Example problems

Chapter 3: The Kinetic Theory of Gases

Homework: 13, 18, 20, 23, 25, 27
(p. 531-532)

9. An automobile tire has a volume of 1.64×10^2 m^3 and contains air at a gauge pressure (above atmospheric pressure) of 165 kPa when the temperature is 0.00 ^0C. What is the gauge pressure of the air in the tires when its temperature rises to 27.0 ^0C and its volume increases to 1.67×10^2 m^3? Assume atmospheric pressure is 1.01×10^5 Pa.

$$pV = nRT$$

At state i:

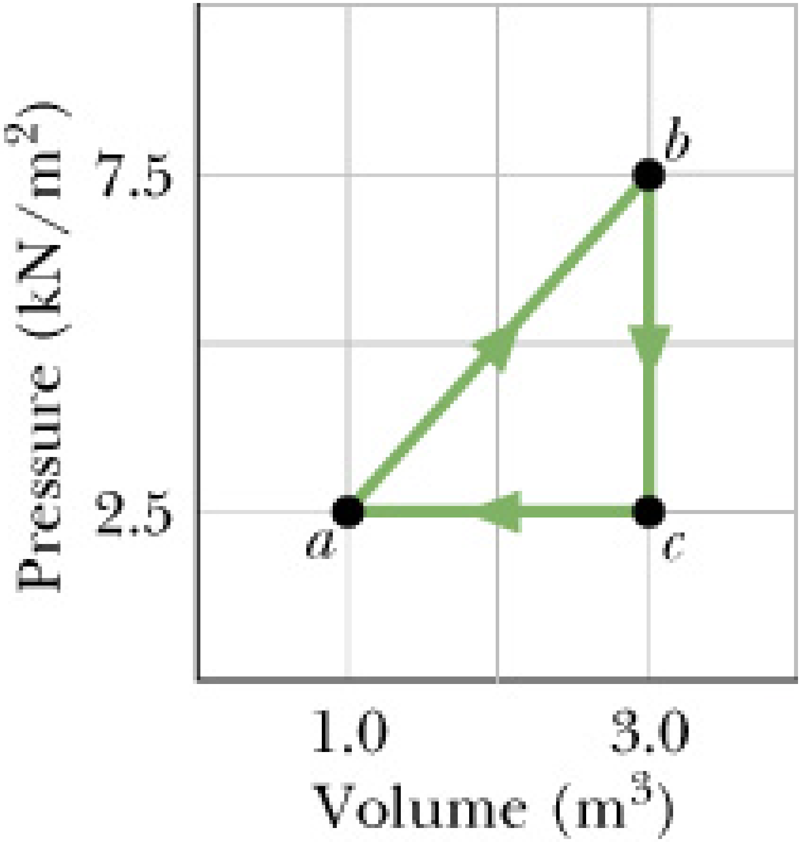
$$p_i V_i = nRT_i$$

At the final state f:

$$p_f V_f = nRT_f \quad \Rightarrow p_f = \frac{p_i V_i T_f}{T_i V_f}$$

Gauge pressure: $p_G = p_f - p_o$

13. A sample of an ideal gas is taken through the cyclic process abca shown in the figure below; at point a, T=200 K. (a) How many moles of gas are in the sample? What are (b) the temperature of the gas at point b, (c) the temperature of the gas at point c, and (d) the net energy added to the gas as heat during the cycle?



(a) Applying the equation of state:

$$pV = nRT \Rightarrow n = \frac{pV}{RT}$$

At point a, p=2.5 kN/m² or 2500 N/m²; V=1 m³.

$$n = \frac{2500\times 1}{8.31\times 200} = 1.5\,(\text{mol})$$

(b) $$pV = nRT \Rightarrow \frac{p_a V_a}{T_a} = \frac{p_b V_b}{T_b} = nR = 12.5$$

At point b, p=7.5 kN/m² or 7500 N/m²; V=3 m³.

$$T_b = \frac{p_b V_b}{nR} = \frac{7500\times 3}{12.5} = 1800\,(\text{K})$$

(c) see part b; T_c=600 K;

