Part B: Laws of Conservation

Chapter 3: Work and Mechanical Energy

3.1. Kinetic Energy and Work

3.2. Work-Kinetic Energy Theorem

3.3. Power

3.4. Potential Energy and Work

3.5. Conservative and Non-conservative Forces.

3.6. Work Done on a System by an External Force. Conservation of Energy.

3.7. Conservation of Mechanical Energy

What is energy?

Energy is the capacity of a system to do work.

There are many forms of energy:

- Mechanical energy:
  - Potential energy, stored in a system
  - Kinetic energy, from the movement of matter
- Radiant energy (solar energy)
- Thermal energy
- Chemical energy (chemical bonds of molecules)
- Electrical energy (movement of electrons)
- Electromagnetic energy (from X-rays to radio waves)
- Nuclear energy

Energy can be transformed from one type to another and transferred from one object to another, but the total amount is always the same (the principle of energy conservation).

3.1. Kinetic Energy and Work

3.1.1. Kinetic Energy

Kinetic energy is energy associated with the state of motion of an object.

$$K = \frac{1}{2}mv^2$$

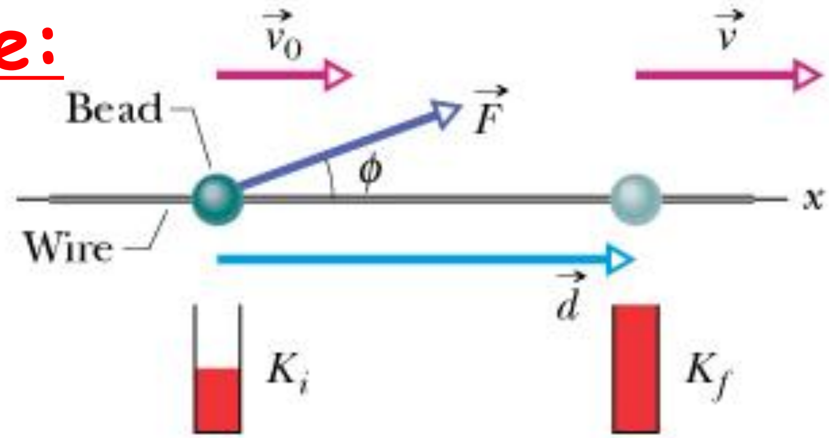
Unit: joule (J)

1 joule = 1 J = 1 kg. m^2/s^2

Work is energy transferred to or from an object by means of a force acting on the object.
Energy transferred to the object is positive work and energy transferred from the object is negative work.

3.1.2. Work done by a force

A. Work done by a constant force:



•To establish an expression for work, we consider a constant force F that accelerates a bead along a wire:

$$\begin{cases} F_x = ma_x \\ v^2 = v_0^2 + 2a_x d \end{cases}$$

$$\Rightarrow \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = F_x d$$

Therefore, the work W done on the bead by F is:

$$W = F_x d = Fd\cos\phi$$

$$\boxed{W = \vec{F}\vec{d}}$$

Work done by a constant force

Work can be **positive** or **negative**

- **Positive if the force and the displacement are in the same direction**
- **Negative if the force and the displacement are in the opposite direction**

Example 1: lifting a cement block…

Work done by the person:

is positive when lifting the box

is negative when lowering the box

Example 2: … then moving it horizontally

Work done by gravity:

is negative when lifting the box

is positive when lowering the box

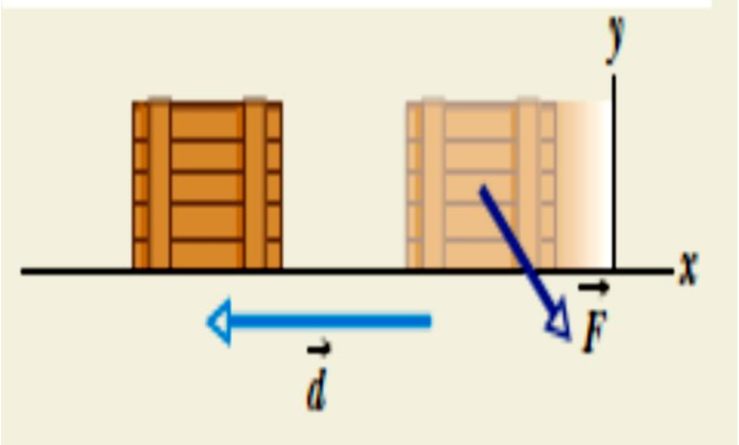
is zero when moving it horizontally

$$\underline{Total\ work}: W = W_1 + W_2 + W_3 = \underbrace{-mgh}_{\text{lifting}} + \underbrace{mgh}_{\text{lowering}} + \underbrace{0}_{\text{moving}} = \underbrace{0}_{\text{total}}$$

Example:

During a storm, a crate of crepe is sliding across a slick, oily parking lot through a displacement $\vec{d} = (-3.0m)\hat{\imath}$ while a steady wind pushes against the crate with a force $\vec{F} = (2.0N)\hat{\imath} + (-6.0N)\hat{\jmath}$. The situation and coordinate axes.

How much work does this force do on the crate during the displacement?



Calculations: We write

$$W = \vec{F} \cdot \vec{d} = [(2.0\text{ N})\hat{i} + (-6.0\text{ N})\hat{j}] \cdot [(-3.0\text{ m})\hat{i}].$$

Of the possible unit-vector dot products, only $\hat{i} \cdot \hat{i}$, $\hat{j} \cdot \hat{j}$, and $\hat{k} \cdot \hat{k}$ are nonzero (see Appendix E). Here we obtain

$$W = (2.0\text{ N})(-3.0\text{ m})\hat{i} \cdot \hat{i} + (-6.0\text{ N})(-3.0\text{ m})\hat{j} \cdot \hat{i}$$

$$= (-6.0\text{ J})(1) + 0 = -6.0\text{ J}. \qquad \text{(Answer)}$$

B. Work done by a general variable force:

One-dimensional analysis:

- Choose Δx small enough, work done by the force in the j*th* interval:

$$\Delta W_j = F_{j,avg}\Delta x$$

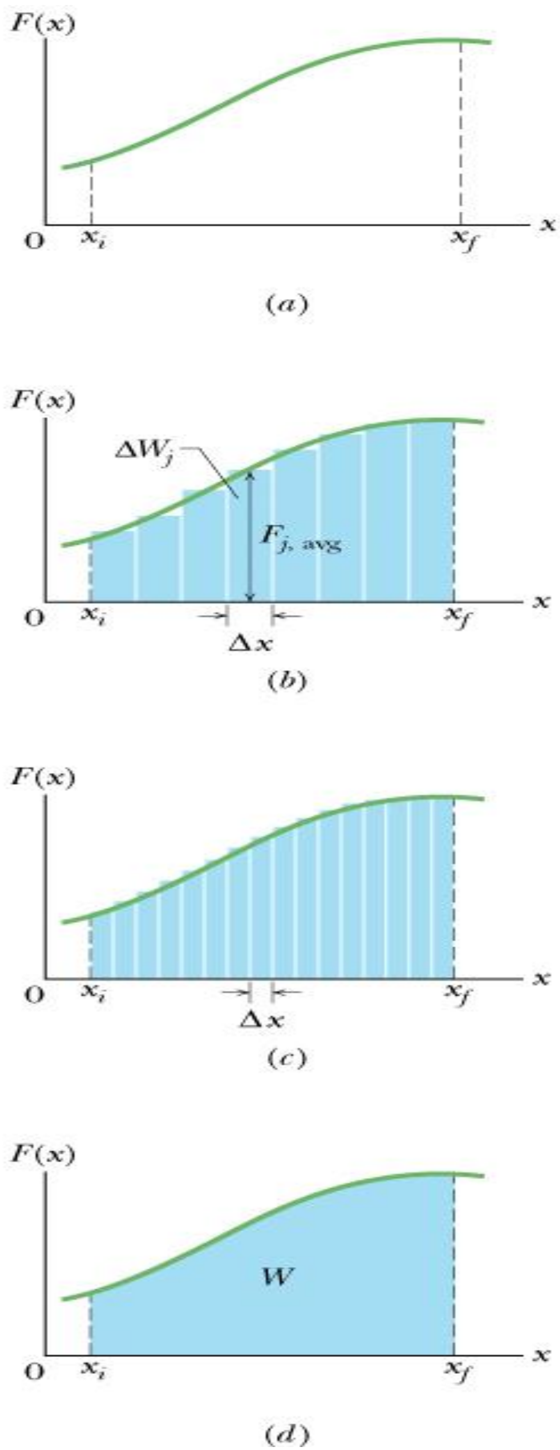
- The total work:

$$W = \sum \Delta W_j = \sum F_{j,avg}\Delta x$$

$$W = \lim_{\Delta x \to 0} \sum F_{j,avg}\Delta x$$

$$\boxed{W = \int_{x_i}^{x_f} F(x)dx}$$

Work done by a variable force



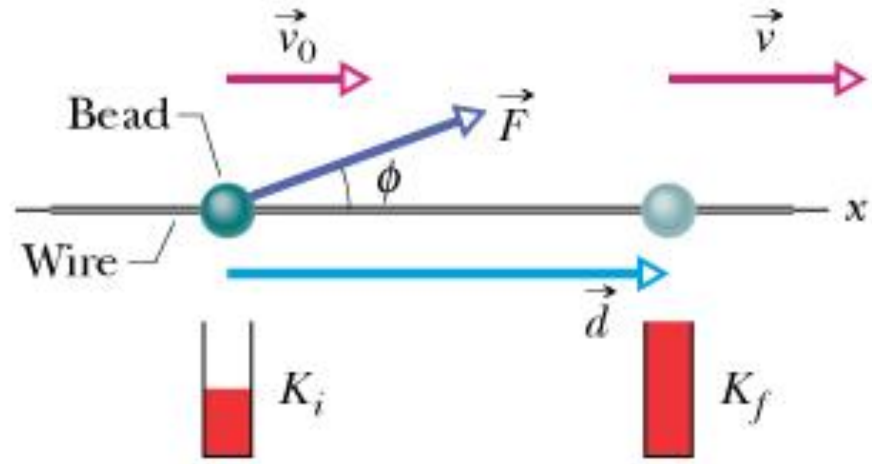
Three-dimensional analysis:

$$dW = \vec{F}d\vec{r} = F_x dx + F_y dy + F_z dz$$

$$\boxed{W = \int_{r_i}^{r_f} dW = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz}$$

3.2. Work-Kinetic Energy Theorem

Let ΔK be the change in the kinetic energy of the bead.



$$\Delta K = K_f - K_i = W$$

This can be read as follows:

$$\begin{pmatrix}\text{change in the kinetic} \\ \text{energy of an object}\end{pmatrix} = \begin{pmatrix}\text{net work done on} \\ \text{the object}\end{pmatrix}$$

or $$K_f = K_i + W$$

$$\begin{pmatrix}\text{kinetic energy after} \\ \text{the net work is done}\end{pmatrix} = \begin{pmatrix}\text{kinetic energy} \\ \text{before the net work}\end{pmatrix} + \begin{pmatrix}\text{the net} \\ \text{work done}\end{pmatrix}$$

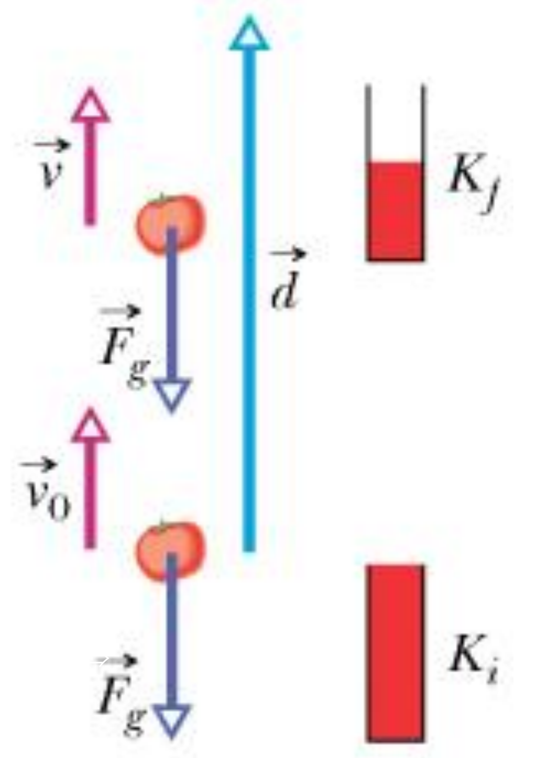
Example1:

Work done by the gravitational force:

$$W = F_x d = Fd\cos\phi$$

For a rising object, Φ=180⁰: $W = -mgd$

For a falling object, Φ=0⁰: $W = +mgd$



Work done in lifting and lowering an object

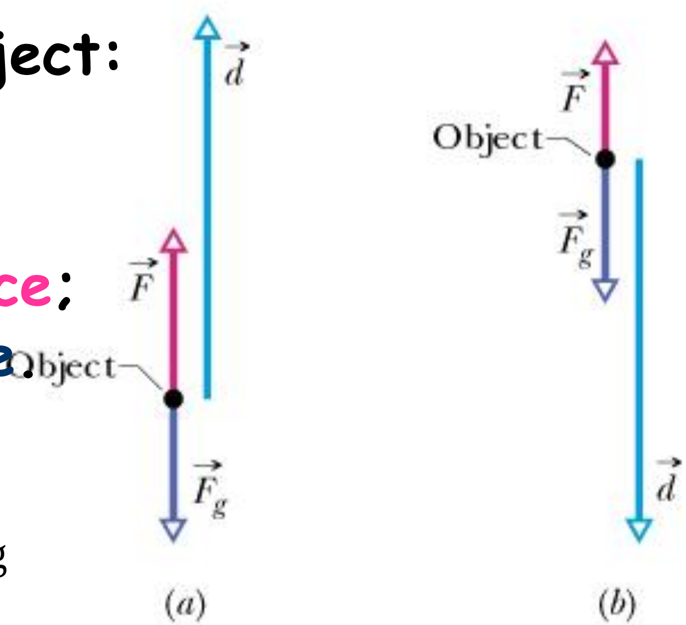
Gravity and an applied force acting on the object:

$$\Delta K = K_f - K_i = W_a + W_g$$

where W_a is the work done by the applied force;
W_g is the work done by the gravitational force.
If initial and final velocities are zero:

$$K_f = K_i = 0 \Rightarrow \Delta K = 0 \Rightarrow W_a = -W_g$$

$$\Rightarrow W_a = -mgd\cos\phi$$



Example 2:

Work done by a spring force:

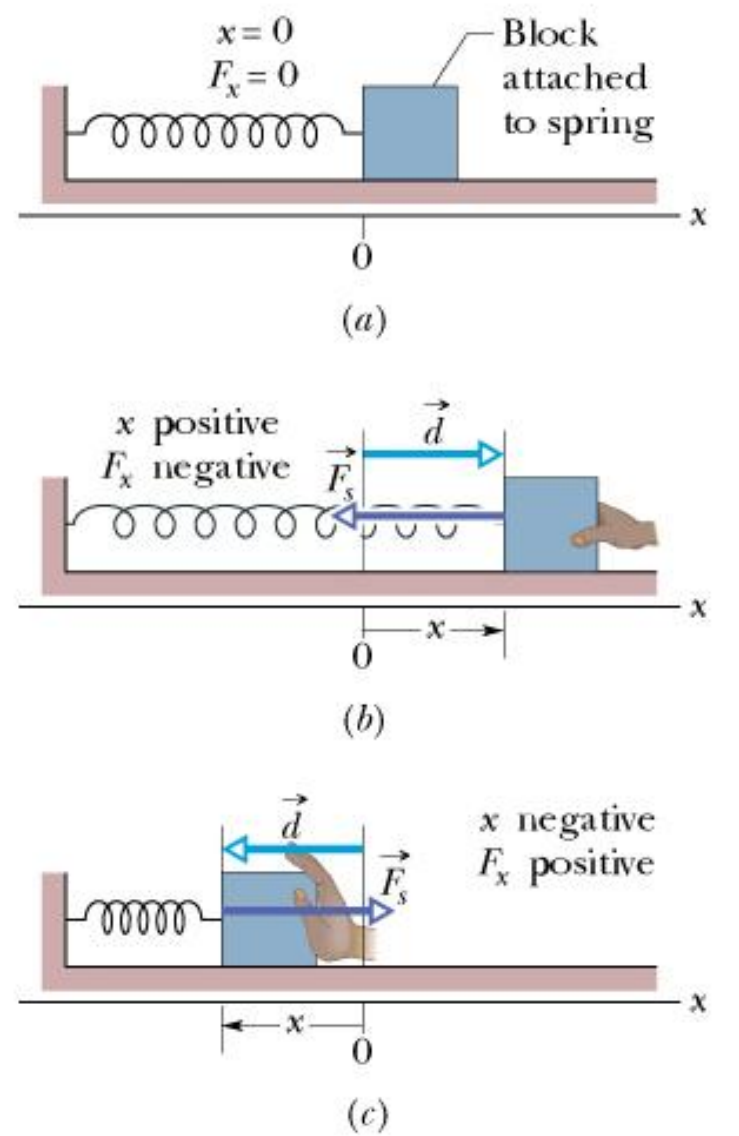
The spring force is computed by:

$$\vec{F}_s = -k\vec{d} \text{ (Hooke's law)}$$

k: the spring constant (or force constant)

in an x axis is parallel to the length of the spring:

$$F_x = -kx \text{ (Hooke's law)}$$



➔A spring force is a variable force F=F(x)

To find the work done by the spring force,
We have to make assumptions:
(1) the spring is massless;
(2) It is an ideal spring (obeys Hooke's law).

$$W_s = \lim_{\Delta x \to 0} \sum \vec{F}_{xj} \Delta\vec{x} = \lim_{\Delta x \to 0} \sum F_{xj} \Delta x \cos\theta = \lim_{\Delta x \to 0} \sum - F_{xj} \Delta x$$

Note: $\theta = 180^0$ and F_{xj} is the magnitude of the spring force

$$W_s = \int_{x_i}^{x_f} -|F_x| dx = \int_{x_i}^{x_f} -kx dx$$

$$\boxed{W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2}$$

$$\text{If } x_i = 0, x_f = x : W_s = -\frac{1}{2}kx^2$$

W_s the work done by the spring force

Work done by an applied force W_a :

$$\Delta K = K_f - K_i = W_a + W_s$$

If the block is stationary before and after the displacement, ΔK=0:

$$W_a = -W_s$$

3.3. Power

Power is the rate at which work is done.

