# Example problems

Chapter 2: **Temperature, heat, and the $1^{st}$ law of Thermodynamic**

**Homework:** 2, 3, 4, 5, 6, 10, 15, 19, 21

(pages 500-501)

2. (Page 500) Two constant-volume gas thermometers are assembled, one with nitrogen and the other with hydrogen. Both contain enough gas so that $p_3$=80 kPa. (a) What is the difference between the pressures in the two thermometers if both bulbs are in boiling water? (b) Which gas is at higher pressure?

$$\frac{T}{p} = \frac{T_3}{p_3} \Rightarrow p = p_3 \times \frac{T}{T_3}$$

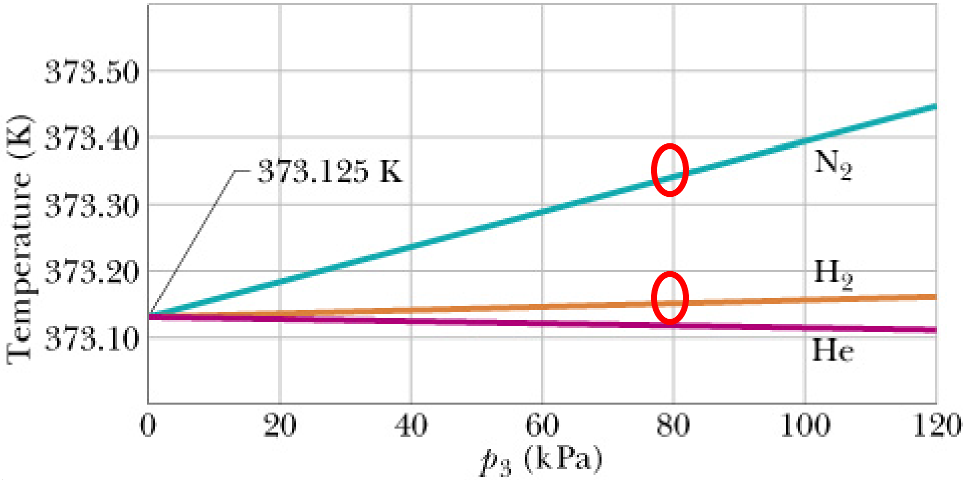



Fig. 18-6 (page 480)

(a) For the nitrogen thermometer:

$T_N \approx$373.35 (K)

$$p_N = 80 \times \frac{373.35}{273.16} = 109.343\,(\text{kPa})$$

For the hydrogen thermometer:

$T_H \approx$373.15 (K)

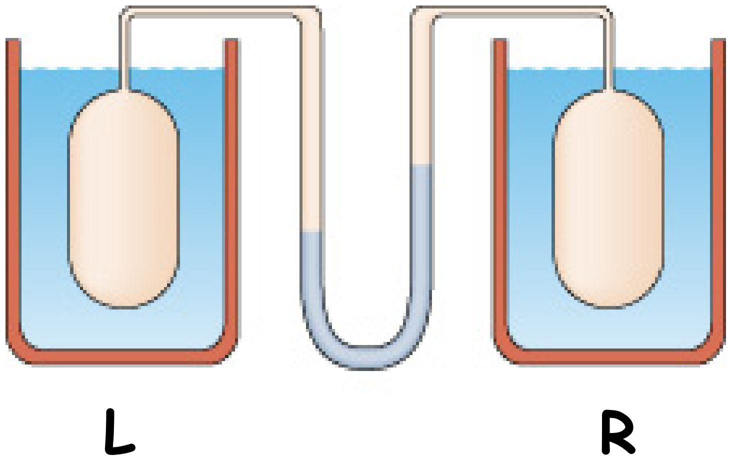
$$p_H = 80 \times \frac{373.15}{273.16} = 109.284\,(\text{kPa})$$

$$\Delta p = 0.059\ \text{kPa or}\ \Delta p = 59\ \text{Pa}$$

(b) $P_N > P_H$

3. A gas thermometer is constructed of two gas-containing bulbs, each in a water bath, as shown in the figure below. The pressure difference between the two bulbs is measured by a mercury manometer as shown. Appropriate reservoirs, not shown in the diagram, maintain constant gas volume in the two bulbs. There is no difference in pressure when both baths are at the triple point of water. The pressure difference is 120 torr when one bath is at the triple point and the other is at the boiling point of water. It is 90.0 torr when one bath is at the triple point and the other is at an unknown temperature to be measured. What is the unknown temperature?