(d) Applying the first law of thermodynamics:

$$\Delta E = Q - W$$

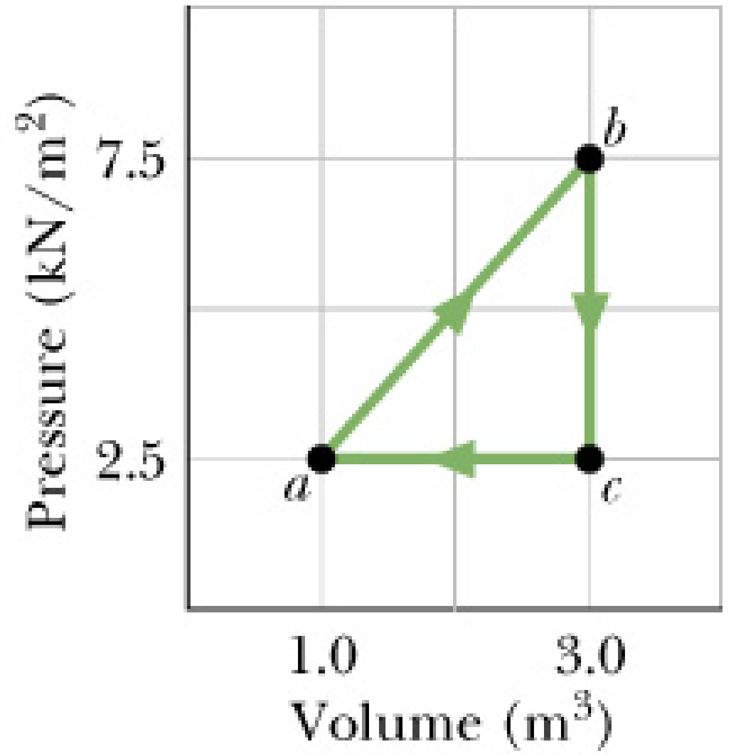
W: work done by the system.

For a closed cycle, ΔE=0:

$$Q = W$$

$$W = \frac{1}{2}(p_b - p_c)(V_b - V_a)$$

$$W = \frac{1}{2} \times 5000.0 \times 2 = 5 \times 10^3 \text{ (J)}$$



14. In the temperature range 310 K to 330 K, the pressure p of a certain nonideal gas is related to volume V and temperature T by:

$$p = (24.9\ J/K)\frac{T}{V} - (0.00662\ J/K^2)\frac{T^2}{V}$$

How much work is done by the gas if its temperature is raised from 315 K to 330 K while the pressure is held constant?

- **Work done by the gas is computed by the following formula:**

$$W = \int_{V_i}^{V_f} p dV = p(V_f - V_i)$$

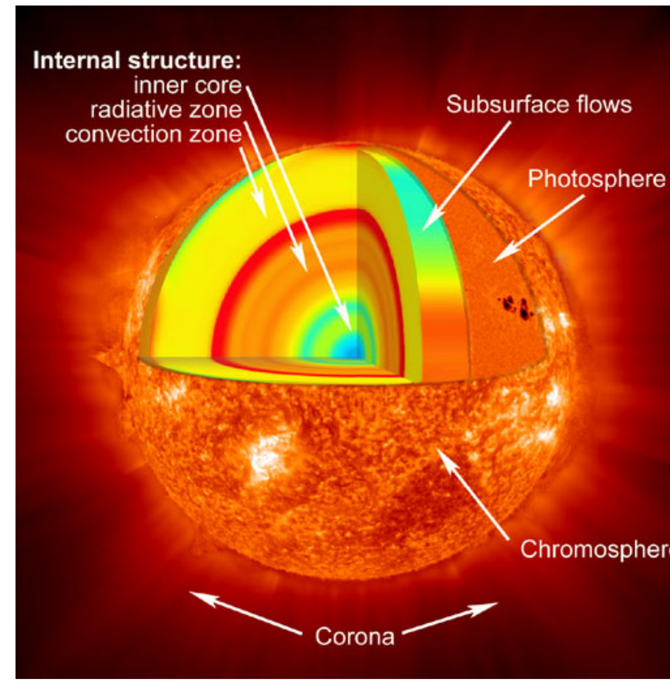
$$W = pV_f - pV_i = 24.9(T_f - T_i) - 0.00662(T_f^2 - T_i^2)$$