Average power:

$$P_{avg} = \frac{W}{\Delta t}$$

Instantaneous power:

$$P = \frac{dW}{dt}$$

Unit: watt (W)

$$1\,\text{watt} = 1\,\text{W} = 1\,\text{J/s}$$

$$1\,\text{horsepower} = 1\,\text{hp} = 550\,\text{ft.lb/s} = 746\,\text{W}$$

$$1\,\text{kilowatt-hour} = 1\,\text{kW.h} = (10^3\,\text{W})(3600\text{s}) = 3.6\times 10^6\,\text{J} = 3.6\,\text{MJ}$$

F=constant: $$P = \frac{dW}{dt} = \frac{F\cos\phi\, dx}{dt} = F\cos\phi\left(\frac{dx}{dt}\right) = Fv\cos\phi$$

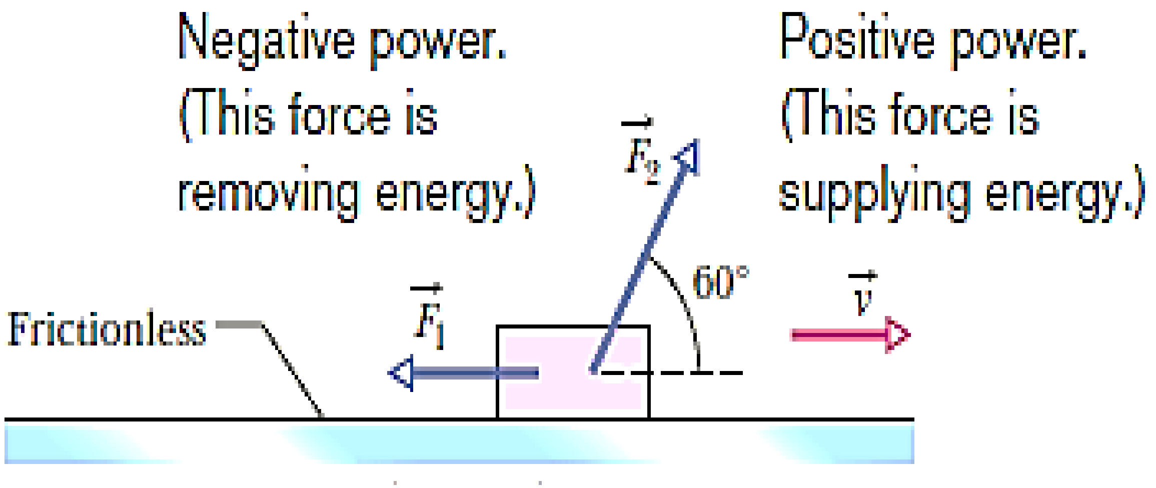
Instantaneous power:

$$P = \vec{F}\vec{v}$$

Example:

Two constant forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ acting on a box as the box slides rightward across a frictionless floor. Force $\overrightarrow{F_1}$ is horizontal with magnitude 2.0 N; force $\overrightarrow{F_2}$ is angled upward by 60° to the floor and has magnitude 4.0 N. The speed v of the box at a certain instant is 3.0 m/s. What is the **power** due to each force acting on the box at that instant, and what is the **net power**? Is the net power changing at that instant?

$$P = \vec{F} . \vec{v}$$



Homework:

1, 8, 15, 24, 26, 29, 36, 43, 48 (p. 159-163)

Part B: Laws of Conservation

Chapter 3: Work and Mechanical Energy

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3.2. Work-Kinetic Energy Theorem

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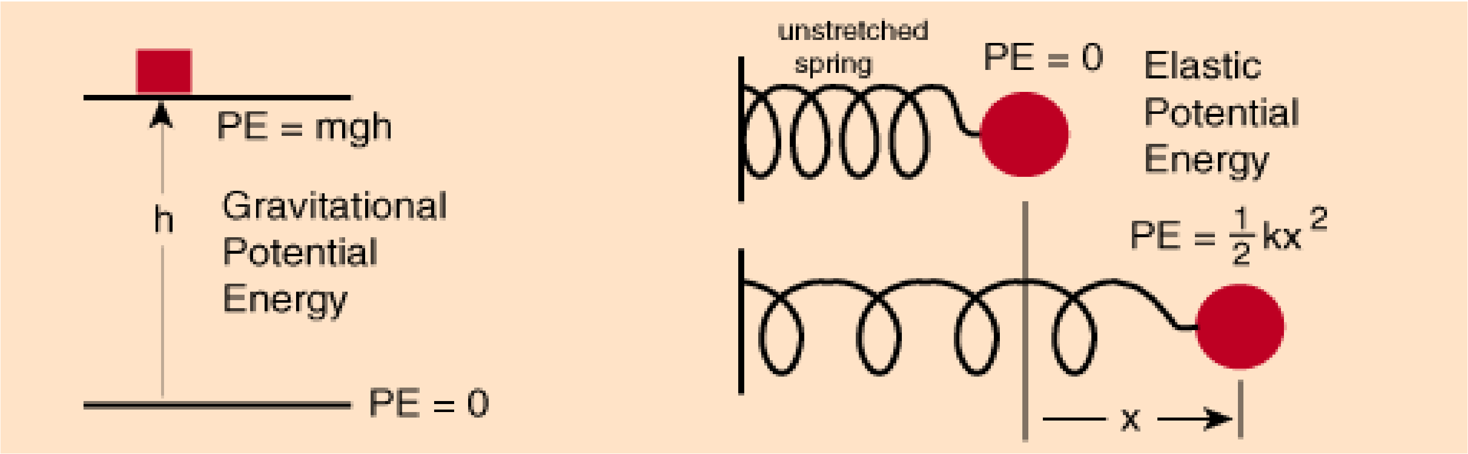
3.7. Conservation of Mechanical Energy

3.4. Potential Energy

Potential energy (U) is the energy associated with the configuration of a system of objects that exert forces on one another (a STORED ENERGY because of its position relative to other objects).

