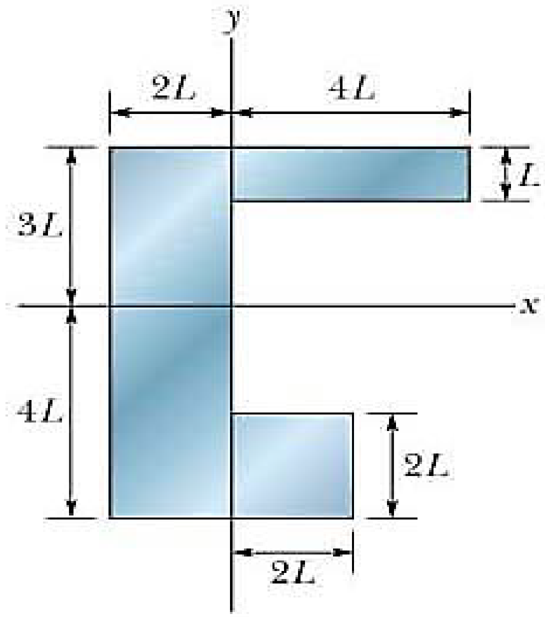
Chapter 4:

Linear Momentum and Collisions

Problems: 2, 5, 13, 14, 22, 25, 38
(pages 230-233)

5. What are (a) the x coordinate and (b) the y coordinate of the center of mass for the uniform plate shown in the figure below if L=5.0 cm?

$$x_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i, \quad y_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i$$



three pieces:
1. L x 4L; 2. 2L x 7L; and
3. 2L x 2L.

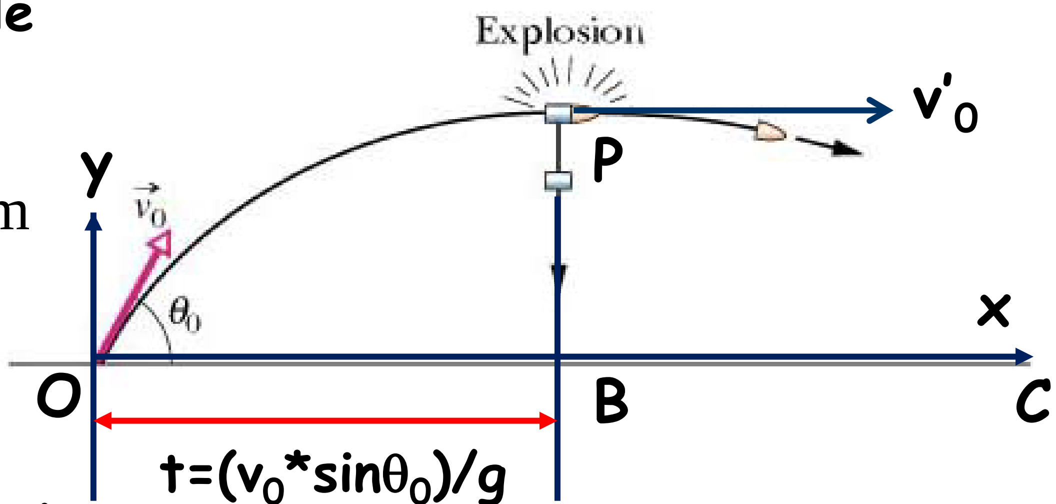
x_1=4L/2=10 cm; y_1=2.5L=12.5 cm;
x_2=-1L=-5 cm; y_2=-0.5L=-2.5 cm;
x_3=1L=5 cm; y_3=-3.0L=-15 cm;

$$\frac{m_1}{M} = \frac{m_1}{m_1+m_2+m_3} = \frac{\rho \times thickness \times area_1}{\rho \times thickness \times (area_1 + area_2 + area_3)} = \frac{4}{4+14+4} = 0.182$$

$$\frac{m_2}{M} = 0.636; \frac{m_3}{M} = 0.182$$

$$x_{com} = -0.45\text{ cm}; \ y_{com} = -2.01\text{ cm}$$

13. A shell is shot with an initial velocity v_0 of 20 m/s, at an angle of $\theta_0 = 60^0$ with the horizontal. At the top of the trajectory, the shell explodes into 2 fragments of equal mass (see figure). One fragment, whose speed immediately after the explosion is zero, falls vertically. How far from the gun does the other fragment land, assuming that the terrain is level and that air drag is negligible?