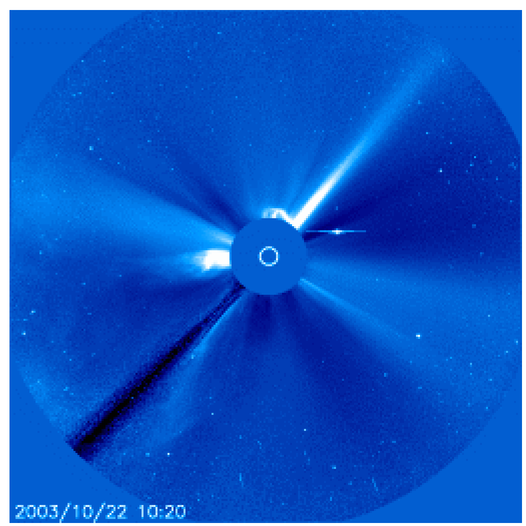
$$T_f = 330K; T_i = 315K \Rightarrow W \approx 310\,(\text{J})$$

18. The temperature and pressure in the Sun's atmosphere are 2.00x10⁶ K and 0.0300 Pa. Calculate the rms speed of free electrons (mass 9.11x10⁻³¹ kg) there, assuming they are an ideal gas.

$$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3RT}{mN_A}}$$

$$v_{rms} = \sqrt{\frac{3\times 8.31\times 2\times 10^6}{9.11\times 10^{-31}\times 6.023\times 10^{23}}} = 9.5\times 10^6 \text{ (m/s)}$$





20. Calculate the rms speed of helium atoms at 1000 K, the molar mass of helium atoms is 4.0026 g/mol.

$$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\times 8.31\times 1000}{4.0026\times 10^{-3}}} = 2.5\times 10^{3}\ \text{(m/s)}$$

24. At 273 K and 1.0 x 10^{-2} atm, the density of a gas is 1.24 x 10^{-5} g/cm^3. (a) Find v_{rms} for the gas molecules. (b) Find the molar mass of the gas and (c) identify the gas (hint: see Table 19-1).

(a) Root-mean-square speed:

$$v_{rms} = \sqrt{\frac{3RT}{M}} \qquad (1)$$

$$\rho = \frac{M_{gas}}{V} = \frac{nM}{V} \Rightarrow M = \frac{\rho V}{n} \qquad (2)$$

(1) and (2):

$$v_{rms} = \sqrt{\frac{3nRT}{\rho V}} = \sqrt{\frac{3p}{\rho}}$$

$$\rho = 1.24\times10^{-5}\,\text{g/cm}^3 = 1.24\times10^{-2}\,\text{kg/m}^3$$

$$p = 1.0\times10^{-2}\,\text{atm} = 1.01\times10^{3}\,\text{Pa}$$

$$v_{rms} \approx 494\ \text{m/s}$$

(b)

$$M = \frac{\rho V}{n} \quad (2)$$

Equation of state:

$$pV = nRT \quad (3)$$

$$\Rightarrow M = \frac{\rho V}{n} = \frac{\rho RT}{p}$$

$$\Rightarrow M \approx 0.028 \text{ kg/mol} = 28 \text{ g/mol}$$

(c)

From Table 19.1, the gas is nitrogen (N_2)

25. Determine the average value of the translational kinetic energy of the molecules of an ideal gas at (a) 0.00^{0}C and (b) 100^{0}C. What is the translational kinetic energy per mole of an ideal gas at (c) 0.00^{0}C and (d) 100^{0}C?