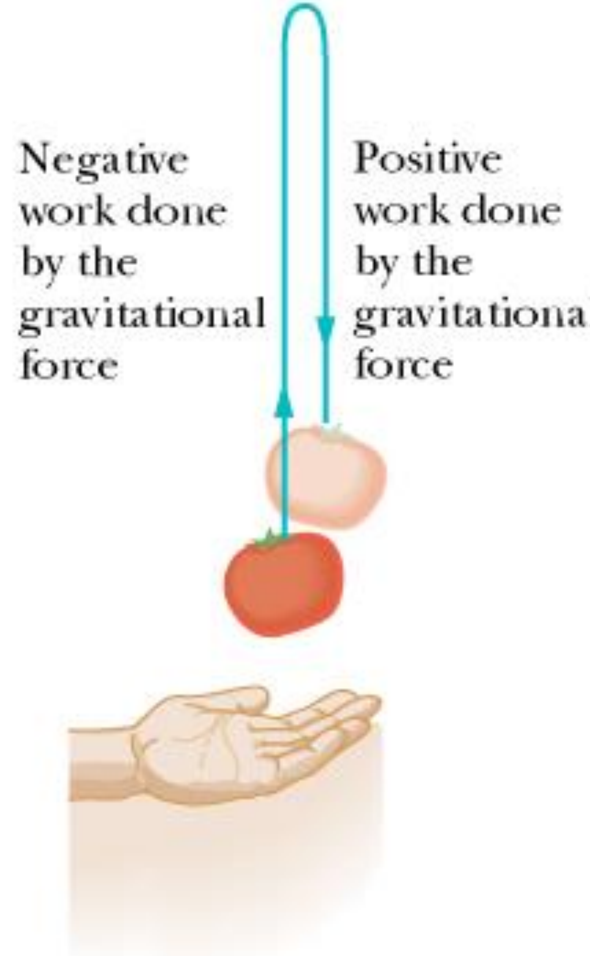
<u>We study here two kinds of potential energy</u>:

1. **Gravitational potential energy**: The energy is associated with the state of separation between two objects that attract each other, e.g., a jumper and the Earth.
2. **Elastic potential energy**: The energy is associated with the state of compression or extension of an elastic object, e.g., the bungee cord.



Example: throw a tomato upward.

When the tomato goes upward, the gravitational force does negative work, decreasing its kinetic energy: the kinetic energy is transferred by the gravitational force to the gravitational potential energy of the tomato-Earth system.



When it goes downward, the gravitational force does positive work, increasing its kinetic energy: the gravitational PE is transferred by the gravitational force to the KE of the tomato.

We define that **the change ΔU in gravitational PE** is equal to the negative of the work done on the object (e.g., the tomato) by the gravitational force:

$$\Delta U = -W$$

Note: This equation also applies to block-spring systems.

3.5. Conservative and Non-conservative Forces.

3.5.1. Conservative and Non-conservative Forces

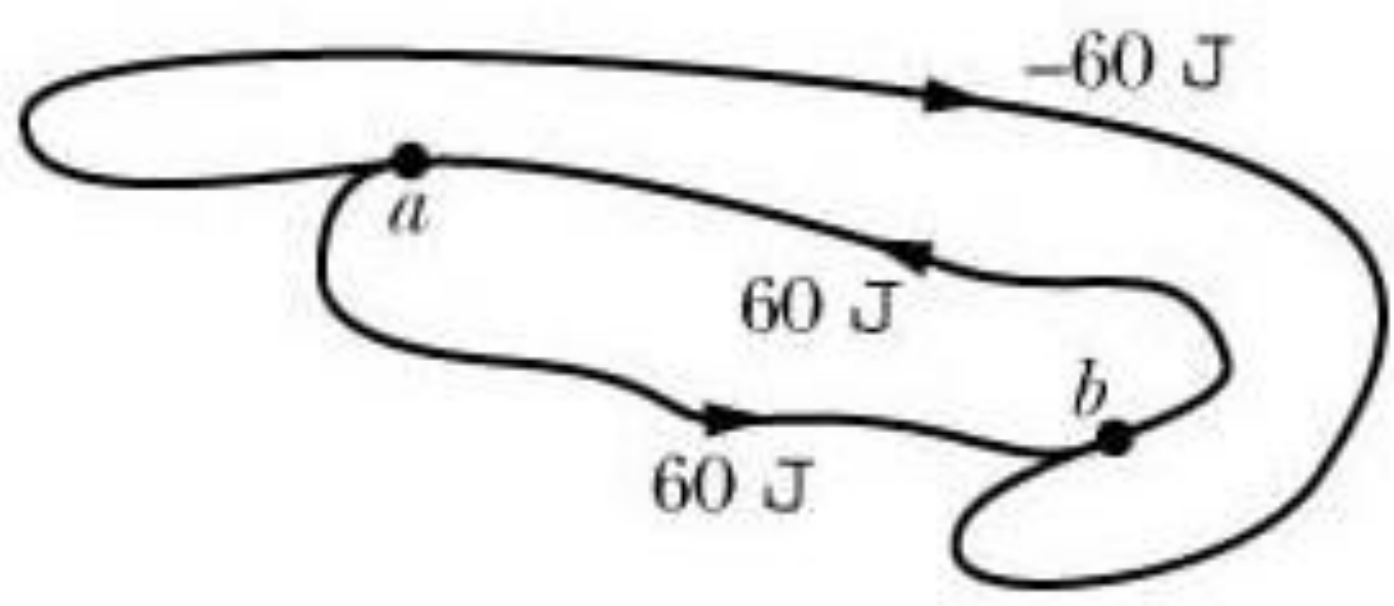
- A force is conservative if the work done by the force on an object moving between two points is independent of the path taken between the two points, e.g., gravitational force, spring force.

- Nonconservative force: the work done by a non-conservative force depends on the path taken.

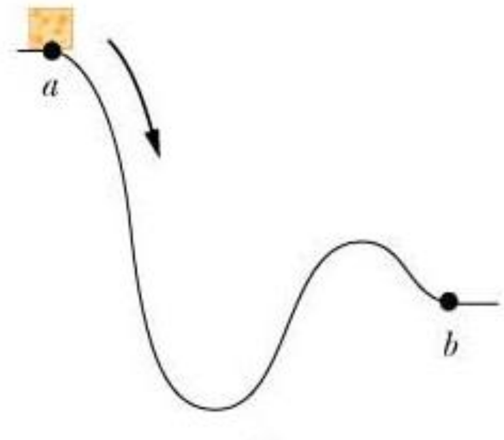
E.g.: frictional force F(x)

- A net work done by a conservative force on a particle moving around any closed path is zero.