$$\frac{T}{p} = \frac{T_3}{p_3} \Rightarrow p = p_3 \times \frac{T}{T_3}$$




$$p_L = p_3 \times \frac{T_L}{T_3}; \qquad p_R = p_3 \times \frac{T_R}{T_3}$$

- When one bath (L) is at the triple point and the other (R) is at the boiling point of water:

$$T_L = T_3 = 273.16\,(\text{K}) \text{ and } p_L = p_3$$

$$T_R = T_{boiling} = 373.125\,(K)$$

$$p_R - p_L = p_3\left(\frac{T_R}{T_3} - 1\right)$$

- When one bath (L) is at the triple point and the other (R) is at an unknown temperature $X = T'_R$:

$$p'_R - p_L = p_3\left(\frac{T'_R}{T_3} - 1\right)$$

$$\Rightarrow \frac{p_R - p_L}{p'_R - p_L} = \frac{T_R - T_3}{X - T_3} = \frac{120}{90}$$

$$\frac{373.125 - 273.16}{X - 273.16} = \frac{4}{3}$$

$$\boxed{X \approx 348\,(K)}$$

5. At what temperature is the Fahrenheit scale reading equal to (a) twice that of the Celsius scale and (b) half that of the Celsius scale?

$$T_F = \frac{9}{5}T_C + 32^0$$

$$T_F = 2T_C :$$

$$T_C = 160^0 C; T_F = 320^0 F$$

$$T_F = \frac{1}{2}T_C :$$

$$T_C \approx -24.6^0 C; T_F = -12.3^0 F$$

6. On a linear X temperature scale, water freezes at $-125.0^0X$ and boils at $360.0^0X$. On a linear Y temperature scale, water freezes at $-70.0^0Y$ and boils at $-30.0^0Y$. A temperature of $50.0^0Y$ corresponds to what temperature on the X scale?

For linear scales, the relationship between X and Y can be written by:

$$y = ax + b$$

$$-70 = -125\,a + b$$

$$-30 = 360\,a + b$$

$$\Rightarrow a, b$$

$$x = \frac{y - b}{a} = 1330\,^0X$$

**10. An aluminum flagpole is 33m high. By how much does its length increase as the temperature increases by 15 $C^0$?**

**For a linear expansion:**

$$\Delta L = L\alpha\Delta T = 33 \times 23 \times 10^{-6} \times 15 = 0.011\,(m)$$

$$\Delta L = 1.1\,\text{cm}$$

**Note: $\alpha$=23x10$^{-6}$/$C^0$ is the coefficient of linear expansion of aluminum (see Table 18-2, page 483).**

**15. A steel rod is 3.0 cm in diameter at 25.0$^0$C. A brass ring has an interior diameter of 2.992 cm at 25.0$^0$C. At what common temperature will the ring just slide onto the rod?**

**For a linear expansion of the steel rod:**

$$D_{steel} = D_{steel,0} + D_{steel,0}\alpha_s \Delta T$$

**For a linear expansion of the brass ring:**

$$D_{brass} = D_{brass,0} + D_{brass,0}\alpha_b \Delta T$$

**If the ring just slides onto the rod, so $D_{steel} = D_{brass}$**

$$\Delta T = \frac{D_{steel,0} - D_{brass,0}}{D_{brass,0}\alpha_b - D_{steel,0}\alpha_s}$$

$$\Delta T = \frac{3.0 - 2.992}{2.992\times 19\times 10^{-6} - 3.0\times 11\times 10^{-6}} = 335.5^0$$

$$T = 25 + 335.5 = 360.5^0 C$$

**19. A 1.28m-long vertical glass tube is half filled with a liquid at $20^0C$. How much will the height of the liquid column change when the tube is heated to $30^0C$? Take $\alpha_{glass}=1.0\times10^{-5}/K$ and $\beta_{liquid}=4.0\times10^{-5}/K$.**

**Here, we need to consider the cross-sectional area expansion of the glass and the volume expansion of the liquid:**

$$\Delta A = A_0(2\alpha)\Delta T$$

$$\Delta V = V_0\beta\Delta T$$

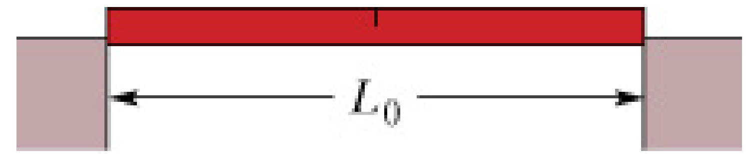
$$h = \frac{V}{A} = \frac{V_0+\Delta V}{A_0+\Delta A} = \frac{V_0(1+\beta\Delta T)}{A_0(1+2\alpha\Delta T)} = h_0\frac{(1+\beta\Delta T)}{(1+2\alpha\Delta T)}$$

$$\Delta h = h - h_0 = h_0\left[\frac{(1+\beta\Delta T)}{(1+2\alpha\Delta T)} - 1\right]$$

$$h_0 = \frac{1.28}{2} = 0.64\ (m);\ \Delta T = 30^0C - 20^0C = 10^0$$

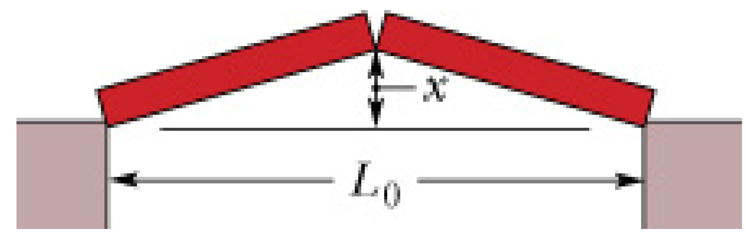
$$\boxed{\Delta h = 1.28\times10^{-4}(m)}$$