(a) The translational kinetic energy per molecule:

$$\overline{K} = \frac{3}{2}kT$$

$$T = 0 + 273 = 273\,\mathrm{K}:$$

$$\overline{K} = \frac{3}{2} \times 1.38 \times 10^{-23} \times 273 = 5.65 \times 10^{-21}\ \mathrm{(J)}$$

(b) see (a):

$$\overline{K} = \frac{3}{2} \times 1.38 \times 10^{-23} \times 373 = 7.72 \times 10^{-21}\ \mathrm{(J)}$$

(c) The translational kinetic energy per mole: $K_{mole} = \overline{K} \times N_A$

$$K_{mole} = 5.65 \times 10^{-21} \times 6.02 \times 10^{23} = 3.4 \times 10^{3}\ \mathrm{(J)}$$

(d)

$$K_{mole} = 4.7 \times 10^{3}\ \mathrm{(J)}$$

Note: If a sample of gas has n moles (or N molecules), its total translational kinetic energy is:

$$K_{total} = n \times K_{mole} = n \times N_A \times \overline{K}$$

$$K_{total} = n \times K_{mole} = n \times N_A \times \frac{3}{2}kT = \frac{3}{2}nRT$$

$$\boxed{K_{total} = \frac{3}{2}nRT}$$

Homework: 28, 32, 33, 40 (page 532)

28. At what frequency would the wavelength of sound in air be equal to the mean free path of oxygen molecules at 1.0 atm pressure and 0.0^{0}C? take the diameter of an oxygen molecule to be 3.0 x 10^{-8} cm.

Mean Free Path:

$$\lambda_{\text{MFP}} = \frac{kT}{\sqrt{2}\pi d^2 p}$$

$$k = 1.38\times10^{-23}\ \text{J/K};\ T = 273\ \text{K};\ p = 1.01\times10^{5}\ \text{Pa};$$

$$d = 3\times10^{-8}\,\text{cm} = 3\times10^{-10}\,\text{m}$$

Frequency of sound in air:

$$f_{\text{sound}} = \frac{v_{\text{soundin air}}}{\lambda_{\text{sound}}} = \frac{v_{\text{soundin air}}}{\lambda_{\text{MFP}}}$$

$$v_{\text{soundin air}} = 343\ \text{m/s}:$$

$$\lambda_{\text{MFP}} = 9.33\times10^{-8}\,\text{m}$$

$$f_{\text{sound}} = \frac{343}{9.33\times10^{8}} \approx 3.68\times10^{9}\,(\text{Hz})\ \text{or}\ 3.68\,\text{GHz}$$

32. At 20^0C and 750 torr pressure, the mean free paths for argon gas (Ar) and nitrogen (N_2) are $\lambda_{Ar}=9.9\times10^{-6}$ cm and $\lambda_{N2}=27.5\times10^{-6}$ cm. (a) Find the ratio of the diameter of an Ar atom to that of an N_2 molecule. What is the mean free path of Ar at (b) 20^0C and 150 torr, and (c) -40^0C and 750 torr?

Mean Free Path:

$$\lambda = \frac{kT}{\sqrt{2}\pi d^2 p}$$

(a) The ratio d_{Ar} to d_{N2}:

$$\frac{d_{Ar}}{d_{N_2}} = \sqrt{\frac{\lambda_{N_2}}{\lambda_{Ar}}}$$

(b):

$$\lambda_1 = \frac{kT_1}{\sqrt{2}\pi d^2 p_1}; \lambda_2 = \frac{kT_2}{\sqrt{2}\pi d^2 p_2}$$

$$\lambda_2 = \frac{T_2}{T_1} \times \frac{p_1}{p_2} \times \lambda_1$$

33. The speeds of 10 molecules are 2.0, 3.0, 4.0,..., 11 km/s. What are their (a) average speed and (b) rms speed?

(a)

$$\bar{v} = \frac{\sum_{i=1}^{N} v_i}{N} = \frac{2+3+4+...+11}{10} = \frac{65}{10} = 6.5\,(\text{km/s})$$

(b)

$$v_{rms} = \sqrt{\left(v^2\right)_{avg}} = \sqrt{\frac{\sum_{i=1}^{N} v_i^2}{N}} = 7.1\,(\text{km/s})$$

40. Two containers are at the same temperature. The first contains gas with pressure p_1, molecular mass m_1, and rms speed v_{rms1}. The second contains gas with pressure $1.5p_1$, molecular mass m_2, and average speed $v_{avg2}=2.0v_{rms1}$. Find the mass ratio m_1/m_2.

