

Chapter 6

Equilibrium and Elasticity

6.1. Equilibrium

6.2. The Center of Gravity and Conditions for Stable Equilibrium

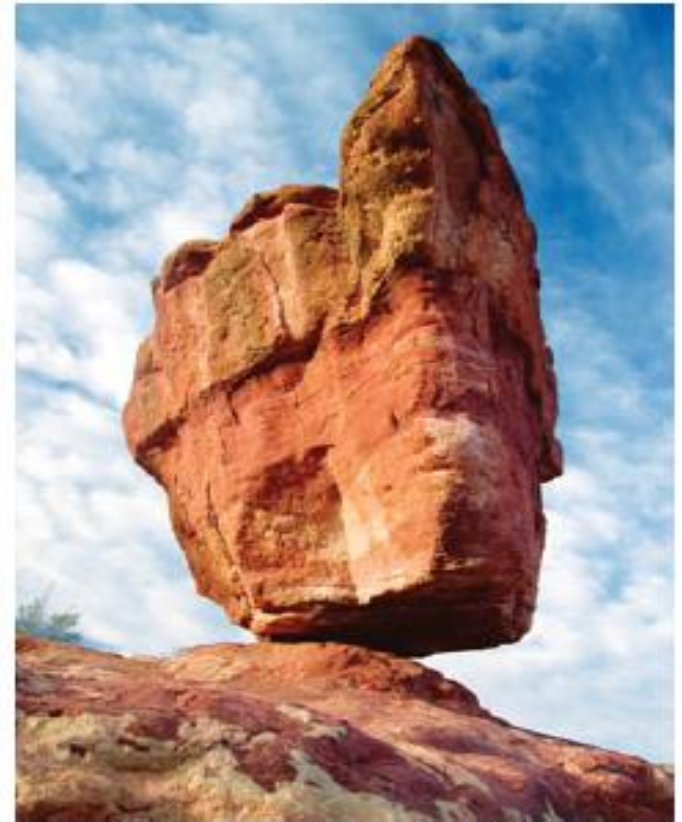
6.3. Elasticity

6.1. Equilibrium

The term **equilibrium** implies either that the object is at rest or that its center of mass moves with constant velocity

Examples:

- (1) a book resting on a table,
- (2) a hockey puck sliding with constant velocity across a frictionless surface,
- (3) the rotating blades of a ceiling fan, and
- (4) the bicycle traveling along a straight path at constant speed.



Kanwarjit Singh Boparai/Shutterstock

A balancing rock

a. The requirements of Equilibrium of the objects:

1. The linear momentum \vec{P} of its center of mass (COM) is constant.

\vec{P} is a constant

$$\vec{F}_{net} = \frac{d\vec{P}}{dt} = 0$$

2. Its angular momentum \vec{L} about its COM, or about any other point is also a constant.

\vec{L} is a constant

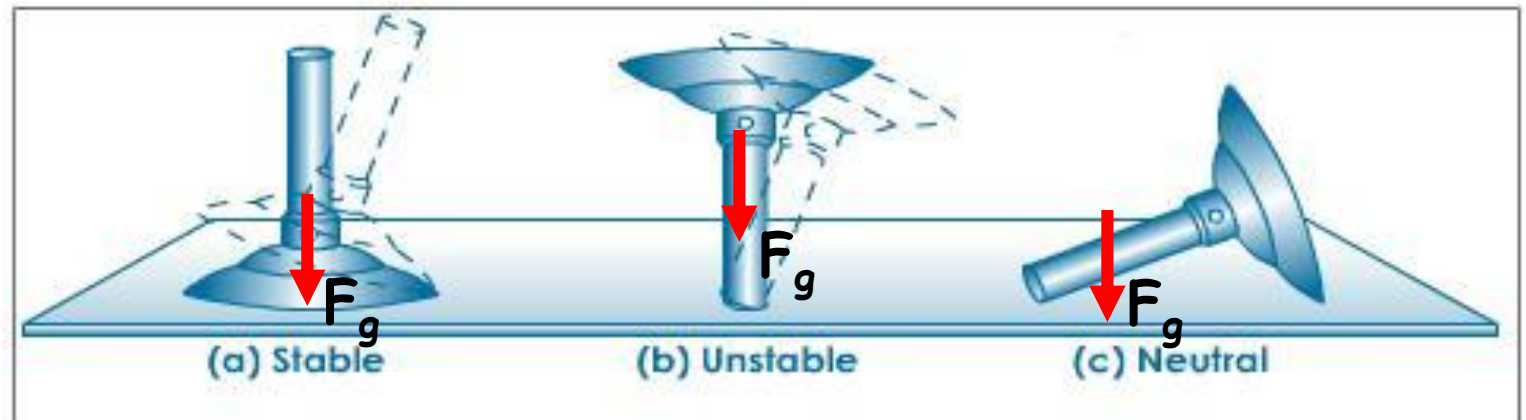
$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt} = 0$$

