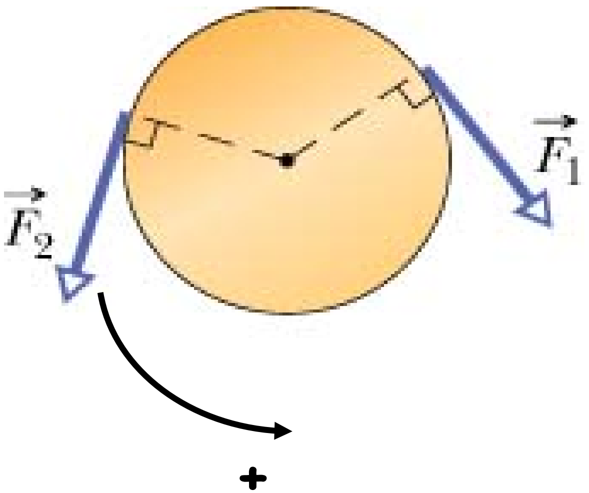
Sample Problems

Chapter 5: Rotation of a Rigid Body About a Fixed Axis

Problems: 53, 56 (p. 270-271); 26, 38 (p. 299-300)

53. The figure below shows a uniform disk that can rotate around its center like a merry-go-round. The disk has a radius of 2.0 cm and a mass of 20.0 grams and is initially at rest. Starting at time t=0, two forces are to be applied tangentially to the rim as indicated, so that at time t=1.25 s the disk has an angular velocity of 250 rad/s counterclockwise. Force F_1 has a magnitude of 0.1 N. What is magnitude F_2?

$$\tau_{net} = I\alpha$$



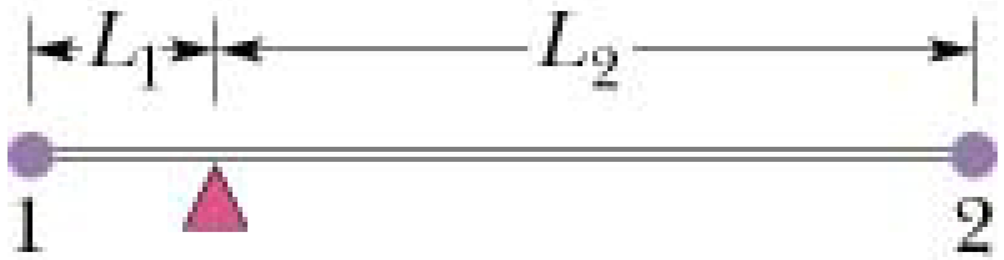
$$\tau_{net} = F_2R - F_1R = I\alpha \Rightarrow F_2 = \frac{I\alpha}{R} + F_1$$

For a uniform disk: $I = \frac{1}{2}MR^2$

For rotation: $\omega = \omega_0 + \alpha t = \alpha t \Rightarrow \alpha = \frac{\omega}{t}$

$$F_2 = \frac{MR\omega}{2t} + F_1 = \frac{(20.0\times10^{-3})(2.0\times10^{-2})(250)}{2\times1.25} + 0.1 = 0.14\,(\text{N})$$

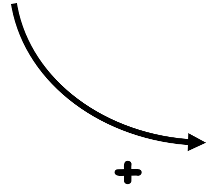
56. The figure shows particles 1 and 2, each of mass m, attached to the ends of a rigid massless rod of length L_1+L_2, with $L_1=20$ cm and $L_2=80$ cm. The rod is held horizontally on the fulcrum and then released. What are the magnitudes of the initial accelerations of (a) particle 1 and (b) particle 2?