RMS speed: $$v_{rms1} = \sqrt{\frac{3RT_1}{m_1}}$$

Average speed: $$\bar{v}_2 = \sqrt{\frac{8RT_2}{\pi m_2}}$$

$$T_1 = T_2 \quad \Rightarrow \frac{m_1}{m_2} = \frac{3\pi}{8}\left(\frac{\bar{v}_2}{v_{\mathrm{rms1}}}\right)^2 = 4.71$$

Homework: 42, 44, 46, 54, 56, 78 (p. 533-535)

42. What is the internal energy of 2.0 mol of an ideal monatomic gas at 273 K?

$$E = nC_V T$$

$$C_V = \frac{3}{2} R = 12.5 \text{ J mol}^{-1} \text{ K}^{-1}$$

$$E = 2.0 \times 12.5 \times 273 = 6825 \text{ (J)}$$

$$E \approx 6.8 \text{ (kJ)}$$

44. One mole of an ideal diatomic gas goes from a to c along the diagonal path in Fig. 19-25. The scale of the vertical axis is set by p_{ab} = 5.0 kPa and p_c = 2.0 kPa, and the scale of the horizontal axis is set by V_{bc} = 4.0 m^3 and V_a = 2.0 m3. During the transition,

(a) what is the change in internal energy of the gas,

(b) how much energy is added to the gas as heat?

(c) How much heat is required if the gas goes from a to c along the indirect path abc?

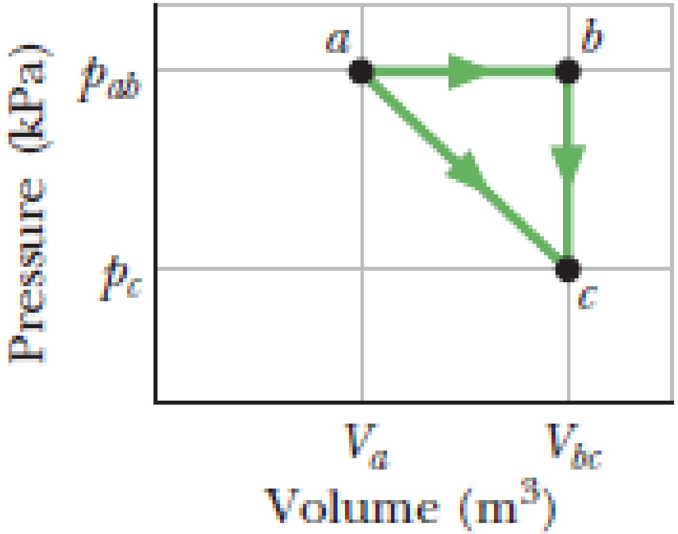


Figure 19-25 Problem 44.

a) $$\Delta E_{\text{int}} = nC_V \Delta T = \frac{5}{2} nR\Delta T$$
$$= \frac{5}{2}(P_c V_c - P_a V_a) = -5000 J$$

b) $$Q = \Delta E_{\text{int}} + W = -5000 + 7000 = 2000 J$$

c) $$Q = \Delta E_{\text{int}} + W_{ac}$$

46. Under constant pressure, the temperature of 3.0 mol of an ideal monatomic gas is raised 15.0 K. What are (a) the work W done by the gas, (b) the energy transferred as heat Q, (c) the change ΔE_{int} of the gas, and (d) the change ΔK in the average KE per atom?

