 19, 23, 27, 31, 35, 44, 54, 56, 59  
(page 598-603)

1. Sketch qualitatively the electric field lines both between and outside two concentric conducting spherical shells when a uniform positive charge  $q_1$  is on the inner shell and a uniform negative charge  $-q_2$  is on the outer. Consider the cases  $q_1 > q_2$ ,  $q_1 = q_2$ , and  $q_1 < q_2$ .

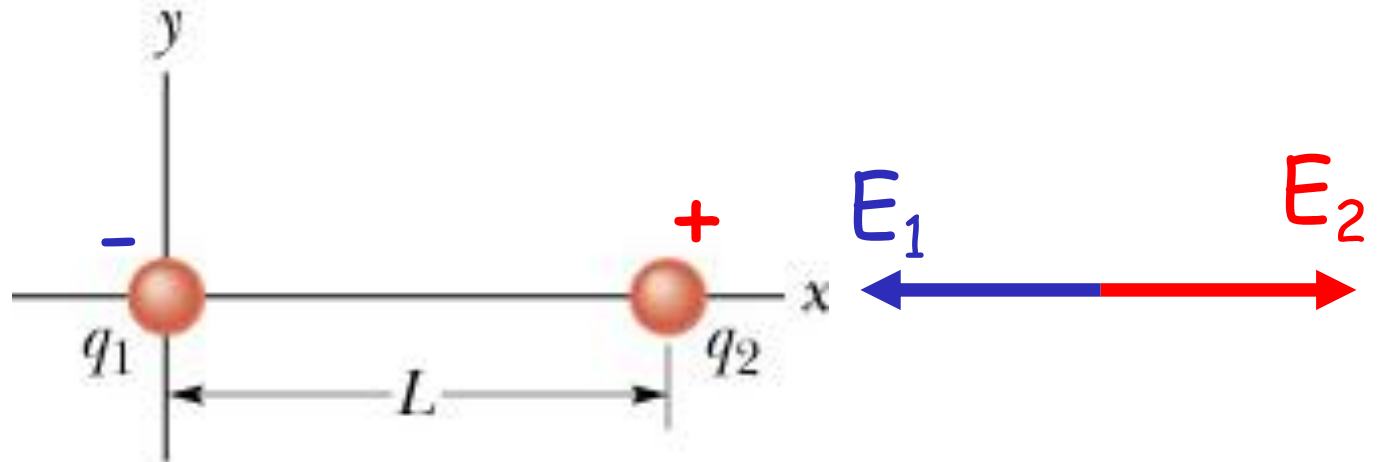


5. What is the magnitude of a point charge whose electric field 50 cm away has the magnitude 2.0 N/C?

$$E = \frac{k|q|}{r^2}$$

$$|q| = \frac{Er^2}{k} = \frac{2 \times 0.5^2}{8.99 \times 10^9} = 5.6 \times 10^{-11} \text{ (C)}$$

**14.** See the figure, particle 1 of charge  $q_1 = -4.0q$  and particle 2 of charge  $q_2 = +2.0q$  are fixed to an x axis. (a) As a multiple of distance  $L$ , at what coordinate on the axis is the net electric field of the particles zero? (b) Sketch the net electric field lines.



$$E_1 = \frac{k|q_1|}{x^2} = E_2 = \frac{k|q_2|}{(x-L)^2}$$

$$\frac{(x-L)^2}{x^2} = \frac{2}{4} \Rightarrow x = 3.41L$$

**15.** The three particles are fixed in place and have charges  $q_1 = q_2 = +e$  and  $q_3 = +2e$ . Distance  $a = 6.0 \mu\text{m}$ . What are the (a) magnitude and (b) direction of the net electric field at point P due to the particles?

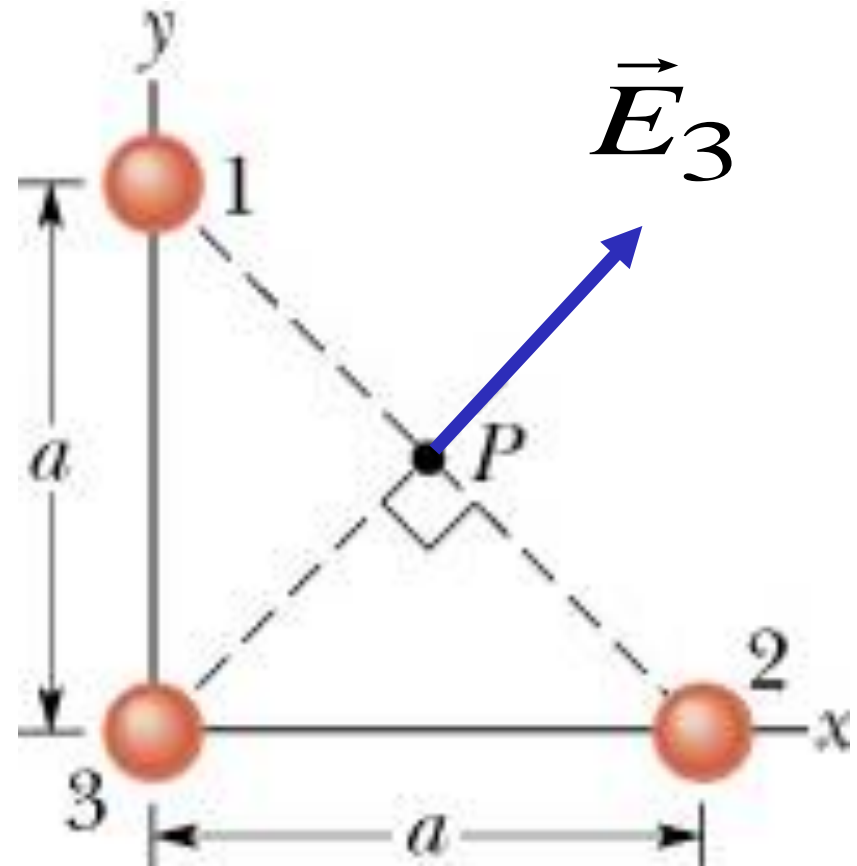
$$\vec{E}_1 + \vec{E}_2 = 0$$

$$\vec{E}_{net} = \vec{E}_3$$

$$E_3 = k \frac{2e}{OP^2}; OP = \frac{a\sqrt{2}}{2}$$

$$E_3 = 8.99 \times 10^9 \frac{4 \times 1.6 \times 10^{-19}}{6.0^2 \times 10^{-12}}$$

$$E_3 = 160(N/C); (\vec{E}_3, Ox) = 45^\circ$$



19. The figure shows an electric dipole. What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the dipole's electric field at point P, located at distance  $r \gg d$ ?