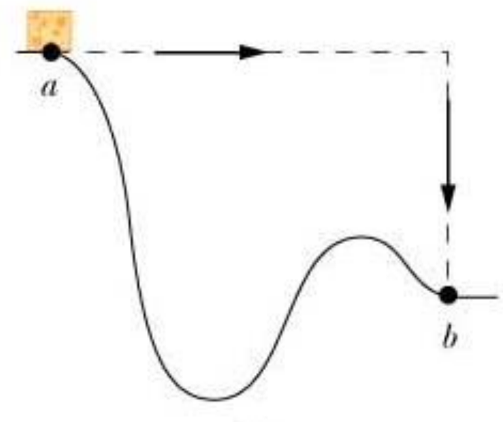
Checkpoint: The figure shows three paths connecting points a and b. A single force F does the indicated work on a particle moving along each path in the indicated direction. On the basis information, is the force F conservative?



Sample Problem (p. 170): A 2 kg block of slippery cheese slides along a frictionless track from point a to point b. The cheese travels through a total distance of 2.0 m along the track, and a net vertical distance of 0.8 m. How much work is done on the cheese by the gravitational force during the slide?



The gravitational force F_g is a conservative force → choose a simple path to calculate the work done by F_g.



$$W = W_{horizontal} + W_{vertical}$$

$$W_{horizontal} = F_g \Delta x \cos\theta = 0 \text{ because } \theta = (F, \Delta x) = 90^0$$

$$W_{vertical} = F_g \Delta y \cos\theta = mg\Delta y \ \ (\theta = (F, \Delta y) = 0^0)$$

$$= 2 \times 9.8 \times 0.8 \approx 15.7 \ \ (J)$$

$$W = W_{horizontal} + W_{vertical} = 15.7 \ \ (J)$$

3.5.2. Conservative Forces and Potential Energy

The work done by a force F=F(x):

$$\mathrm{W} = \int_{x_i}^{x_f} F(x)dx$$

The change of potential energy

$$\Delta\mathrm{U} = -\mathrm{W} = -\int_{x_i}^{x_f} F(x)dx$$

a. Gravitational Potential Energy

$$\Delta U = -W = -\int_{y_i}^{y_f} (-mg)dy = mg(y_f - y_i)$$

At a certain height y: $\Delta U = U - U_i = mg(y - y_i)$

If we take U_i to be the gravitational PE of the system when it is in a reference configuration in which the object is at a reference point y_i and U_i=0 and y_i=0, we have:

$$\boxed{U(y) = mgy}$$

Gravitational Potential Energy

→ **U(y) depends only on the vertical position y of the object relative to the reference point y=0.**

b. Elastic Potential Energy

$$\Delta U = -\int_{x_i}^{x_f} (-kx)dx = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

If we choose the reference configuration to be when the spring at its relaxed length, x_i=0 and U_i=0:

$$U = \frac{1}{2}kx^2 \quad \text{(elastic potential energy)}$$

3.6. Conservation of Mechanical Energy

The mechanical energy of a system:

$$E_{mec} = K + U$$

If we only consider conservative forces that cause energy transfers within the system and assume that the system is isolated (i.e., no external forces):

$$\left.\begin{array}{l} \Delta K = W \\ \Delta U = -W \end{array}\right\} \Rightarrow \Delta K = -\Delta U$$

$$K_2 - K_1 = -(U_2 - U_1) \quad \textbf{or} \quad \boxed{K_2 + U_2 = K_1 + U_1}$$

In an isolated system, the kinetic energy and potential energy can change but the mechanical energy of the system is a constant.

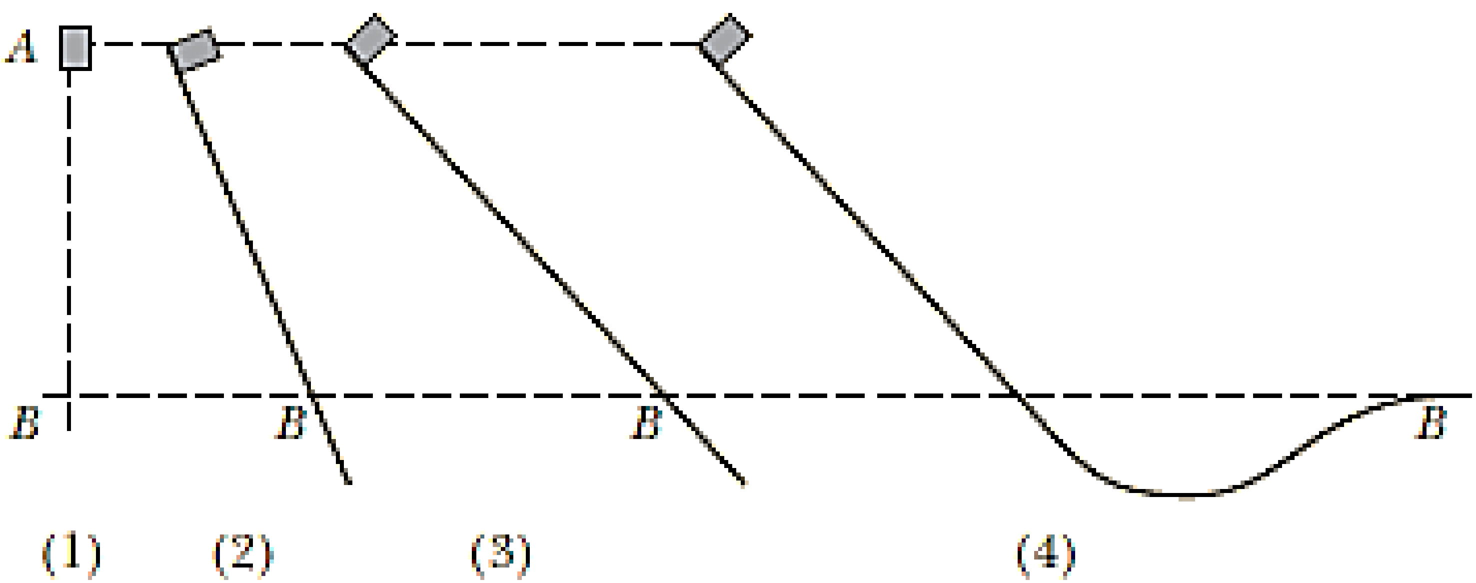
$$\Delta E_{mec} = \Delta K + \Delta U = 0$$

Checkpoint:

The figure shows four situations—one in which an initially stationary block is dropped and three in which the block is allowed to slide down frictionless ramps.