(a) At constant pressure:

$$W = p\Delta V = nR\Delta T = 3.0\times 8.31\times 15.0 \approx 374 \text{ (J)}$$

(b)

$$Q = nC_p\Delta T = n\times\frac{5}{2}R\times\Delta T = \frac{5}{2}W \approx 935 \text{ (J)}$$

(c) We use the first law of thermodynamics:

$$\Delta E_{\text{int}} = Q - W \quad (\text{or } \Delta E_{\text{int}} = nC_V\Delta T = \frac{3}{2}nR\Delta T)$$

$$\Delta E_{\text{int}} = 935 - 374 = 561 \text{ (J)}$$

(d) For a monatomic gas: $K_{\text{avg}} = \frac{3}{2}kT \Rightarrow \Delta K_{\text{avg}} = \frac{3}{2}k\Delta T$

$$\Delta K_{\text{avg}} = \frac{3}{2}\times 1.38\times 10^{-23}\times 15.0 \approx 3.1\times 10^{-22} \text{ (J)}$$

54. We know that for an adiabatic process $pV^\gamma = \text{constant}$. Evaluate "constant" for an adiabatic process involving exactly 2.0 mol of an ideal gas passing through the state having exactly p=1.5 atm and T=300 K. Assume a diatomic gas whose molecules rotate but do not oscillate.

$$1\,\text{atm} = 1.01\times10^5\ (\text{Pa})$$

Equation of state:

$$pV = nRT$$

$$V = \frac{nRT}{p} = \frac{2.0\times8.31\times300}{1.5\times1.01\times10^5} \approx 0.033\ (\text{m}^3)$$

$$\gamma = \frac{C_p}{C_V} = \frac{C_V + R}{C_V} = \frac{\frac{f}{2}R + R}{\frac{f}{2}R}$$

For a diatomic gas, f=5:

$$\gamma = \frac{7}{5}$$

$$\text{constant} = pV^\gamma = 1.5\times1.01\times10^5\times0.033^{\frac{7}{5}} = 1.28\times10^3(\text{N/m}^2\times(\text{m}^3)^{1.4})$$

$$\text{constant} = 1.28\times10^3(\text{N m}^{2.2})$$

56. Suppose 1.0L of a gas with γ=1.30, initially at 285 K and 1.0 atm, is suddenly compressed adiabatically to half its initial volume. Find its final (a) pressure and (b) temperature. (c) If the gas is then cooled to 273 K at constant pressure, what is its final volume?

$$p_i V_i^{\gamma} = p_f V_f^{\gamma}; \quad V_f = \frac{1}{2} V_i$$

$$p_f = p_i \left(\frac{V_i}{V_f} \right)^{\gamma}$$

$$T_f = T_i \left(\frac{V_i}{V_f} \right)^{\gamma - 1}$$

$$pV = nRT, \, p = \text{constant} \Rightarrow \frac{V'_f}{V_f} = \frac{T'_f}{T_f}$$

78. (a) An ideal gas initially at pressure p_0 undergoes a free expansion until its volume is 3.0 times its initial volume. What then is the ratio of its pressure to p_0? (b) The gas is next slowly and adiabatically compressed back to its original volume. The pressure after compression is $(3.0)^{1/3}p_0$. Is the gas monatomic, diatomic, or polyatomic? (c) What is the ratio of the average kinetic energy per molecule in this final state to that in the initial state?

(a) $$p_0V_0 = p_1V_1; V_1 = 3V_0 \Rightarrow p_1 = \frac{1}{3}p_0$$

(b) $$p_1V_1^{\gamma} = p'_1V_0^{\gamma}$$

$$p'_1 = p_1\left(\frac{V_1}{V_0}\right)^{\gamma} = \frac{1}{3}p_0 3^{\gamma} = 3^{\gamma-1}p_0$$

$$\Rightarrow \gamma - 1 = \frac{1}{3} \Rightarrow \gamma = \frac{4}{3} = \frac{C_p}{C_V} = \frac{C_V + R}{C_V} = \frac{\frac{f}{2}+1}{\frac{f}{2}} = \frac{f+2}{f}$$

$$f = 6: \text{polyatomic}$$

(c)

$$K_{avg} = \frac{3}{2} kT$$

$$r = \frac{K'_{avg}}{K_{avg}} = \frac{T'_1}{T_0}$$

$$r = \frac{T'_1}{T_0} = \frac{p'_1 V_1^{'}}{p_0 V_0} = \frac{p'_1}{p_0} = 3^{1/3} = 1.44 \text{ (since } V_1^{'} = V_0)$$