21. As a result of a temperature rise of 32⁰C, a bar with a crack as its center buckles upward. If the fixed distance $L_0$ is 3.77 m and the coefficient of linear expansion of the bar is 25 x $10^{-6}/C^0$, find the rise x of the center.




For a linear expansion:

$$L - L_0 = L_0 \alpha \Delta T$$




$$x^2 = l^2 - l_0^2 = (l_0 + l_0 \alpha \Delta T)^2 - l_0^2$$

where l = L/2; $l_0$ = L/2

$$x^2 = l_0^2 (1 + \alpha \Delta T)^2 - l_0^2 \approx 2 l_0^2 \alpha \Delta T$$

(using the binomial theorem, see Appendix E)

$$x = l_0 \sqrt{2\alpha \Delta T} = \frac{3.77}{2} \sqrt{2 \times 25 \times 10^{-6} \times 32} = 75.4 \times 10^{-3} (m) = 75.4 (mm)$$

**Homework:**

**25, 30, 32, 43, 44, 46, 47, 48, 49, 50, 51, 59, 60 (pages 501 - 504)**

25. A certain diet doctor encourages people to diet by drinking ice water. His theory is that the body must burn off enough fat to raise the temperature of the water from $0.00^0C$ to the body temperature of $37.0^0C$. How many liters of ice water would have to be consumed to burn off 454 g (about 1 lb) of fat, assuming that burning this much fat requires 3500 Cal be transferred to the ice water? Why is it not advisable to follow this diet? (One liter : $10^3$ $cm^3$. The density of water is 1.00 $g/cm^3$.)

1 food calorie (Cal)= 1000 cal

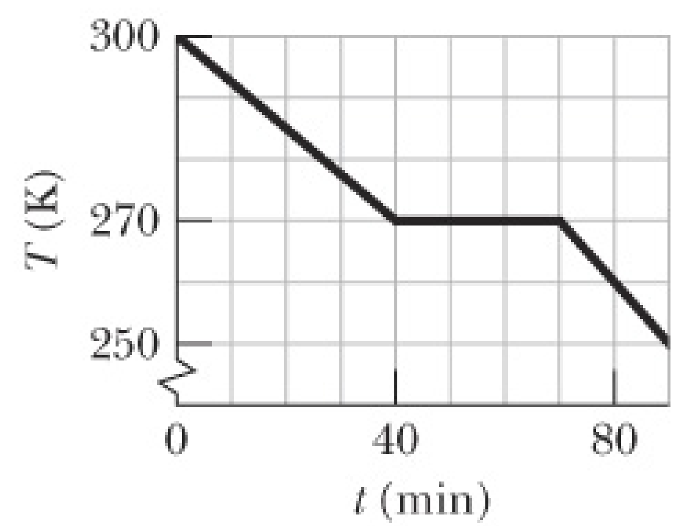
The mass of water needs to drink:

$$m = \frac{Q}{c\Delta T} = \frac{3500 \times 1000\,(cal)}{1(cal/g.K) \times (37-0)^0 C} = 94.6 \times 10^3 (g)$$

$$V = \frac{m}{\rho} = \frac{94.6 \times 10^3 g}{1000\, g/liter} = 94.6(liters)$$

too much water to drink!!!

**30. A 0.4 kg sample is placed in a cooling apparatus that removes energy as heat at a constant rate. The figure below gives T of the sample vs. time t; the sample freezes during the energy removal. The specific heat of the sample in its initial liquid phase is 3000 J/kg $K^{-1}$. What are (a) the sample's heat of fusion and (b) its specific heat in the frozen phase?**




**Key issue:** **The cooling apparatus removes energy as heat at a constant rate.**

**The rate of removed energy as heat (per minute):**

$$R = \frac{Q_{cooling}}{t_{cooling}} = \frac{cm\Delta T}{t_{cooling}}$$

$$R = 900\ (J/min)$$

**(a)**

$$Q_{freezing} = 900\ (J/min) \times 30\ (min) = 27000\ (J) \text{ or } 27\ (kJ)$$

$$Q_{freezing} = L_F m \Rightarrow L_F = 67.5\ (kJ/kg)$$

**(b)** $$Q_{frozen} = cm\Delta T \Rightarrow c = \frac{Q_{frozen}}{m\Delta T} = \frac{R \times 20(min)}{m \times 20(^0)} = 2250\left(\frac{J}{kg.K}\right)$$

