Example problems

Chapter 4: Entropy and the second Law of thermodynamics

Homework: 1, 3, 7, 10, 14, 15, 18 (page 556)

1/ Suppose 4.0 mol of an ideal gas undergoes a reversible isothermal expansion from volume V1 to volume V2=2.0V1 at temperature T=400K. Find (a) the work done by the gas and (b) the entropy change of the gas. (c) If the expansion is reversible and adiabatic instead of isothermal, what is the entropy change of the gas?

(a)

$$W = nRT \ln \frac{V_2}{V_1}$$

$$W = 4.0 \times 8.31 \times 400 \times \ln \frac{2.0V_1}{V_1} = 9216\,(\text{J})$$

(b)

$$\Delta S = \int_i^f \frac{dQ}{T}$$

For an isothermal process: T=constant

$$\Delta S = \frac{Q}{T}; \Delta E_{\text{int}} = Q - W = 0 \Rightarrow Q = W$$

$$\Delta S = \frac{W}{T} = \frac{9216}{400} = 23.0\,(\text{J/K})$$

(c) $Q = 0 \Rightarrow \Delta S = 0$

2/ An ideal gas undergoes a reversible isothermal expansion at 77.0 °C, increasing its volume from 1.30 L to 3.90 L. The entropy change of the gas is 22.0 J/K. How many moles of gas are present?

$$\Delta S = S_f - S_i = nR\ln\frac{V_f}{V_i} + nC_V \ln\frac{T_f}{T_i}$$

$$\Delta S = nR\ln\frac{V_f}{V_i} \Rightarrow n = \frac{\Delta S}{R\ln\frac{V_f}{V_i}} = \frac{22.0}{8.31\times\ln\frac{3.9}{1.3}}$$

$$n = 2.41\,(\text{mol})$$

3/

A 2.5 mol sample of an ideal gas expands reversibly and isothermally at 360 K until its volume is doubled. What is the increase in entropy of the gas?

$$\Delta S = S_f - S_i = nR\ln\frac{V_f}{V_i} + nC_V \ln\frac{T_f}{T_i}$$

For an isothermal process, T=constant:

$$\Delta S = S_f - S_i = nR\ln\frac{V_f}{V_i}$$

$$\Delta S = 2.5\times 8.31\times \ln 2 = 14.4\,(\mathrm{J/K})$$

5. Find (a) the energy absorbed as heat and (b) the change in entropy of a 2.0 kg block of copper whose temperature is increased reversibly from 25.0 ^{0}C to 100 ^{0}C. The specific heat of copper is 386 J.kg/K.

(a) Energy absorbed as heat to increase the copper temperature:

$$Q = cm\Delta T$$

$$Q = 386 \times 2 \times 75 = 57900 \text{ (J)}$$

(b) The change in entropy:

$$\Delta S = \int_{T_1}^{T_2} \frac{dQ}{T} = \int_{T_1}^{T_2} \frac{cmdT}{T} = cm \ln \frac{T_2}{T_1}$$

$$T_1 = 25 + 273.15 = 298.15^0\text{K};\ T_2 = 100 + 273.15 = 373.15^0\text{K}$$

$$\Delta S = 386 \times 2 \times \ln \frac{373.15}{298.15} = 173.2 \text{ (J/K)}$$

8. At very low temperatures, the molar specific heat CV of many solids is approximately CV=AT3, where A depends of the particular substance. For aluminum, A = 3.15 x 105 J mol-1 K-4. Find the entropy change for 4.0 mol of aluminum when its temperature is raised from 5.0 K to 10.0 K.

$$\Delta S = \int_i^f \frac{dQ}{T}$$

We assume that the volume change is negligible:

$$dQ = nC_V dT$$

$$\Delta S = \int_i^f \frac{nC_V dT}{T} = nA\int_{5.0}^{10.0} T^2 dT =$$

$$= \frac{1}{3} nAT^3 \Big|_{5.0}^{10.0} = \frac{1}{3} \times 4.0 \times 3.15 \times 10^{-5} \left(10^3 - 5^3\right) = 0.037\ (\mathrm{J/K})$$

Homework: 25, 29, 33, 40 (page 558)

25/ A Carnot engine has an efficiency of 22.0%. It operates between constant-temperature reservoirs differing in temperature by 75.0 C .What is the temperature of the
(a) lower-temperature and (b) higher-temperature reservoir?