b. Static equilibrium:

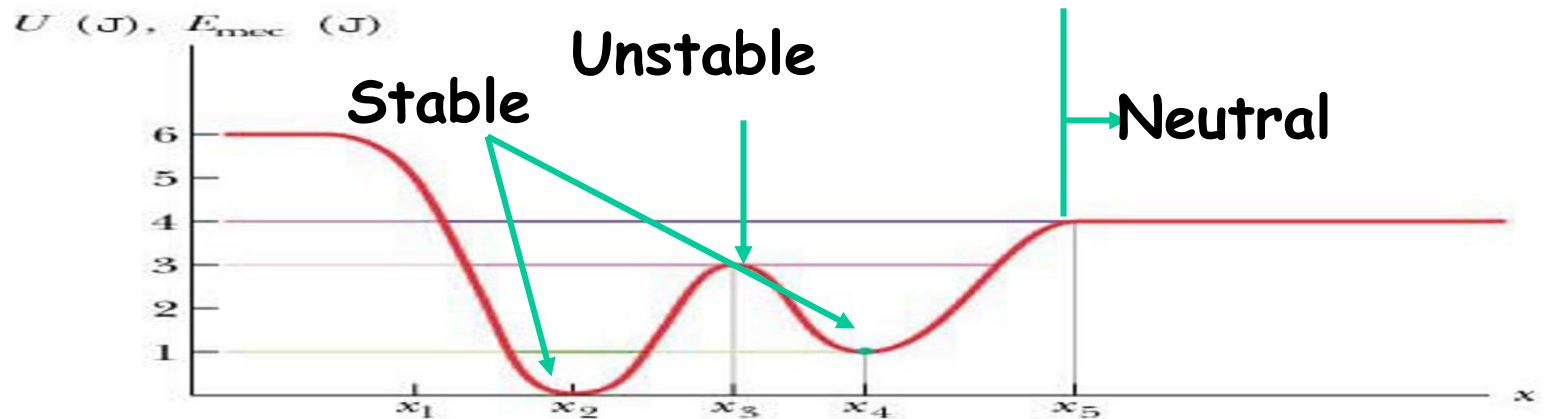
Objects are not moving either in translation or in rotation.

Three states of static equilibrium: neutral, stable, and unstable:

- **Stable Equilibrium:** An object can return to its equilibrium position after it has been displaced *slightly*.
- **Unstable Equilibrium:** An object does not return to its equilibrium position and does not remain in the displaced position after it has been displaced slightly.
- **Neutral Equilibrium:** A body stays in the displaced position after it has been displaced slightly.



Equilibrium of a Bunsen burner



Potential Energy Curve

6.2. The Center of Gravity and Conditions for Stable Equilibrium

• **Center of Gravity (COG):** The point of a body at which the gravitational force can be considered to act.

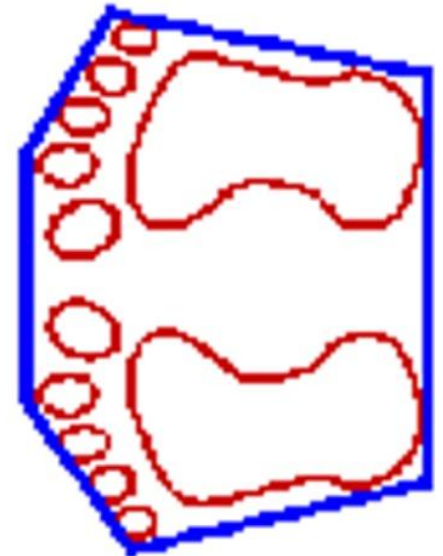
If the gravitational acceleration g is the same for all elements of a body, then the body's COG is coincident with its COM.