**32. The specific heat of a substance varies with temperature according to $c=0.20+0.14T+0.023T^2$, with T in $^0C$ and c in cal/g $K^{-1}$. Find the energy required to raise the temperature of 1.0 g of this substance from $5^0C$ to $15^0C$.**

$$Q = cm\Delta T$$

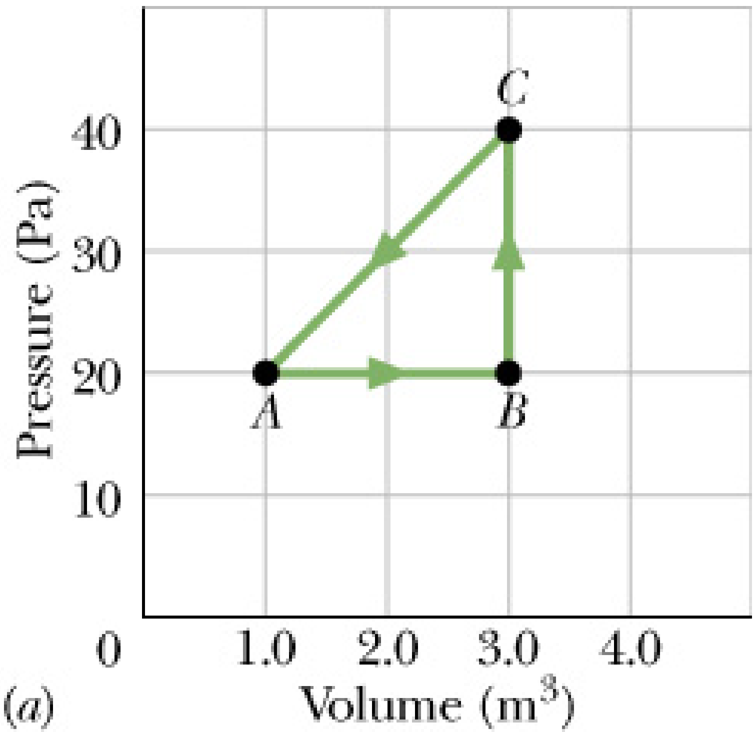
**In the case here:**

$$c = c(T)$$

$$dQ = cmdT$$

$$Q_{total} = \int_{T_1}^{T_2} cmdT = m\int_{T_1}^{T_2} cdT = m\int_{T_1}^{T_2} (0.20 + 0.14T + 0.023T^2)dT$$

44. A thermodynamic system is taken from state A to state B to state C, and then back to A, as shown in the p-V diagram of Fig.a. (a)-(g) Complete the table in Fig.b by inserting a plus sign, a minus sign, or a zero in each indicated cell. (h) What is the net work done by the system as it moves once through the cycle ABCA?




(a)

| | $Q$ | $W$ | $\Delta E_{int}$ |
|---|---|---|---|
| $A \longrightarrow B$ | (a) + | (b) + | + |
| $B \longrightarrow C$ | + | (c) 0 | (d) + |
| $C \longrightarrow A$ | (e) – | (f) – | (g) – |

(b)

$$\Delta E_{int} = Q - W$$

$$W = -\frac{1}{2} AB \times BC = -\frac{1}{2} 2 \times 20 = -20(J)$$

46. Suppose 200 J of work is done on a system and 80.0 cal is extracted from the system as heat. In the sense of the first law of thermodynamics, what are the values (including algebraic signs) of (a) W, (b) Q, and (c) $\Delta E_{int}$?

(a)

$$W_{on} = 200\,J$$

Work done on the gas = - work done by the gas

$W_{on}$ = - W

$$W = -W_{on} = -200(J)$$

(b) the gas released energy as heat, so Q<0:

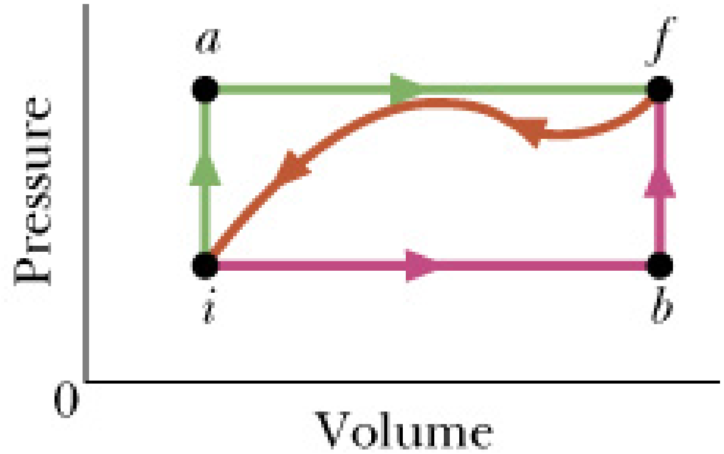
$$Q = -80\,cal = -80 \times 4.19 = -335.2(J)$$