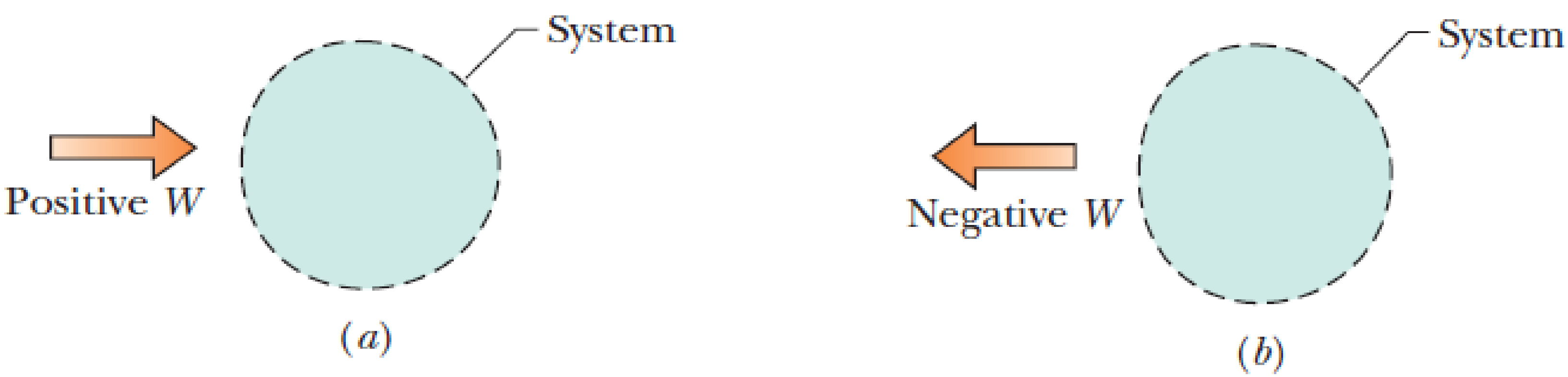
(a) Rank the situations according to the kinetic energy of the block at point B, greatest first.

(b) Rank them according to the speed of the block at point B, greatest first.



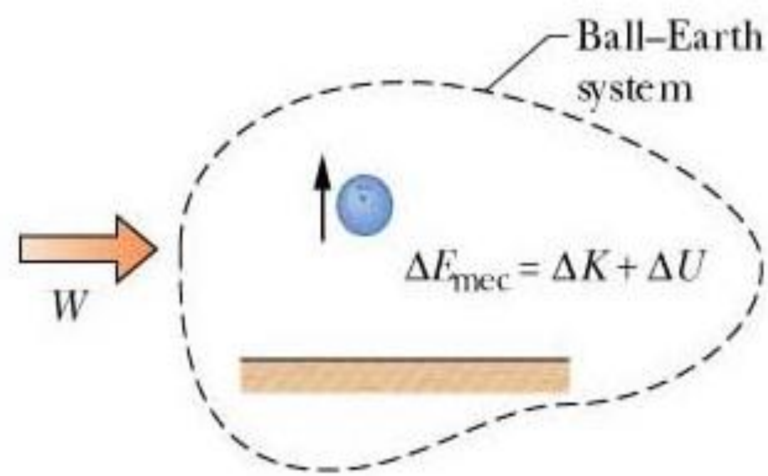
3.7. Work Done on a System by an External Force.

Work is energy transferred to or from a system by means of an external force acting on that system.



a. No friction involved:

$$W = \Delta E_{mec} = \Delta K + \Delta U$$

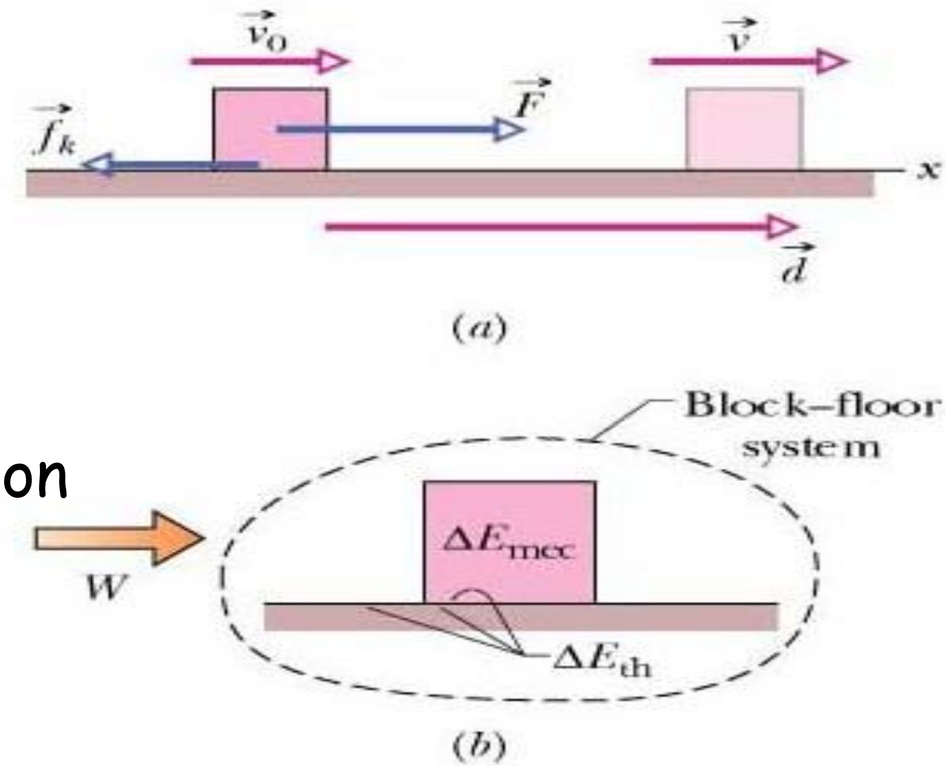


➔**The work done by an external force on a system is equal to the change in the mechanical energy of the system.**

b. Friction involved:

$$F_{net,x} = ma_x$$

$$F - f_k = ma$$



The forces F and the acceleration a are constants.