$$E_{net} = 2k \frac{q}{r^2} \cos \theta = \frac{kqd}{r^3}$$

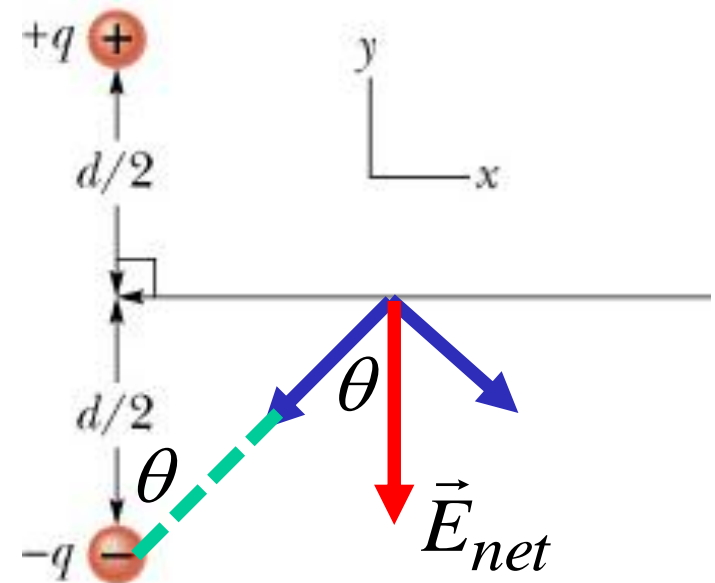
$p = qd$  : electric dipole moment

$$E_{net} = \frac{kp}{\left[ \left( \frac{d}{2} \right)^2 + x^2 \right]^{3/2}}$$

$$E_{net} = \frac{kp}{x^3}$$

• If  $x \gg d$ :

•  $\vec{E}_{net}$  points in the negative direction of the y axis



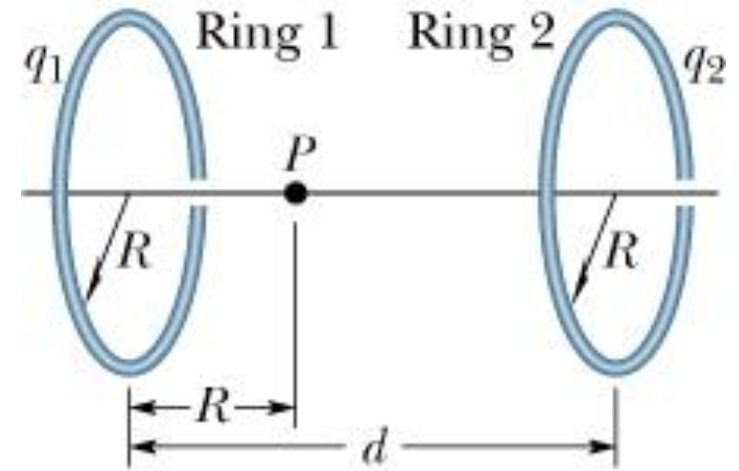


23. The figure shows two parallel nonconducting rings with their central axes along a common line. Ring 1 has uniform charge  $q_1$  and radius  $R$ ; ring 2 has uniform charge  $q_2$  and the same radius  $R$ . The rings are separated by distance  $d = 3.0R$ . The net electric field at point  $P$  on the common line, at distance  $R$  from ring 1, is zero. What is the ratio  $q_1/q_2$ ?

$$E = \frac{qz}{4\pi\epsilon_0(z^2 + R^2)^{3/2}}$$

$$\frac{kq_1R}{(R^2 + R^2)^{3/2}} = \frac{kq_2 2R}{(4R^2 + R^2)^{3/2}}$$

$$\frac{q_1}{q_2} = \frac{4\sqrt{2}}{5\sqrt{5}} = 0.51$$



**Note:**  $q_1$  and  $q_2$  must have the same sign to produce a net electric field equal to zero

**27.** Two curved plastic rods, one of charge  $+q$  and the other of charge  $-q$ , form a circle of radius  $R = 8.5$  cm in an  $xy$  plane. The  $x$  axis passes through both of the connecting points, and the charge is distributed uniformly on both rods. If  $q = 15.0$  pC, what are the (a) magnitude and (b) direction (relative to the positive direction of the  $x$  axis) of the electric field produced at  $P$ , the center of the circle?

Consider a differential charge  $dq$ :

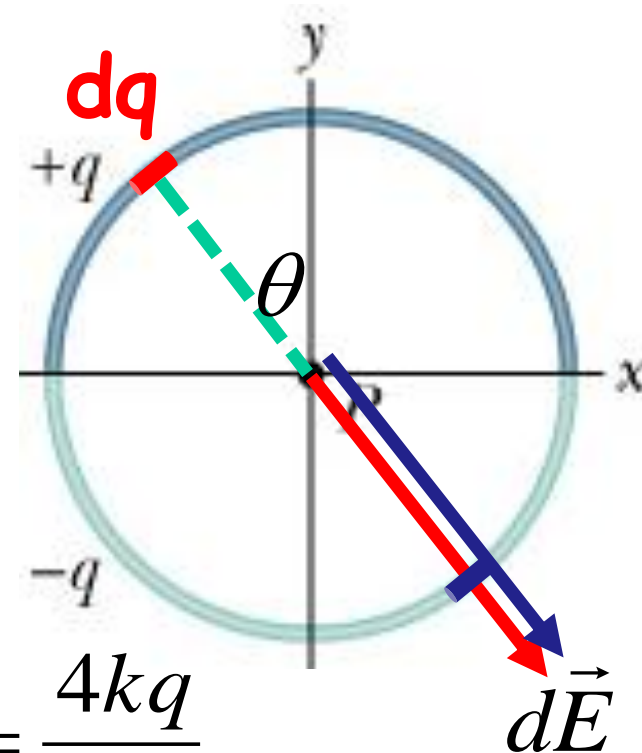
$$dq = \lambda ds = \lambda(Rd\theta)$$

$$dE_{1\text{rod}} = k \frac{(\lambda ds)}{R^2} \cos \theta = \frac{k\lambda}{R} \cos \theta d\theta$$

For two rods:  $dE_{2\text{rods}} = 2 \times \frac{k\lambda}{R} \cos \theta d\theta$