(c) the first law of thermodynamics:

$$\Delta E_{int} = Q - W$$

$$\Delta E_{int} = -335.2 - (-200) = -135.2(J)$$

47. When a system is taken from state i to state f along path iaf in the figure below, Q = 50 cal and W = 20 cal. Along path ibf, Q = 36 cal. (a) What is W along path ibf? (b) If W = -13 cal for the return path fi, what is Q for this path? (c) If $E_{int,i}$ = 10 cal, what is $E_{int,f}$? If $E_{int,b}$ = 22 cal, what is Q for (d) path ib and (e) path bf?




$$\Delta E_{int} = Q - W$$

**(a) For path iaf:**

$$\Delta E_{int,iaf} = E_{int,f} - E_{int,i} = Q_{iaf} - W_{iaf}$$

**For path ibf:**

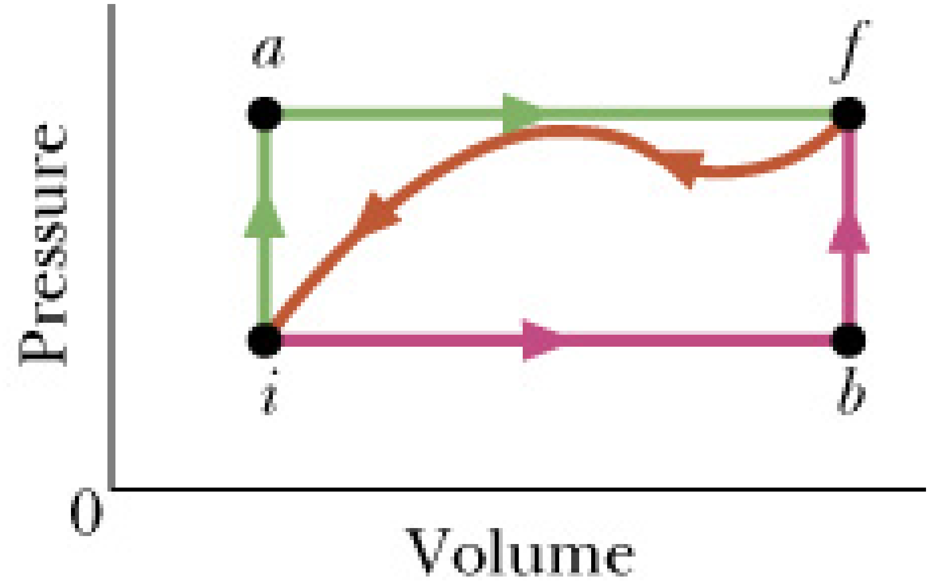
$$\Delta E_{int,ibf} = E_{int,f} - E_{int,i} = Q_{ibf} - W_{ibf} = \Delta E_{int,iaf}$$

$$\Rightarrow W_{ibf} = Q_{ibf} - (Q_{iaf} - W_{iaf}) = 36 - (50 - 20) = 6(\text{cal})$$

**(b) For path fi:** $\Delta E_{int,fi} = E_{int,i} - E_{int,f} = -\Delta E_{int,if} = -30(\text{cal})$

$$Q_{fi} = \Delta E_{int,fi} + W = -30 - 13 = -43(\text{cal})$$

**(c) For path fi:** $E_{int,f} = E_{int,i} - \Delta E_{int,fi} = 10 - (-30) = 40(\text{cal})$




**(d) For path ibf:**

$$W_{ibf} = W_{ib} = 6\,(cal)\text{ as } W_{bf} = 0\text{ (constant volume)}$$

$$\Delta E_{int,ib} = E_{int,b} - E_{int,i} = 22 - 10 = 12(cal)$$

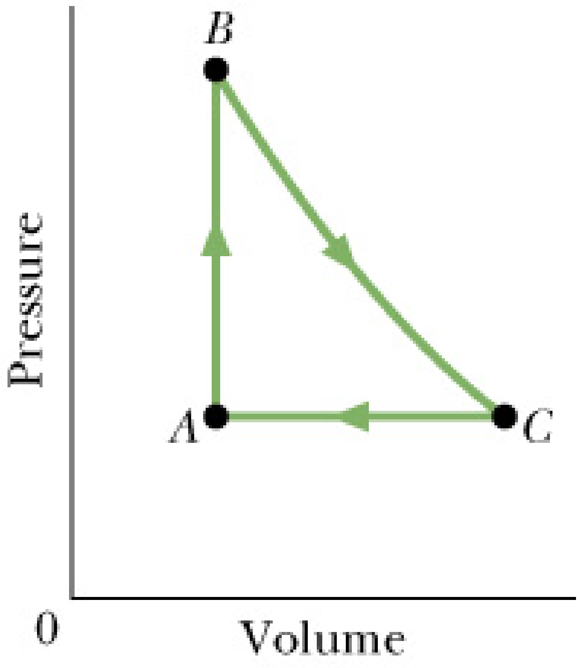
$$Q_{ib} = \Delta E_{int,ib} + W_{ib} = 12 + 6 = 18(cal)$$