29/ Figure 20-27 shows a reversible cycle through which 1.00 mol of a monatomic ideal gas is taken. Assume that $p=2p_0$, $V=2V_0$, $p_0=1.01\times10^5$ Pa, and $V_0=0.0225$ m^3. Calculate

(a) the work done during the cycle,

(b) the energy added as heat during stroke abc, and

(c) The efficiency of the cycle.

(d) What is the efficiency of a Carnot engine operating between the highest and lowest temperatures that occur in the cycle?

(e) Is this greater than or less than the efficiency calculated in (c)?

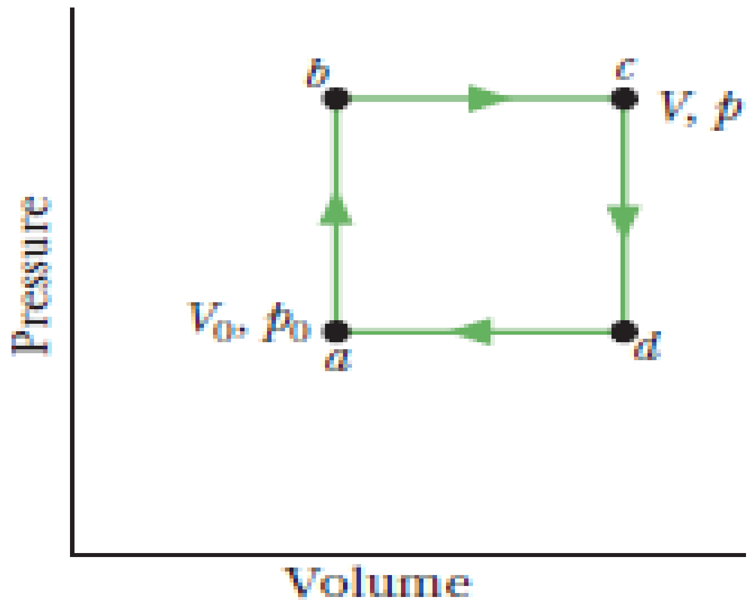

Figure 20-27 Problem 29.

33/. Figure 20-29 shows a reversible cycle through which 1.00 mol of a monatomic ideal gas is taken. Volume $V_c = 8.00V_b$. Process *bc* is an adiabatic expansion, with $p_b = 10.0$ atm and $V_b = 1.00 \times 10^3$ m^3. For the cycle, find

(a) the energy added to the gas as heat,
(b) the energy leaving the gas as heat,
(c) the net work done by the gas, and
(d) The efficiency of the cycle.

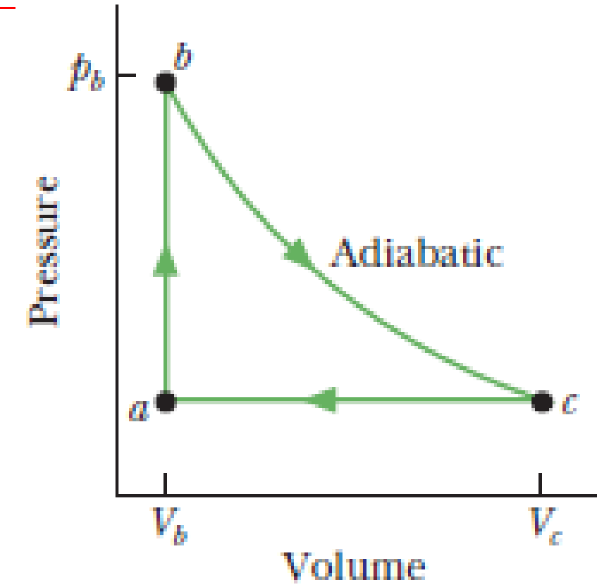

Figure 20-29 Problem 33.