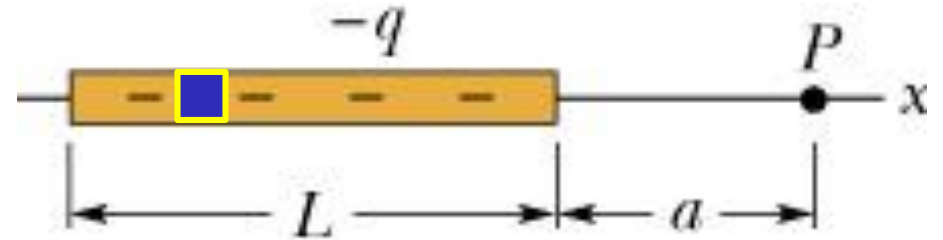
$$E_{2\text{rods}} = 2 \times \frac{k\lambda}{R} \int_{-90}^{90} \cos \theta d\theta = \frac{4k\lambda}{R} = \frac{4kq}{\pi R^2}$$

$$E_{2\text{rods}} = \frac{4 \times 8.99 \times 10^9 \times 15 \times 10^{-12}}{3.14 \times 8.5^2 \times 10^{-4}} = 23.8 (N/C)$$



31. A nonconducting rod of length  $L=8.15$  cm has charge  $-q=-4.23$  fC uniformly distributed along its length. (a) What is the linear charge density of the rod? What are the (b) magnitude and (c) direction (relative to the positive direction of the  $x$  axis) of the electric field produced at point  $P$ , at  $a = 12.0$  cm from the rod? What is the electric field magnitude at  $a = 50$  m by (d) the rod and (e) a particle of charge  $-q = -4.23$  fC that replaces the rod?

(a)



(b)

$$\lambda = \frac{-q}{L} = \frac{-4.23 \times 10^{-15}}{8.15 \times 10^{-2}} = -0.519 \times 10^{-13} \text{ (C/m)}$$

$$dE_x = k \frac{\lambda dx}{(L + a - x)^2}$$

$$E_x = -k \frac{q}{a(L + a)} = -1.6 \times 10^{-3} \text{ (N/C)}$$

(c) the negative direction of the  $x$  axis

$L \ll a$ :

$$E_x = -k \frac{q}{a^2}$$

$$E_x = -1.5 \times 10^{-8} \text{ (N / C)}$$

**(d) for a distant point, the rod acts like a point charge, so the electric field of the point charge is the same as that of the rod:**

$$E_x = -1.5 \times 10^{-8} \text{ (N / C)}$$

35. At what distance along the central perpendicular axis of a uniformly charged plastic disk of radius 0.6 m is the magnitude of the electric field equal to one-half the magnitude of the field at the center of the surface of the disk?

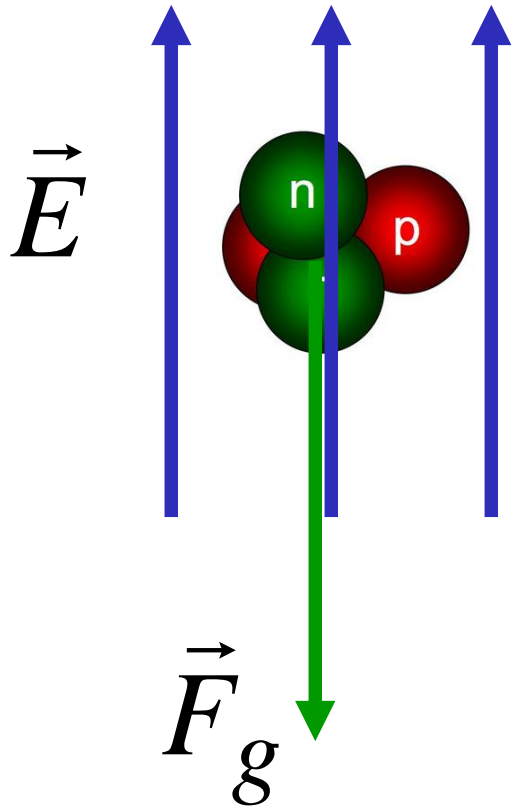
$$E_z = \frac{\sigma}{2\epsilon_0} \left( 1 - \frac{z}{\sqrt{z^2 + R^2}} \right)$$

At the center (very close to the center):

$$E_z = \frac{1}{2} E_c :$$
$$E_c = \frac{\sigma}{2\epsilon_0}$$
$$1 - \frac{z}{\sqrt{z^2 + R^2}} = \frac{1}{2}$$

$$z = R / \sqrt{3} = 0.35(m)$$

**44.** An alpha particle (the nucleus of a helium atom) has a mass of  $6.64 \times 10^{-27}$  kg and a charge of  $+2e$ . What are the (a) magnitude and (b) direction of the electric field that will balance the gravitational force on the particle?



$$qE = mg$$

$$E = \frac{mg}{2e}$$

54. An electron is shot at an initial speed of  $v_0 = 4.0 \times 10^6$  m/s, at angle  $\theta_0 = 40^\circ$  from an x axis. It moves through a uniform electric field  $\vec{E} = (5.0 \text{ N/C})\hat{j}$ . A screen for detecting electrons is positioned parallel to the y axis, at distance  $x = 3.0$  m. In unit vector notation, what is the velocity of the electron when it hits the screen?

$$F_g \approx 9 \times 10^{-30} \text{ (N)}$$

$$F = 8 \times 10^{-19} \text{ (N)}$$

So, we ignore gravity

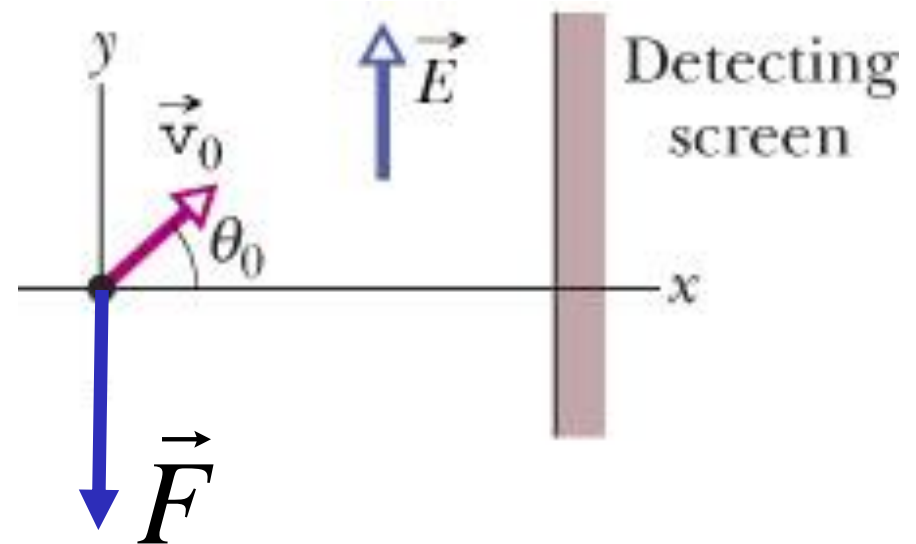
$$a_y = \frac{eE}{m} = \frac{1.6 \times 10^{-19} \times 5.0}{9.11 \times 10^{-31}}$$