**(e) For path ibf:**

$$Q_{bf} = Q_{ibf} - Q_{ib} = 36 - 18 = 18(cal)$$

48. Gas within a chamber passes through the cycle shown in the figure below. Determine the energy transferred by the system as heat during process *CA* if the energy added as heat $Q_{AB}$ during process *AB* is 25.0 J, no energy is transferred as heat during process *BC*, and the net work done during the cycle is 15.0 J.

$$\Delta E_{int} = Q - W$$




For the *ABCA* closed cycle:

$$\Delta E_{int} = 0$$

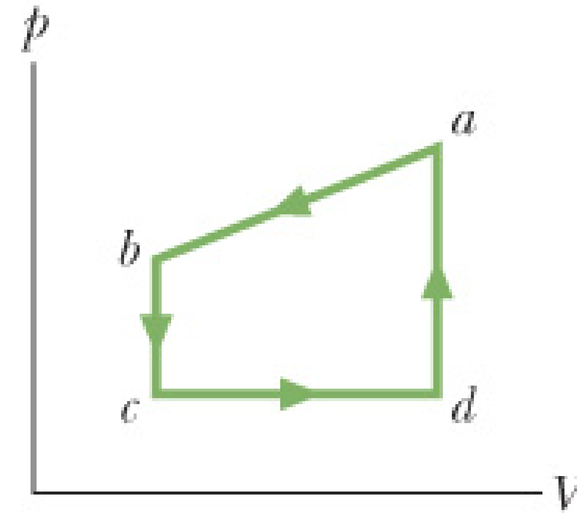
$$Q_{AB} + Q_{BC} + Q_{CA} = W$$

$$Q_{CA} = W - Q_{AB} - Q_{BC}$$

$$Q_{CA} = 15 - 25 - 0 = -10 \text{ (J)}$$

49. The figure below displays a closed cycle for a gas (the figure is not drawn to scale). The change in the internal energy of the gas as it moves from *a* to *c* along the path *abc* is -200 J. As it moves from *c* to *d*, 180 J must be transferred to it as heat. An additional transfer of 80 J as heat is needed as it moves from *d* to *a*. How much work is done by the gas as it moves from *c* to *d*?

$$\Delta E_{int} = Q - W$$




For a closed cycle:

$$\Delta E_{int} = 0$$

$$\Delta E_{abc} + \Delta E_{cd} + \Delta E_{da} = 0$$

$$\Delta E_{abc} = -200\ (J)$$

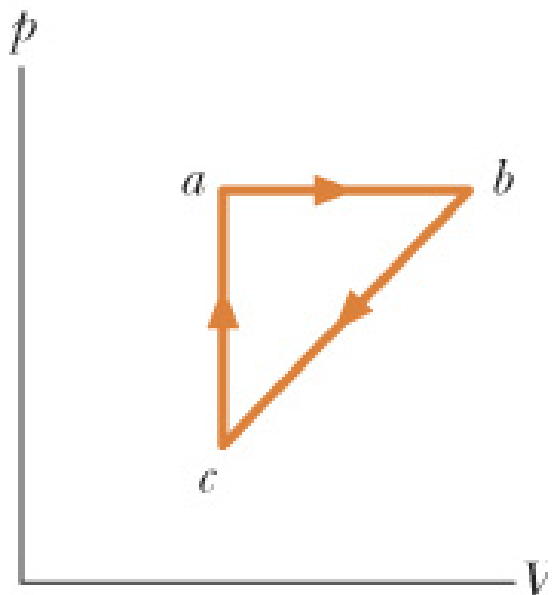
For process da: $\Delta E_{da} = Q - W = 80 - 0 = 80\ (J)$

$$\Delta E_{cd} = 200 - 80 = 120\ (J)$$

$$\Rightarrow W_{cd} = Q_{cd} - \Delta E_{cd} = 180 - 120 = 60\ (J)$$

**50. A sample of gas is taken through cycle *abca* shown in the p-V diagram (see figure). The net work done is +1.5 J. Along path *ab*, the change in the internal energy is +3.0 J and the magnitude of the work done is 5.0 J. Along path *ca*, the energy transferred to the gas as heat is 2.5 J. How much energy is transferred as heat along (a) path *ab* and (b) path *bc*?**