$$\tau_{net} = mgL_1 - mgL_2 = I\alpha$$

$$I = mL_1^2 + mL_2^2$$

$$\Rightarrow \alpha = \frac{g(L_1 - L_2)}{L_1^2 + L_2^2}$$



$$\alpha = \frac{9.8(0.2-0.8)}{0.2^2+0.8^2} = -8.65\,(\text{rad/s}^2)$$

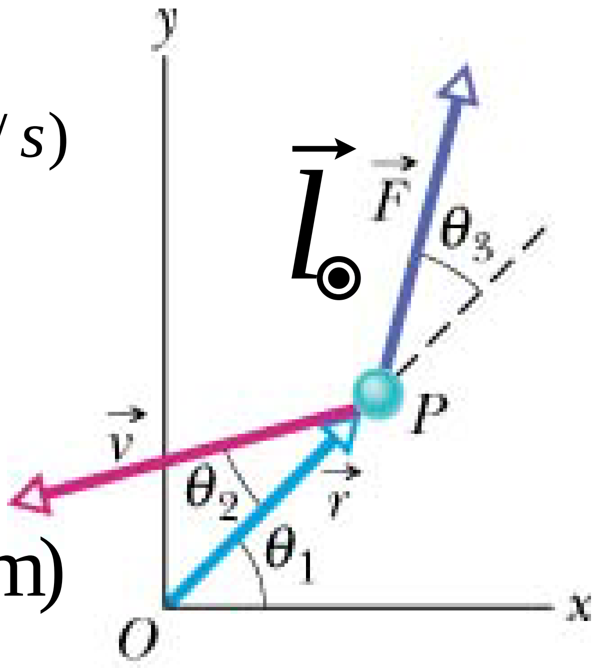
$$\vec{a} = \vec{a}_r + \vec{a}_t;\, a_r = \frac{v^2}{r};\, a_t = \alpha\, r$$

(a) $a_{1t} = \alpha\, L_1 = -8.65\times 0.2 = -1.73\,(\text{m/s}^2)$

(b) $a_{2t} = \alpha\, L_2 = -8.65\times 0.8 = -6.92\,(\text{m/s}^2)$

26. At the instant of the figure below, a 2.0 kg particle P has a position vector $\vec{r}$ of magnitude 5.0 m and angle $\theta_1=45^0$ and a velocity vector $\vec{V}$ of magnitude 4.0 m/s and angle $\theta_2=30^0$. Force $\vec{F}$, of magnitude 2.0 N and angle $\theta_3=30^0$, acts on P. All three vectors lie in the xy plane. About the origin, what are the (a) magnitude and (b) direction of the angular momentum of P and the (c) magnitude and (d) direction of the torque acting on P?

$$\vec{l} = \vec{r} \times \vec{p}$$



(a) $l = rmv\sin\theta_2 = 5.0\times 2.0\times 4.0\times \sin(30) = 20\ (\text{kg m}^2/s)$

(b) Using the right-hand rule, $\vec{l}$ points out of the page and it is perpendicular to the figure plane.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

(c) $\tau = rF\sin\theta_3 = 5.0\times 2.0\times \sin(30) = 5\ (\text{N m})$

(d) $\vec{\tau}$ points out of the page and it is perpendicular to the figure plane.

38. A sanding disk with rotational inertia 8.6×10^{-3} kg.m^2 is attached to an electric drill whose motor delivers a torque of magnitude 16 N.m about the central axis of the disk. About that axis and with the torque applied for 33 ms (milliseconds), what is the magnitude of the (a) angular momentum and (b) angular velocity of the disk?

(a) Using Newton's second law for rotation:

$$\tau_{avg} = \frac{\Delta L}{\Delta t}$$

$$\Rightarrow \Delta L = L - L_0 = L = \tau_{avg}\Delta t$$

$$L = 16\times33\times10^{-3} = 0.528\,(\text{Nms}) \text{ or } 0.528\,(\text{kg m}^2/\text{s})$$

(b)

$$L = I\omega \Rightarrow \omega = \frac{L}{I}$$

$$\omega = \frac{0.528}{8.6\times10^{-3}} = 61.4\,(\text{rad/s})$$