• Recall Center of Mass (COM):
$$\vec{r}_{com} = \frac{1}{M} \sum_{i=1}^n m_i \vec{r}_i$$

• **Base of Support:** It is the area formed by a perimeter around the supporting parts of an object in balance on a surface.

• Conditions for Stable Equilibrium:

- The body should have a broad base of support.
- COG of the body should be as low as possible.
- Vertical line drawn from the center of gravity should fall within the base of support.



Example: a uniform beam, of length L and mass $m = 1.8$ kg, is at rest on two scales. A uniform block, with mass $M = 2.7$ kg, is at rest on the beam, with its center a distance $L/4$ from the beam's left end. What do the scales read?

The forces have no x components, we consider to the y components,

Balance of Force on y component:

$$F_{\text{net},y} = 0$$

$$F_l + F_r - Mg - mg = 0 \quad (1)$$

Balance of Torque:

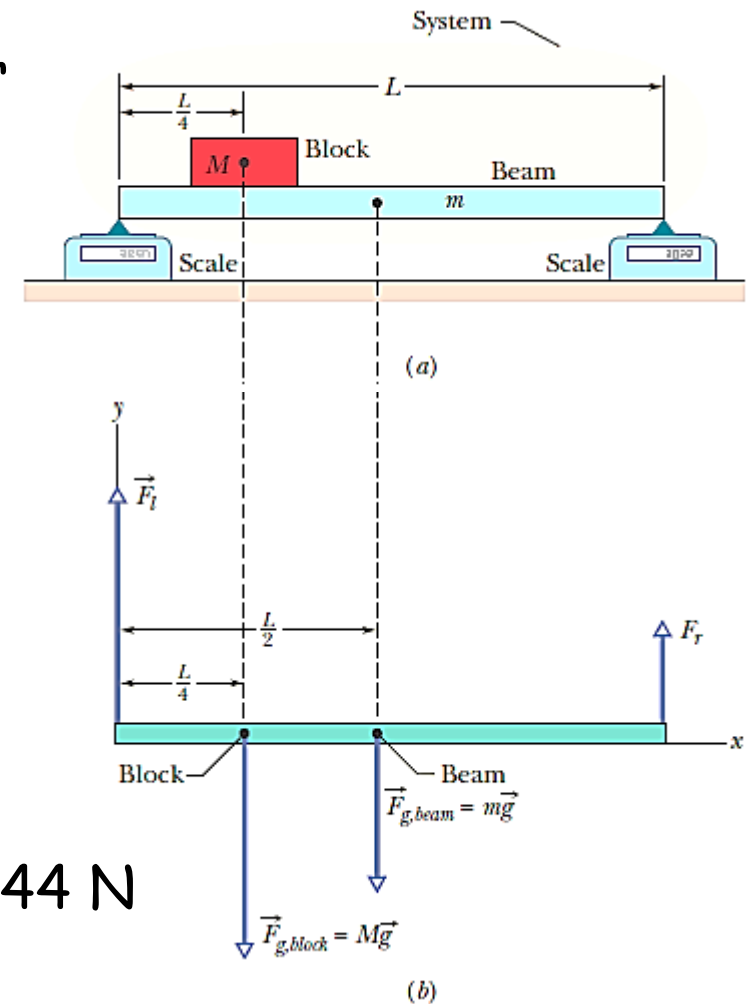
$$\tau_{\text{net},z} = 0$$

$$(0)(F_l) - (L/4)(Mg) - (L/2)(mg) + (L)(F_r) = 0 \quad (2)$$

Therefore:

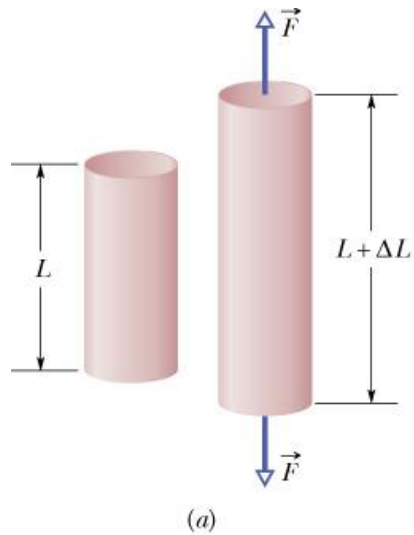
$$F_r = 1/4 Mg + 1/2 mg = \left(\frac{1}{4}2.7 + \frac{1}{2}1.8\right) \times 9.8 = 15.44 \text{ N}$$

$$F_l = (M + m)g - F_r = (2.7 + 1.8) \times 9.8 - 15.44 = 38.66 \text{ N}$$

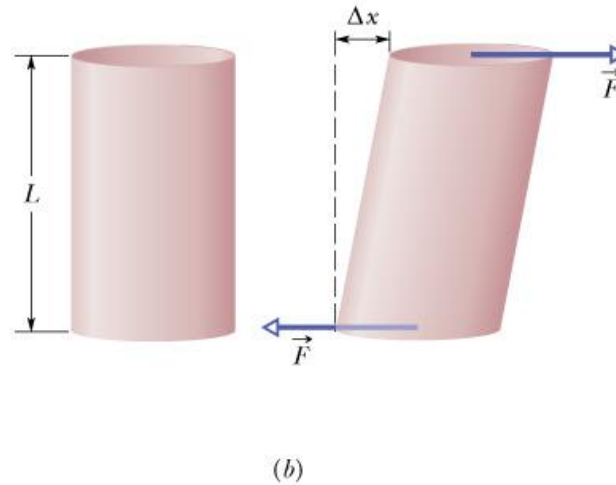


6.3. Elasticity

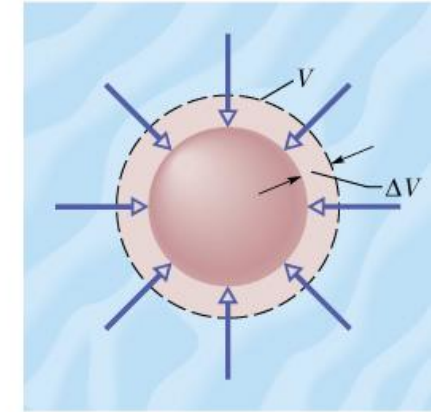
It is the physical property of a material that returns to its original shape once deforming forces (stress) are removed.



Tensile stress



Shearing stress



Hydraulic stress

Read text (p. 315-318)

Chapter 7 Gravitation

7.1. Newton's Law of Gravitation

7.2. Kepler's Laws

7.3. Gravitational Lensing Effect

7.1. Newton's Law of Gravitation

$$F = G \frac{m_1 m_2}{r^2}$$

The tendency of bodies to move toward one another.

$G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$: the gravitational constant