$$\Delta E_{int} = Q - W$$




**(a) This process a→b is an expansion ($V_b > V_a$):**

$$W > 0 \text{ and } W = 5\text{ J}$$

$$Q_{ab} = \Delta E_{int} + W = 3 + 5 = 8\ (\text{J})$$

**(b) We consider a closed cycle abca:**

$$\Delta E_{int} = Q - W = 0$$

$$Q_{ab} + Q_{bc} + Q_{ca} = W_{net}$$

$$Q_{bc} = W_{net} - Q_{ab} - Q_{ca} = 1.5 - 8 - 2.5 = -9.0\ (\text{J})$$

51. A sphere of radius 0.500 m, temperature 27.0°C, and emissivity 0.850 is located in an environment of temperature 77.0°C. At what rate does the sphere (a) emit and (b) absorb thermal radiation? (c) What is the sphere's net rate of energy exchange?

(a)

$$P_{rad} = \sigma\varepsilon A T^4$$

$$A = 4\pi R^2; T = 273 + 27 = 300\ (K)$$

$$P_{rad} \approx 1.23 \times 10^3\ (W)$$

(b)

$$P_{abs} = \sigma\varepsilon A T_{env}^4$$

$$T_{env} = 273 + 77 = 350\ (K)$$

$$P_{abs} \approx 2.27 \times 10^3\ (W)$$

(c) $$P_{net} = P_{abs} - P_{rad} = 2.27 \times 10^3 - 1.23 \times 10^3 = 1.04 \times 10^3\ (W)$$

54. If you were to walk briefly in space without a spacesuit while far from the Sun (as an astronaut does in the movie 2001), you would feel the cold of space – while you radiated energy, you would absorb almost none from your environment. (a) At what rate would you lose energy? (b) How much energy would you lose in 30 s? Assume that your emissivity is 0.90, and estimate other data needed in the calculations.

(a) The heat transfer mechanism is radiation:

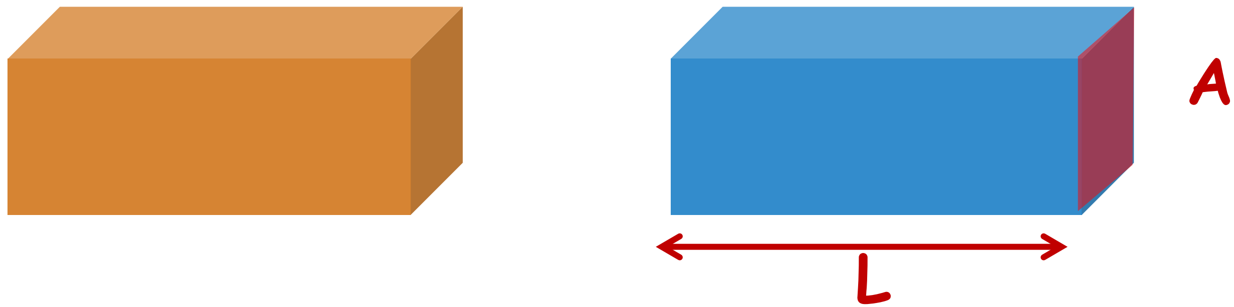
$$P_{rad} = \sigma \varepsilon A T^4$$

$$P_{rad} = 5.67 \times 10^{-8} \times 0.9 \times 2.0 \times 310^4 = 9.4 \times 10^2 \text{ (W)}$$

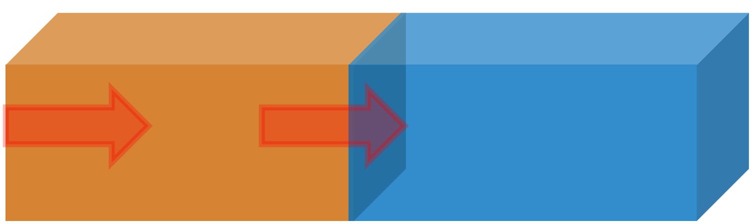
(b) The energy lost in 30 s is:

$$E = P_{rad} \times t = 9.4 \times 10^2 \times 30 = 2.8 \times 10^4 \text{ (J)}$$

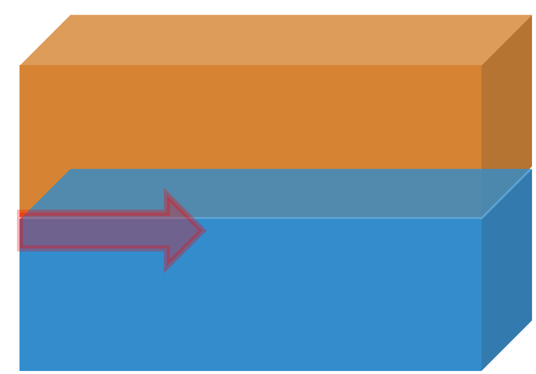
59. In Figure a, two identical rectangular rods of metal are welded end to end, with a temperature of $T_1=0^0C$ on the left side and a temperature of $T_2=100^0C$ on the right side. In 2.0 min, 10 J is conducted at a constant rate from the right side to the left side. How much time would be required to conduct 10 J if the rods were welded side to side as in Figure b.