$$= 8.78 \times 10^{11} \text{ (m/s}^2\text{)}$$

Time to hit the screen:  $t = \frac{x}{v_0 \cos \theta_0} = \frac{3}{4 \times 10^6 \cos 40^\circ} = 0.98 \times 10^{-6} \text{ (s)}$

$$v_y = v_{0y} - a_y t = v_0 \sin \theta - at = 1.71 \times 10^6 \text{ (m/s)}$$

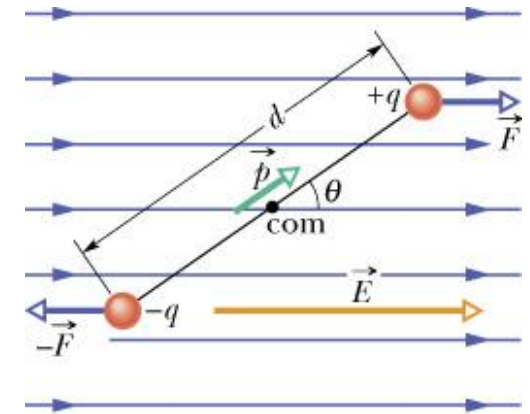
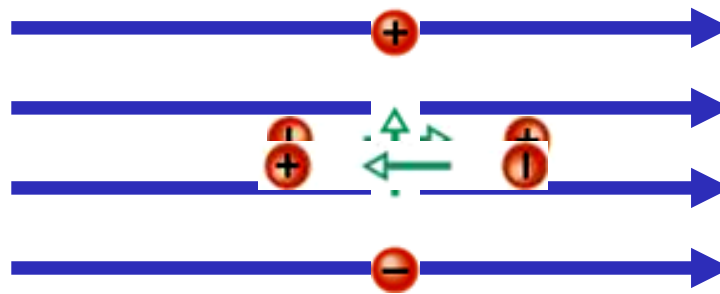
$$\vec{v} = 3.06 \times 10^6 \text{ (m/s)}\hat{i} + 1.71 \times 10^6 \text{ (m/s)}\hat{j}$$



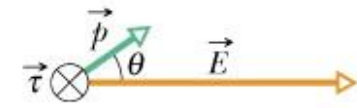
**56.** An electric dipole consists of charges  $+2e$  and  $-2e$  separated by  $0.85 \text{ nm}$ . It is in an electric field of strength  $3.4 \times 10^6 \text{ N/C}$ . Calculate the magnitude of the torque on the dipole when the dipole moment is (a) parallel to, (b) perpendicular to, and (c) antiparallel to the electric field.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = pE \sin \theta$$



(a)



(b)

(a)  $\tau_{net} = 0 \quad (\theta = 0)$

(b)  $\tau_{net} = 2edE = 2 \times 1.6 \times 10^{-19} \times 0.85 \times 10^{-9} \times 3.4 \times 10^6$

$$\tau_{net} = 9.3 \times 10^{-22} \text{ (N.m)}$$

(c)  $\tau_{net} = 0 \quad (\theta = 180^\circ)$



59. How much work is required to turn an electric dipole  $180^\circ$  in a uniform electric field of magnitude  $E = 46.0 \text{ N/C}$  if  $p = 3.02 \times 10^{-25} \text{ C.m}$  and the initial angle is  $64^\circ$ ?

$$U = -pE \cos \theta$$

$$W_{\text{applied}} = -W_{\vec{E}} = \Delta U$$

$$W_{\text{applied}} = U_{\pi+\theta_0} - U_{\theta_0} = -pE \cos(180^\circ + 64^\circ) + pE \cos 64^\circ$$

$$W_{\text{applied}} = 2pE \cos 64^\circ$$

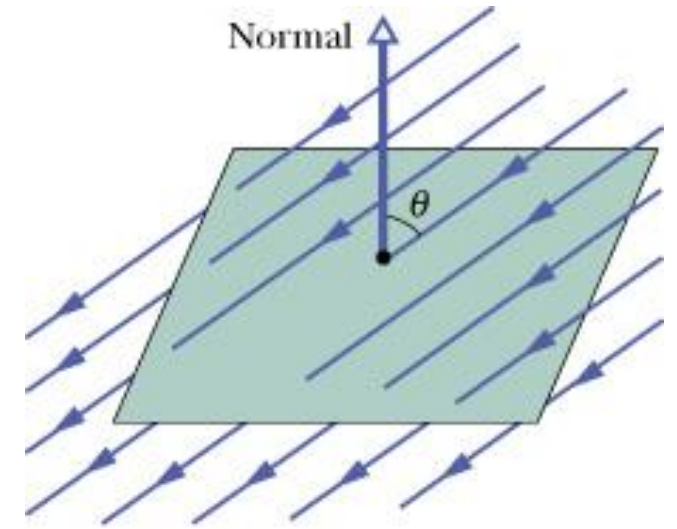
$$W_{\text{applied}} = 2 \times 3.02 \times 10^{-25} \times 46.0 \times 0.438$$

$$W_{\text{applied}} = 1.22 \times 10^{-23} \text{ (J)}$$

### **Homework 3:**

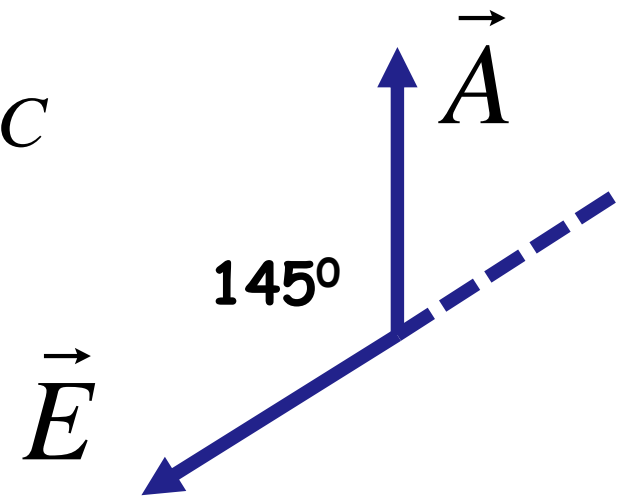
**1, 7, 14, 17, 21, 22, 36, 39, 43, 44, 51, 52**  
**(Page 622-626)**