Gravitation Near Earth's Surface

$$F = G \frac{Mm}{r^2}$$

gravitational acceleration $g = G \frac{M}{r^2}$

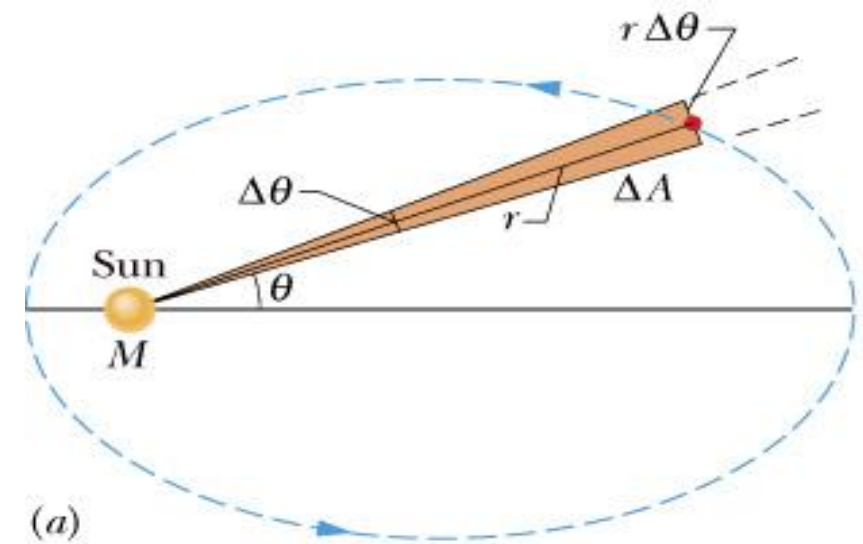
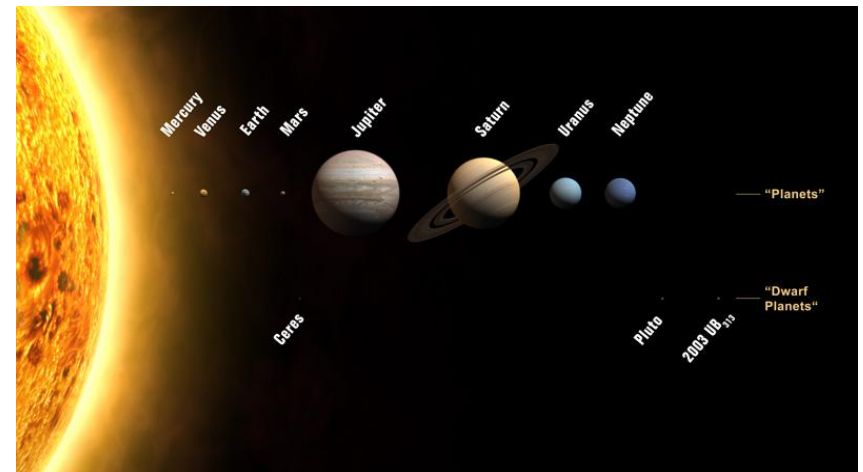
$$\Rightarrow F = mg$$

Gravitational Potential Energy:

$$U = -\frac{GMm}{r}$$

$$U = 0 \text{ for } r = \infty$$

7.2. Kepler's Laws



1. **The law of orbits:** All planets move in elliptical orbits, with the Sun at one focus.

2. **The law of areas:** the rate dA/dt at which it sweeps out area A is constant. (A line that connects a planet to the Sun sweeps out equal areas in the plane of the planet's orbit in equal time intervals)