Work done on the systems

$$W = \Delta E_{mechanical} + \Delta E_{thermal}$$

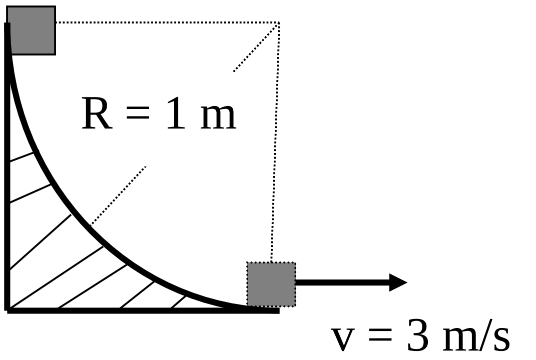
where

$$\Delta E_{thermal} = f_k d \text{ (increase in thermal energy by friction)}$$

f_k: the frictional force

Positive work done on the block-floor system = A change in the block's <u>mechanical energy</u> + A change in <u>the thermal energy</u> of the block and floor.

Example: A block of mass 1.0 kg is released from rest and slides down a rough track of radius R = 1.0 m (Figure 1). If the speed of the block at the bottom of the track is 3.0 m/s, what is the work done by the frictional force acting on the block? (Final exam, June 2014)



$$W = \Delta E_{mechanical} + \Delta E_{thermal}$$

W: work done by external (applied) forces

In this case: W = 0

$$|W_{friction}| = \Delta E_{thermal} = -\Delta E_{mechanical}$$

$$\Delta E_{mechanical} = \Delta K + \Delta U = E_{mechanical\text{, bottom}} - E_{mechanical\text{, top}}$$

$$\Delta E_{mechanical} = \left(0 + \frac{1}{2}mv^2\right) - (mgh - 0)$$ **(We choose U_{bottom} = 0)**

$$|W_{friction}| = mgh - \frac{1}{2}mv^2 = 1.0(9.8 \times 1.0 - \frac{1}{2} \times 3^2) = 5.3\ (J)$$

The law of conservation of energy:

The total energy E of a system can change only by amounts of energy that are transferred to or from the system.

Energy transfer = work done on the system

$$W = \Delta E_{mechanical} + \Delta E_{thermal} + \Delta E_{internal}$$

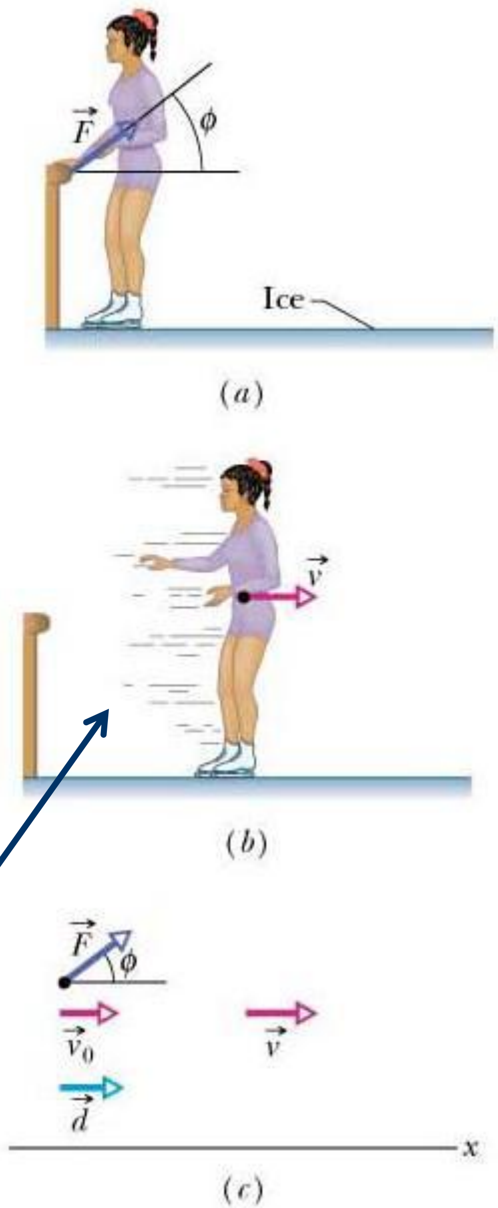
Isolated systems:

No energy transfer to or from the systems.

$$W = 0 \iff \Delta E_{mechanical} + \Delta E_{thermal} + \Delta E_{internal} = 0$$

If **an external force acts on the system but does no work**, i.e. no energy transfer to the system, **the force can change the KE or PE of the system**.

Her KE increases due to internal transfers from the biochemical energy in her muscles.



Homework:

2, 3, 6, 10, 16, 22, 24, 29, 44, 56, 79

(pages 191-197)

Problem-Solving Strategy
with Conservation of Energy

- Define the system
- Select the location of zero gravitational potential energy (U_g)
  - Do *not* change this location while solving the problem
- Determine whether or not non-conservative forces are present
- If only conservative forces are present, apply conservation of energy and solve for the unknown

Review Chapter 3: Work and Mechanical Energy

1. The principle of energy conservation

2. Kinetic energy

$$K = \frac{1}{2} m v^2$$

(Unit: J)

3. Work done (W)

by a constant force

$$W = \vec{F}\,\vec{d} = F d \cos\phi$$

by a general variable force

$$W = \int_{x_i}^{x_f} F(x)\,dx$$

Work can be positive or negative

4. Power (P)

Unit: Watt (W)

Average power: $P_{avg} = \frac{W}{\Delta t}$

Instantaneous power: $P = \frac{dW}{dt}$

If F is constant

$$P = \vec{F}\vec{v} = F v \cos\phi$$

1. Kinetic energy

$$K = \frac{1}{2} mv^2$$

- Work and Kinetic Energy: $\Delta K = K_f - K_i = W$

2. Potential energy

- Gravitational potential energy $U(y) = mgy$
- Elastic potential energy $U = \frac{1}{2} kx^2$

·Mechanical Energy: $E_{mec} = K + U$

- The change of Mechanical energy: $\Delta E_{mec} = \Delta K + \Delta U$

For an isolated system: $K_1 + U_1 = K_2 + U_2$ Or $\Delta E_{mec} = 0$

Work Done on a System by an External Force.

$$\boxed{W = \Delta E_{mechanical} + \Delta E_{thermal}}$$

$$\Delta E_{thermal} = f_k d$$

Isolated systems:

$$W = 0 \iff \Delta E_{mechanical} + \Delta E_{thermal} + \Delta E_{internal} = 0$$