1. The square surface as shown measures 3.2 mm on each side. It is immersed in a uniform electric field with  $E = 1800 \text{ N/C}$  and with field lines at an angle of  $\theta = 35^\circ$  with a normal to the surface. Calculate the electric flux through the surface.



$$\Phi = \vec{E} \cdot \vec{A} = EA \cos \theta = (1800 \text{ N/C}) \times (3.2 \times 10^{-3} \text{ m})^2 \cos(180^\circ - 35^\circ)$$

$$\Phi = -1.51 \times 10^{-2} \text{ Nm}^2 / \text{C}$$



7. A point charge of  $1.8 \mu\text{C}$  is at the center of a cubical Gaussian surface  $55 \text{ cm}$  on edge. What is the net electric flux through the surface?

Using Gauss's law:

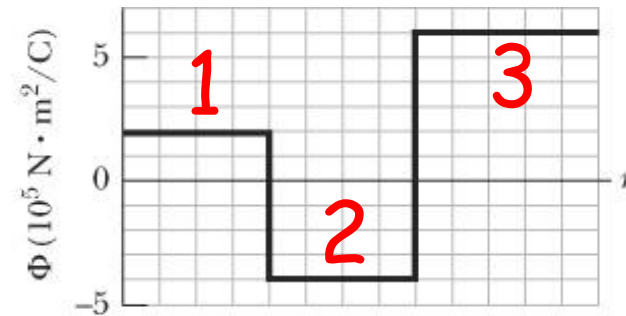
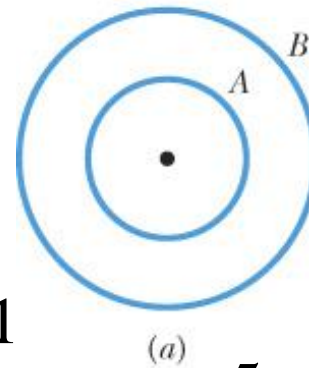
$$\epsilon_0 \Phi = q_{\text{enclosed}}$$

$$\Phi = \frac{q_{\text{enclosed}}}{\epsilon_0} = \frac{1.8 \times 10^{-6} \text{ C}}{8.85 \times 10^{-12} \text{ C}^2 / \text{Nm}^2} = 2.0 \times 10^5 \text{ Nm}^2 / \text{C}$$

14. A charged particle is suspended at the center of two concentric spherical shells that are very thin and made of nonconducting material. Figure a shows a cross section. Figure b gives the net flux  $\Phi$  through a Gaussian sphere centered on the particle, as a function of the radius  $r$  of the sphere. (a) What is the charge of the central particle? What are the net charges of (b) shell A and (c) shell B?

$$\epsilon_0 \Phi = q_{\text{enclosed}}$$

(a) For  $r < r_A$  (region 1):



$$q_{\text{enclosed}1} = q_{\text{particle}} = \epsilon_0 \Phi_1$$

$$q_{\text{particle}} = 8.85 \times 10^{-12} \times 2 \times 10^5 = 1.77 \times 10^{-6} \text{ (C)}$$

$$\approx 1.8 \text{ (}\mu\text{C)}$$

(b) For  $r_A < r < r_B$  (region 2):  $q_{\text{enclosed}2} = q_{\text{particle}} + q_A = \epsilon_0 \Phi_2$

$$\Phi_2 = -4 \times 10^5 \text{ (Nm}^2/\text{C)} \Rightarrow q_A = -5.3 \times 10^{-6} \text{ (C) or } -5.3 \mu\text{C}$$

(c) For  $r_B < r$  (region 3):  $\Phi_3 = 6 \times 10^5 \text{ (Nm}^2/\text{C)} \Rightarrow q_B$

17. A uniformly charged conducting sphere of 1.2 m diameter has a surface charge density of  $8.1 \mu\text{C}/\text{m}^2$ . (a) Find the net charge on the sphere. (b) What is the total electric flux leaving the surface of the sphere?

(a) charge = area x surface density

$$q = 4\pi r^2 \sigma = 4 \times 3.14 \times 0.6^2 \times 8.1 \times 10^{-6} = 3.7 \times 10^{-5} \text{ (C)}$$

(b) We choose a Gaussian surface covers whole the sphere, using Gauss' law:

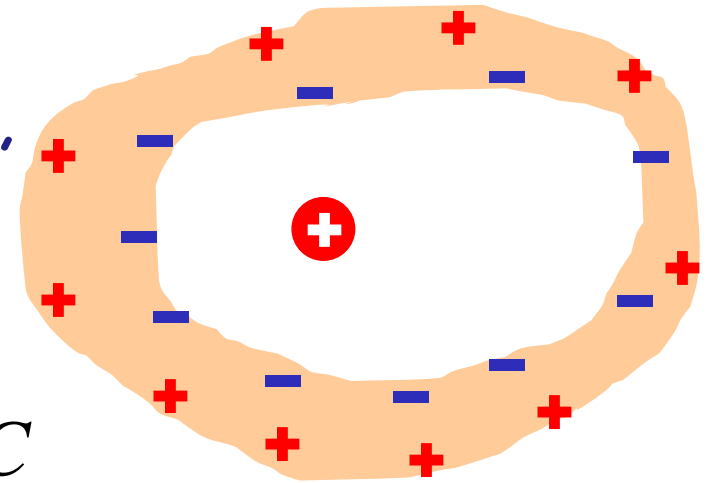
$$\Phi = \frac{q_{\text{enclosed}}}{\epsilon_0} = \frac{3.7 \times 10^{-5}}{8.85 \times 10^{-12}} = 4.2 \times 10^6 \text{ Nm}^2 / \text{C}$$

21. An isolated conductor of arbitrary shape has a net charge of  $+10 \times 10^{-6} \text{ C}$ . Inside the conductor is a cavity within which is a point charge  $q = +3.0 \times 10^{-6} \text{ C}$ . What is the charge (a) on the cavity wall and (b) on the outer surface of the conductor?