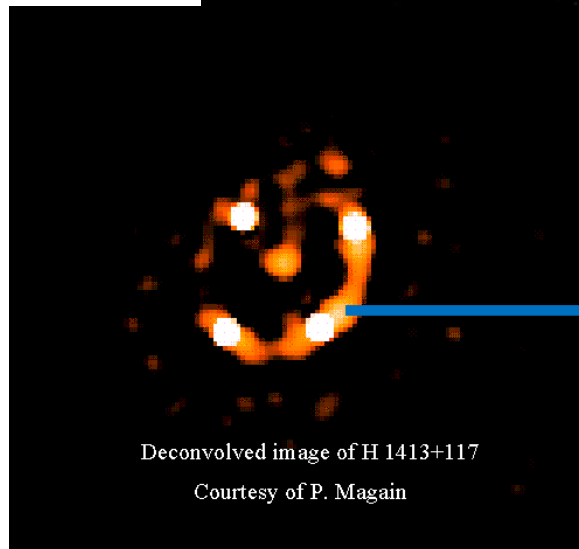
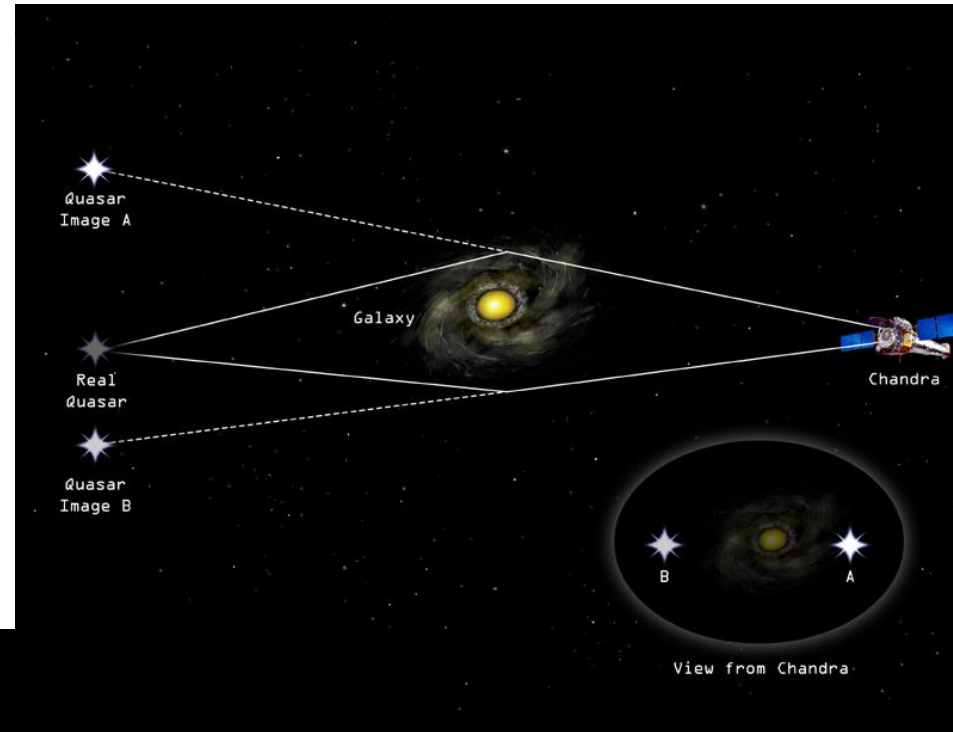
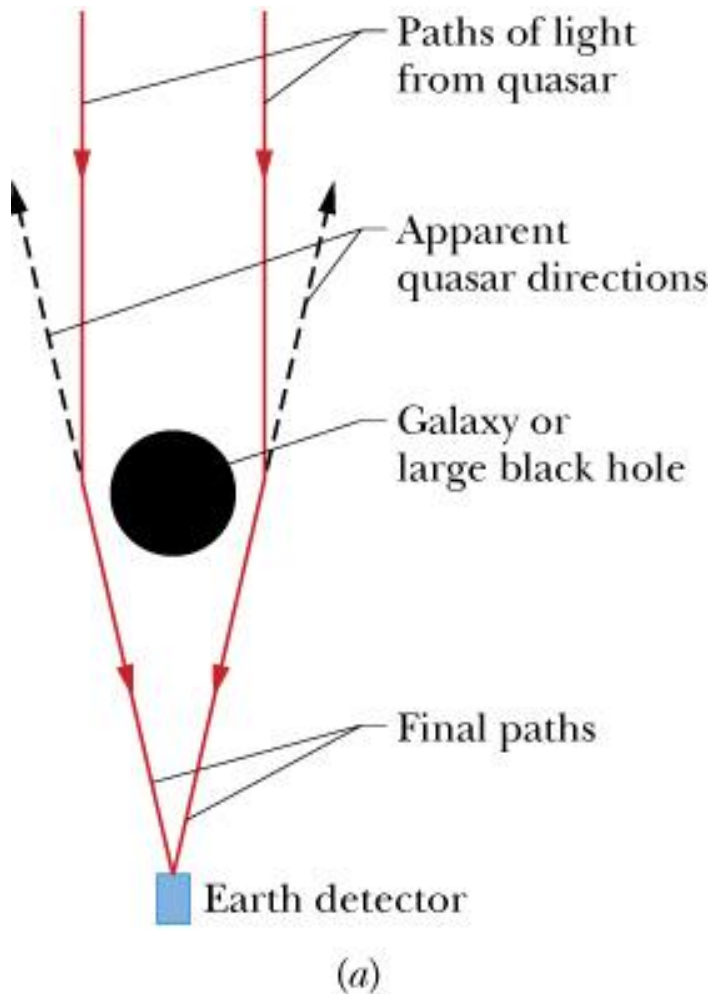
3. **The law of periods:** The square of the period of any planet is proportional to the cube of the semi-major axis of its orbit.

$$T^2 = \left(\frac{4\pi^2}{GM} \right) r^3$$

<https://www.youtube.com/watch?v=s77LJO6USEY>

7.3. Gravitational Lensing Effect

According to Einstein's general theory of relativity, when light passes near a massive object (e.g., Earth), the path of the light bends slightly because of the curvature of space there. This effect is called **Gravitational Lensing**.



Einstein's ring