Example problems

Chapter 1: Fluid Mechanics

A fish maintains its depth in fresh water by adjusting the air content of porous bone or air sacs to make its average density the same as that of the water. Suppose that with its air sacs collapsed, a fish has a density of 1.08 g/cm³. To what fraction of its expanded body volume must the fish inflate the air sacs to reduce its density to that of water?

Let the volume of the expanded air sacs be $V_s$ and that of the fish be $V_f$:

\[
\rho_{\text{fish}} = \frac{m_{\text{fish}}}{V_f} = 1.08 \text{ (g/cm}^3\text{)}
\]

\[
\rho_{\text{water}} = \frac{m_{\text{fish}}}{V_f + V_s} = 1 \text{ (g/cm}^3\text{)}
\]

\[
\Rightarrow \frac{V_s}{V_f + V_s} = \frac{\rho_{\text{fish}} - \rho_{\text{water}}}{\rho_{\text{fish}}} \approx 7.4\%
\]
5. (Page 379)
An office window has dimensions 3.4 m by 2.1 m. As a result of the passage of a storm, the outside air pressure drops to 0.93 atm, but inside the pressure is held at 1.0 atm. What net force pushes out on the window?

\[ F = \Delta P \times A = (1 - 0.93) \times 1.01 \times 10^5 \times (3.4 \times 2.1) \approx 5.1 \times 10^4 \text{ (N)} \]

1 atm = 1.01 x 10^5 Pa; Pa = N/m^2
14. (P.380)
Calculate the hydrostatic difference in blood pressure between the brain and the foot in a person of height 1.83 m. The density of blood is $1.06 \times 10^3$ kg/m$^3$.

The hydrostatic difference in blood pressure is:

\[
\Delta p = \rho gh
\]

\[
= 1.06 \times 10^3 \times 9.8 \times 1.83
\]

\[
= 1.9 \times 10^4 \text{ (Pa)}
\]
A piston of cross-sectional area \( a \) is used in a hydraulic press to exert a small force of magnitude \( f \) on the enclosed liquid. A connecting pipe leads to a larger piston of cross-sectional area \( A \). (a) What force magnitude \( F \) will the larger piston sustain without moving? (b) If the piston diameters are 3.8 cm and 53.0 cm, what force magnitude on the small piston will balance a 20.0 kN force on the large piston.

(a) Applying Pascal’s principle:

\[
\frac{f}{a} = \frac{F}{A} \quad \Rightarrow \quad F = \frac{fA}{a}
\]

(b) We obtain:

\[
f = \frac{Fa}{A} \quad ; \quad f = \frac{F\pi\left(\frac{d}{2}\right)^2}{\pi\left(\frac{D}{2}\right)^2} = F\left(\frac{d}{D}\right)^2
\]

\[
f \approx 103 \text{ (N)} \quad \Rightarrow \quad f \text{ is about 200 smaller than } F
\]
38. (Page 381) A small solid ball is released from rest while fully submerged in a liquid and then its kinetic energy is measured when it has moved 4.0 cm in the liquid. Figure (below) gives the results after many liquids are used: The kinetic energy $K$ is plotted versus the liquid density $\rho_{\text{lid}}$. What are (a) the density and (b) the volume of the ball?

(a) An object, which has the same density as the liquid surrounding, won’t gain any kinetic energy ($K = 0$) after releasing from rest: At $K = 0$, $\rho_{\text{lid}} = 1.5 \text{ g/cm}^3$. So, $\rho_{\text{ball}} = 1.5 \text{ g/cm}^3$ or 1500 kg/m$^3$.

(b) At $\rho_{\text{lid}} = 0$, $K = 1.6 \text{ J}$: In this case, the ball is freely falling in vacuum:

$$v^2 = 2gh; \quad K = \frac{1}{2}mv^2$$

$$m = \frac{K}{gh} = \frac{1.6}{9.8 \times 4.0 \times 10^{-2}} = 4.08 \text{ (kg)} \Rightarrow V_{\text{ball}} = \frac{m}{\rho_{\text{ball}}} = 2.72 \times 10^{-3} \text{ (m}^3)$$
The intake in the figure has cross-sectional area of 0.74 m$^2$ and water flow at 0.40 m/s. At the outlet, distance D = 180 m below the intake, the cross-sectional area is smaller than at the intake and the water flows out at 9.5 m/s. What is the pressure difference between inlet and outlet?

Using Bernoulli’s equation:

$$p_1 + \frac{1}{2} \rho v_1^2 + \rho g D = p_2 + \frac{1}{2} \rho v_2^2$$

$$\Delta p = p_1 - p_2 = \frac{1}{2} \rho (v_2^2 - v_1^2) - \rho g D$$

$$\Delta p = \frac{1}{2} \times 1000 \times \left(9.5^2 - 0.4^2\right) - 1000 \times 9.8 \times 180 = -1.72 \times 10^6 (Pa)$$
In the figure below, water flows through a horizontal pipe and then out into the atmosphere at a speed \( v_1 = 15 \text{ m/s} \). The diameters of the left and right sections of the pipe are 5.0 cm and 3.0 cm. (a) What volume of water flows into the atmosphere during a 10 min period? (b) In the left section of the pipe, what are (b) the speed \( v_2 \) and (c) the gauge pressure?

(a) The volume of water during 10 min:

\[
V = v_1 \times t \times \pi \times \frac{d_1^2}{4} \approx 6.4 (\text{m}^3)
\]

(b) Using the equation of continuity, the speed \( v_2 \) is:

\[
v_2 = \frac{v_1 A_1}{A_2} = \frac{15 \times 3^2}{5^2} = 5.4 (\text{m/s})
\]
The gauge pressure = the absolute pressure - the atmospheric pressure

Using Bernoulli’s equation for a horizontal pipe, we have:

\[ p_1 + \frac{1}{2} \rho v_1^2 = p_2 + \frac{1}{2} \rho v_2^2 \]

\[ p_1 = p_0; \]  where \( p_0 \) is the atmospheric pressure

The gauge pressure of the left section of the pipe is:

\[ p_g = p_2 - p_0 = \frac{1}{2} \rho (v_1^2 - v_2^2) \]

\[ p_g = \frac{1}{2} 10^3 \times (15^2 - 5.4^2) = 0.98 \times 10^5 \text{ (Pa)} \]

or

\[ p_g = \frac{0.98 \times 10^5}{1.01 \times 10^5} = 0.97 \text{ (atm)} \]