(a) Consider a Gaussian surface within the conductor that covers the cavity wall, in the conductor,  $E = 0$ :

$$q_{\text{wall}} + q_{\text{point}} = 0$$

$$q_{\text{wall}} = -q_{\text{point}} = -3 \times 10^{-6} \text{ C or } -3 \mu\text{C}$$



(b) the total charge of the conductor:

$$q_{\text{wall}} + q_{\text{outer}} = 10 \times 10^{-6} \Rightarrow q_{\text{outer}} = 13 \times 10^{-6} \text{ C or } 13 \mu\text{C}$$

22. An electron is released from rest at a perpendicular distance of 9 cm from a line of charge on a very long nonconducting rod. That charge is uniformly distributed, with  $4.5 \mu\text{C}$  per meter. What is the magnitude of the electron's initial acceleration?

Electric field at point P:

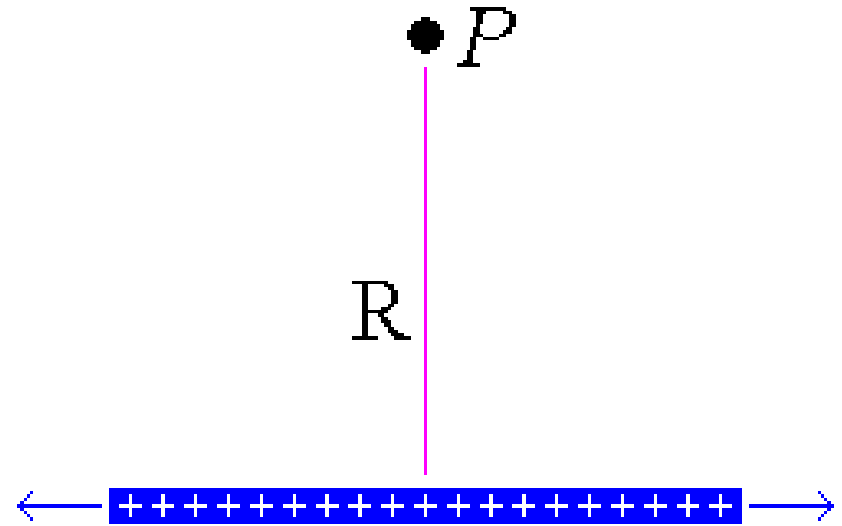
$$E = \frac{\lambda}{2\pi\epsilon_0 R}$$

Force acting on the electron:

$$F = eE = \frac{e\lambda}{2\pi\epsilon_0 R} = ma \Rightarrow a = \frac{e\lambda}{2\pi\epsilon_0 m R}$$

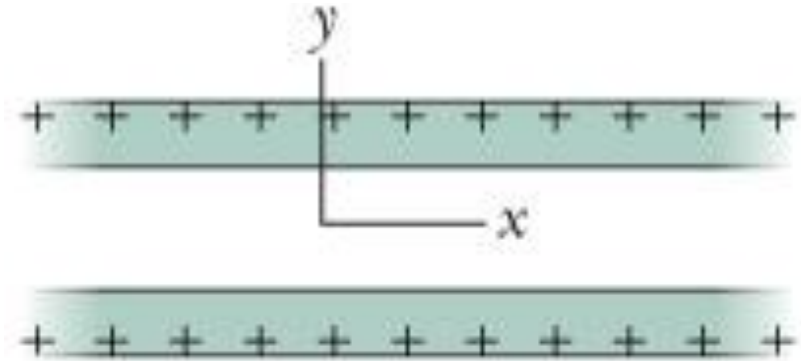
$$R = 9 \text{ cm} = 0.09 \text{ m}$$

$$\lambda = 4.5 \mu\text{C/m} = 4.5 \times 10^{-6} \text{ C/m}$$





36. The figure shows cross sections through two large, parallel, nonconducting sheets with identical distributions of positive charge with surface charge density  $\sigma = 2.31 \times 10^{-22} \text{ C/m}^2$ . In unit-vector notation, what is  $\vec{E}$  at points (a) above the sheets, (b) between them, and (c) below them?



For **one** non-conducting sheet:

$$E = \frac{\sigma}{2\epsilon_0}$$

Using the superposition to calculate E due to **two** sheets:

(a)

$$E = 2 \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0} = \frac{2.31 \times 10^{-22}}{8.85 \times 10^{-12}} = 2.61 \times 10^{-11} \text{ (N/C)}$$

The net electric field direction is upward  $\vec{E} = 2.61 \times 10^{-11} \text{ (N/C)} \hat{j}$

(b)  $E = 0$

(c)  $\vec{E} = -2.61 \times 10^{-11} \text{ (N/C)} \hat{j}$

The direction is downward

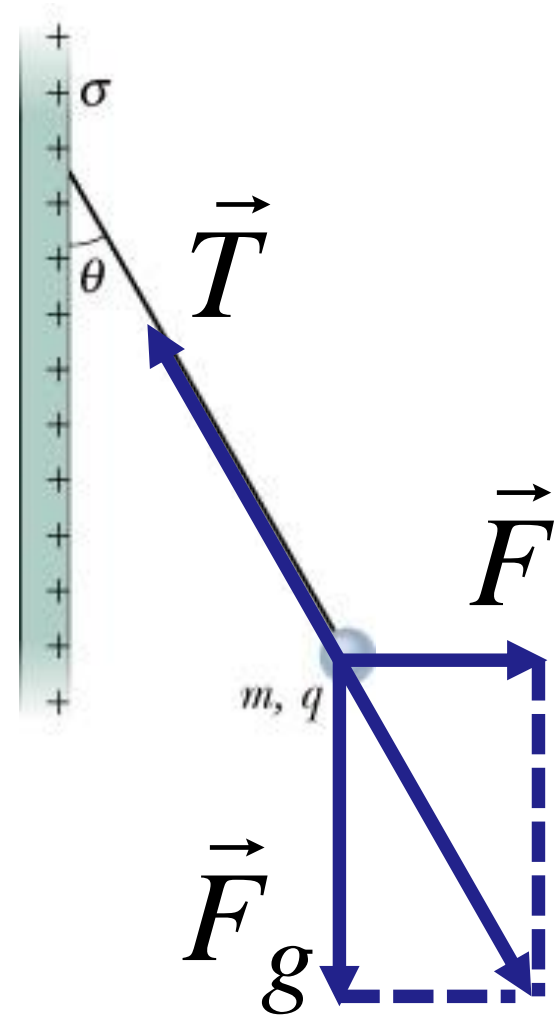
39. A small, nonconducting ball of mass  $m = 1 \text{ mg}$  and charge  $q = 2 \times 10^{-8} \text{ C}$  hangs from an insulating thread that makes an angle  $\theta = 30^\circ$  with a vertical, uniformly charged nonconducting sheet. Considering the gravitational force on the ball and assuming the sheet extends far vertically and into and out of the page, calculate the surface charge density  $\sigma$  of the sheet.