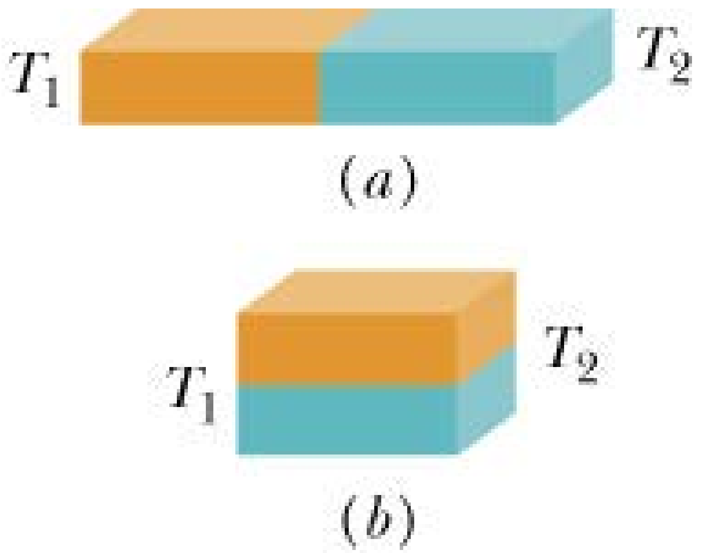

(a)



(b)



**The heat transfer mechanism is conduction:**




$$P_{cond} = kA\frac{T_H - T_C}{L}$$

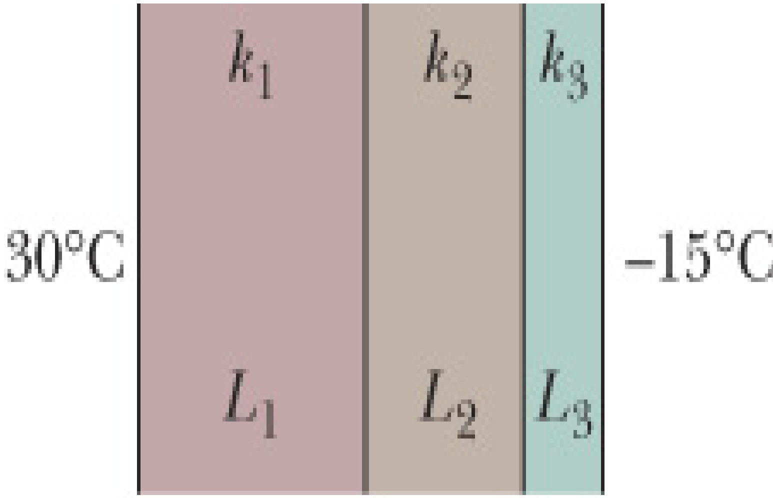
$$P_{cond,\,a} = kA_a\frac{T_H - T_C}{L_a}$$

$$P_{cond,\,b} = kA_b\frac{T_H - T_C}{L_b}$$

$$\Rightarrow P_{cond,\,b} = \frac{A_b}{A_a}\frac{L_a}{L_b}P_{cond,\,a} = 2\times 2\times P_{cond,\,a} = 4P_{cond,\,a}$$

**so, the requested time is 2.0/4=0.5 min or 30 s.**

60. The figure below shows the cross section of a wall made of three layers. The thicknesses of the layers are $L_1$, $L_2=0.750L_1$, and $L_3=0.350L_1$. The thermal conductivities are $k_1$, $k_2=0.900k_1$, and $k_3=0.800k1$. The temperatures at the left and right sides of the wall Are $30.0^0C$ and $-15.0^0C$, respectively. Thermal conduction through the wall has reached the steady state. (a) What is the temperature difference $\Delta T_2$ across layer 2 (between the left and right sides of the layer)? If $k_2$ were, instead, equal to $1.1k_1$, (b) would the rate at which energy is conducted through the wall be greater than, less than, or the same as previously, and (c) what would be the value of $\Delta T_2$?




(a)

$$P_{cond} = \frac{A(T_H - T_C)}{\sum(L/k)} = \frac{A\Delta T_2}{L_2/k_2}$$

$$\Delta T_2 = \frac{(L_2/k_2)(T_H - T_C)}{\sum(L/k)} \approx 16.5^0 C$$

(b) conductivity k increases → conduction rate increases.

(c) Repeat the calculation in part (a):

$$\Delta T_2 \approx 14.5^0 C$$