If the ball is in equilibrium:

$$\vec{F} + \vec{F}_g + \vec{T} = 0$$

$$\tan\theta = \frac{F}{F_g} = \frac{qE}{mg} = \frac{q}{mg} \times \frac{\sigma}{2\epsilon_0}$$

$$\sigma = \frac{2\epsilon_0 mg \tan\theta}{q} = 5 \times 10^{-9} \text{ (C/m}^2\text{)}$$



44. The figure gives the magnitude of the electric field inside and outside a sphere with a positive charge distributed uniformly throughout its volume. What is the charge on the sphere?

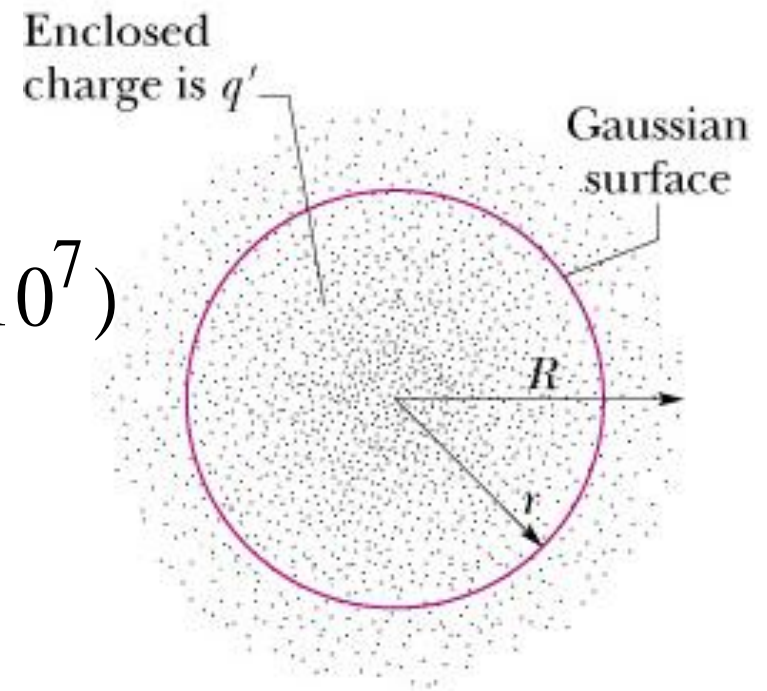
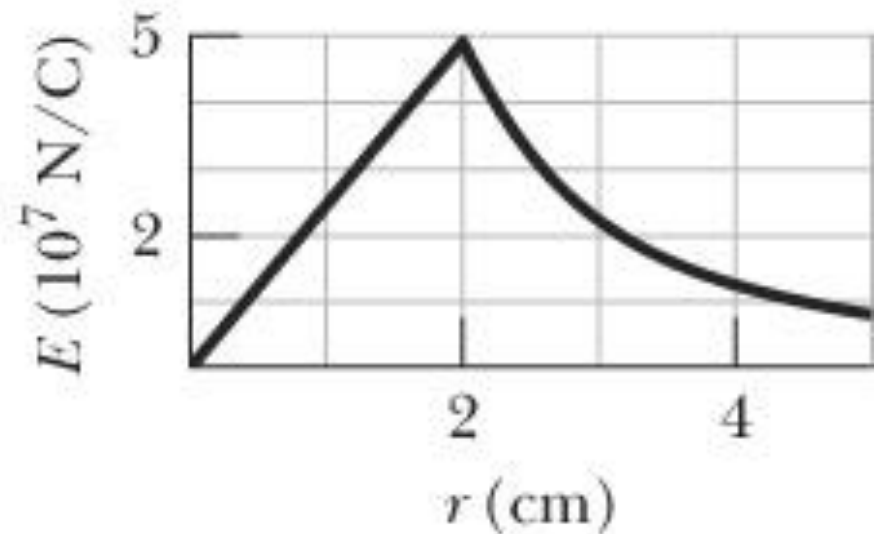
$$E = \left( \frac{q}{4\pi\epsilon_0 R^3} \right) r \quad (r \leq R)$$

- At  $r = 2$  cm,  $E$  is maximum, so  $R = 2$  cm

$$E = \frac{q}{4\pi\epsilon_0 R^2}$$

$$q = 4 \times 3.14 \times 8.85 \times 10^{-12} \times (0.02)^2 \times (5 \times 10^7)$$

$$= 2.2 \times 10^{-6} \text{ (C)}$$



51. A nonconducting spherical shell of inner radius  $a = 2$  cm and outer radius  $b = 2.4$  cm has a positive volume charge density  $\rho = A/r$ , where  $A$  is a constant and  $r$  is the distance from the center of the shell. In addition, a small ball of charge  $q = 45$  fC is located at that center. What value should  $A$  have if the electric field in the shell ( $a \leq r \leq b$ ) is to be uniform?

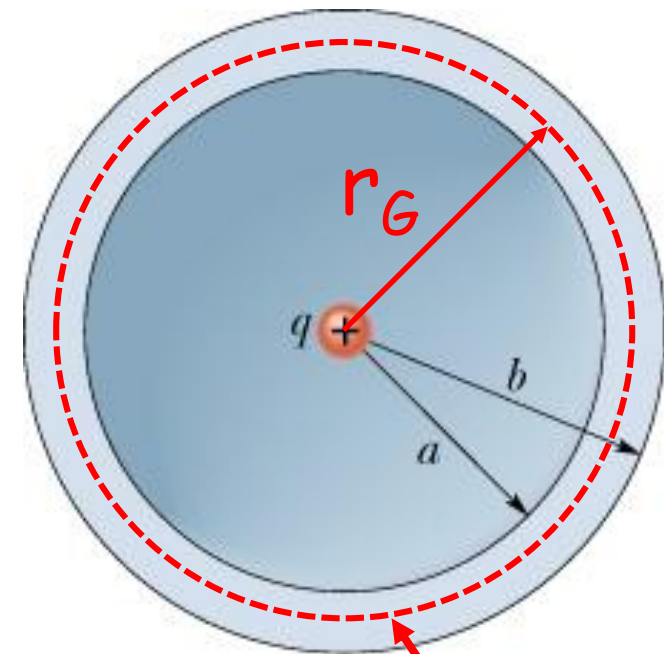
**Key idea:** First, we need to calculate  $E$  inside the shell, if the field is uniform, so  $E$  is independent of distance from the center

$$E_r = \frac{1}{4\pi\epsilon_0} \frac{q_{\text{total}}}{r^2}$$

$$q_{\text{total}} = q + q_{\text{shell}}$$

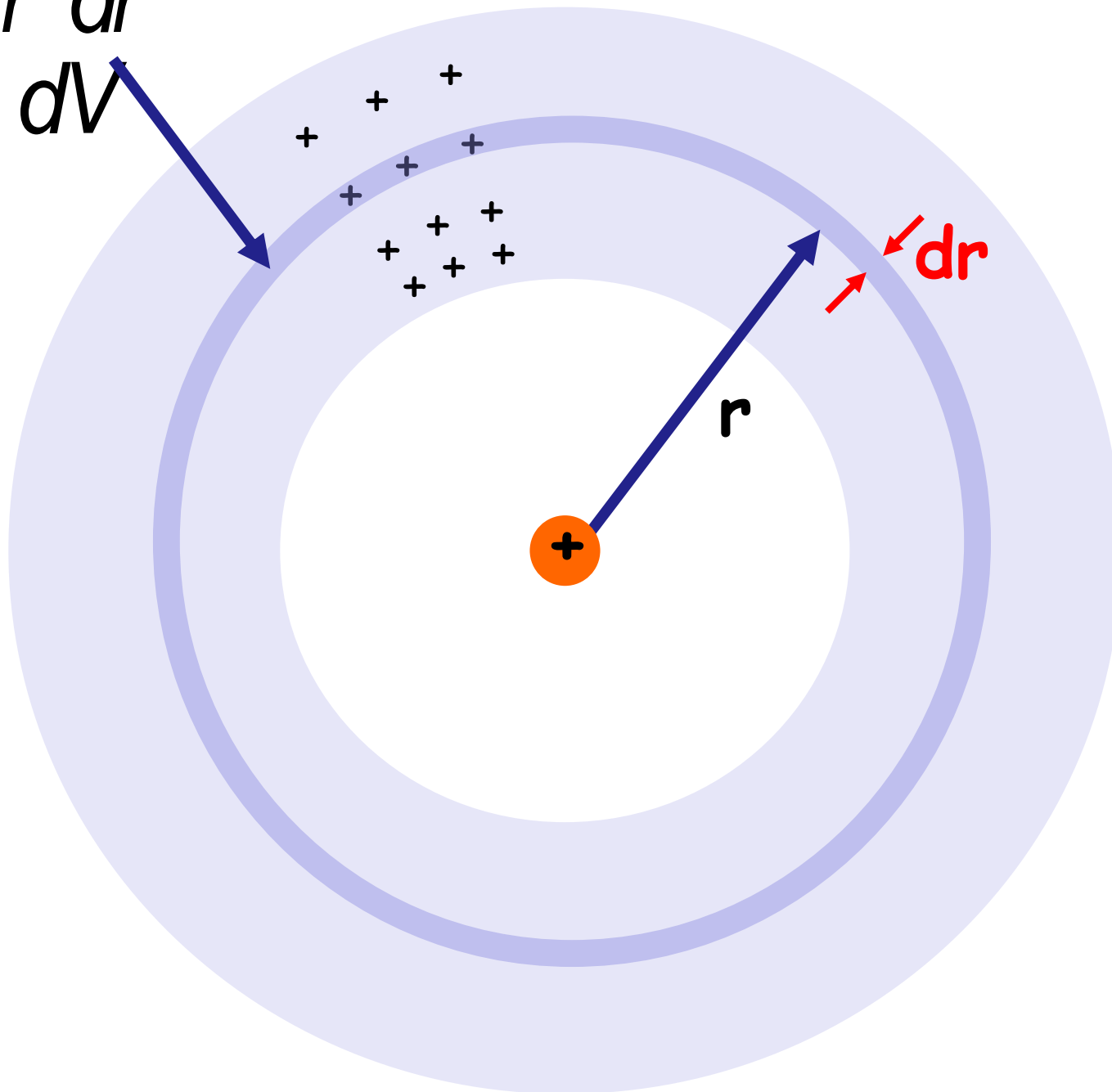
$q_{\text{shell}}$  is the enclosed charge in the shell of thickness  $r_G - a$ :  $dq_{\text{shell}} = \rho \times dV = \rho \times 4\pi r^2 dr$

$$q_{\text{shell}} = 4\pi \int_a^{r_G} \frac{A}{r} r^2 dr = 2\pi A (r_G^2 - a^2)$$



Gaussian surface

$$dV = 4\pi r^2 dr$$
$$dq = \rho dV$$



Using Gauss' law:  $\epsilon_0 \Phi = q_{\text{total}}$

$$\epsilon_0 E 4\pi r_G^2 = q_{\text{total}}$$

$$E = \frac{q_{\text{total}}}{\epsilon_0 4\pi r_G^2} = \frac{q + 2\pi A(r_G^2 - a^2)}{4\pi\epsilon_0 r_G^2}$$

We rewrite:

$$E = \frac{A}{2\epsilon_0} + \frac{1}{2\epsilon_0} \left( \frac{q}{2\pi} - Aa^2 \right) \times \frac{1}{r_G^2}$$

If E is uniform in the shell:

$$\frac{q}{2\pi} - Aa^2 = 0 \Rightarrow A = \frac{q}{2\pi a^2}$$

$$A = \frac{45 \times 10^{-15} \text{ C}}{2 \times 3.14 \times (0.02 \text{ m})^2} = 1.79 \times 10^{-11} (\text{C} / \text{m}^2)$$

52. The figure below shows a spherical shell with uniform volume charge density  $\rho = 1.56 \text{ nC/m}^3$ , inner radius  $a = 10 \text{ cm}$ , and outer radius  $b = 2a$ . What is the magnitude of the electric field at radial distances (a)  $r = 0$ , (b)  $r = a/2$ , (c)  $r = a$ , (d)  $r = 1.5 a$ , (e)  $r = b$ , and (f)  $r = 3b$ ?

For (a), (b), (c) using Gauss's law, we find

$$E = 0$$

For (d), (e)  $a \leq r \leq b$ :

The enclosed charge:

$$q_{enc} = \rho \times V = \rho \left( \frac{4}{3} \pi r^3 - \frac{4}{3} \pi a^3 \right)$$

The electric field:

$$E = \frac{1}{4\pi\epsilon_0} \frac{q_{enc}}{r^2}$$

For (f):

$$E = \frac{1}{4\pi\epsilon_0} \frac{q_{total}}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{\rho \times \frac{4}{3} \pi (b^3 - a^3)}{r^2}$$

$$E = \frac{\rho}{3\epsilon_0} \frac{b^3 - a^3}{r^2}$$