- First stage, from O to P is a projectile motion with mass M, v_0 and angle = θ_0:

$$x_B = (v_0 \cos\theta_0)t = \frac{v_0^2}{g}\sin\theta_0 \cos\theta_0 = 17.7 \text{ m}$$

$$y_P = \frac{v_0^2}{2g}\sin^2\theta_0$$

- Second stage, from P to C is a projectile motion with mass M/2, v'_0 and angle = 0^0:

$$Mv_0\cos\theta_0 = \frac{1}{2}M \times 0 + \frac{1}{2}Mv_0' \Rightarrow v_0' = 2v_0\cos\theta_0$$

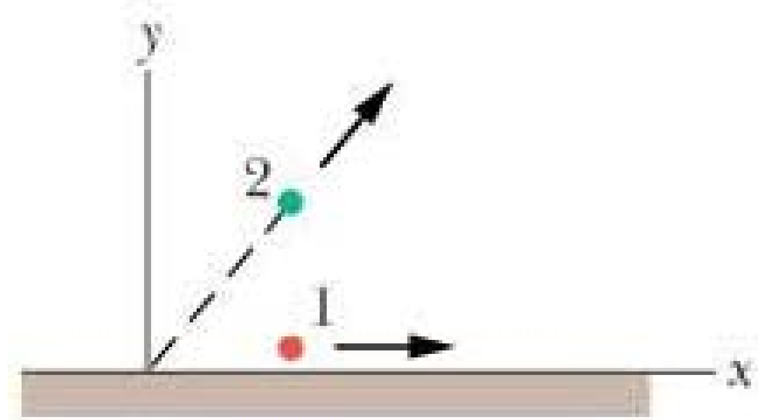
- time for the other fragment flies from P to C:

$$t_{PC} = \sqrt{\frac{2y_B}{g}} = \frac{v_0\sin\theta_0}{g} = t_{OP}$$

$$x_c = (v_0\cos\theta_0)t + (2v_0\cos\theta_0)t = 3x_B = 53{,}1 \text{ m}$$

14. In the figure below, two particles are launched from the origin of the coordinates system at time t=0. Particle 1 of mass m_1=5.0 g is shot directly along the x axis, where it moves with a constant speed of 10 m/s. Particle 2 of mass m_2=3.0 g is shot with a velocity of magnitude 20.0 m/s, at an upward angle such that it always stays directly above particle 1 during its flight. (a) What is the maximum height H_{max} reached by the com of the two particle system? In unit-vector notation, what are the (b) velocity and (c) acceleration of the com when the com reaches H_{max}?

$$v_{2,y}^2 - v_{2,y0}^2 = -2gy$$



(a) At the maximum height:

$$-v_{2,y0}^2 = -2gy_{\max}$$

Particle 2 always stays directly above P.1:

$$v_{2,x} = v_{1,x}$$

$$\Rightarrow v_{2,y0} = \sqrt{v_2^2 - v_{2,x}^2} = \sqrt{v_2^2 - v_{1,x}^2} = 17.3\,(\text{m/s})$$

$$y_{\max} = 15.3\,(\text{m}) \;\Rightarrow H_{\max} = \frac{m_2 y_{\max}}{m_1 + m_2} = 5.74\,(\text{m})$$

(b)

$$\vec{v}_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i\vec{v}_i = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

At the maximum height, $v_{2,y}$=0:

$$v_{com,y} = 0;\ v_{com,x} = \frac{m_1 v_{1,x} + m_2 v_{2,x}}{m_1 + m_2} = v_{1,x}$$

$$\vec{v}_{com} = (10\ \text{m/s})\,\hat{\text{i}}$$

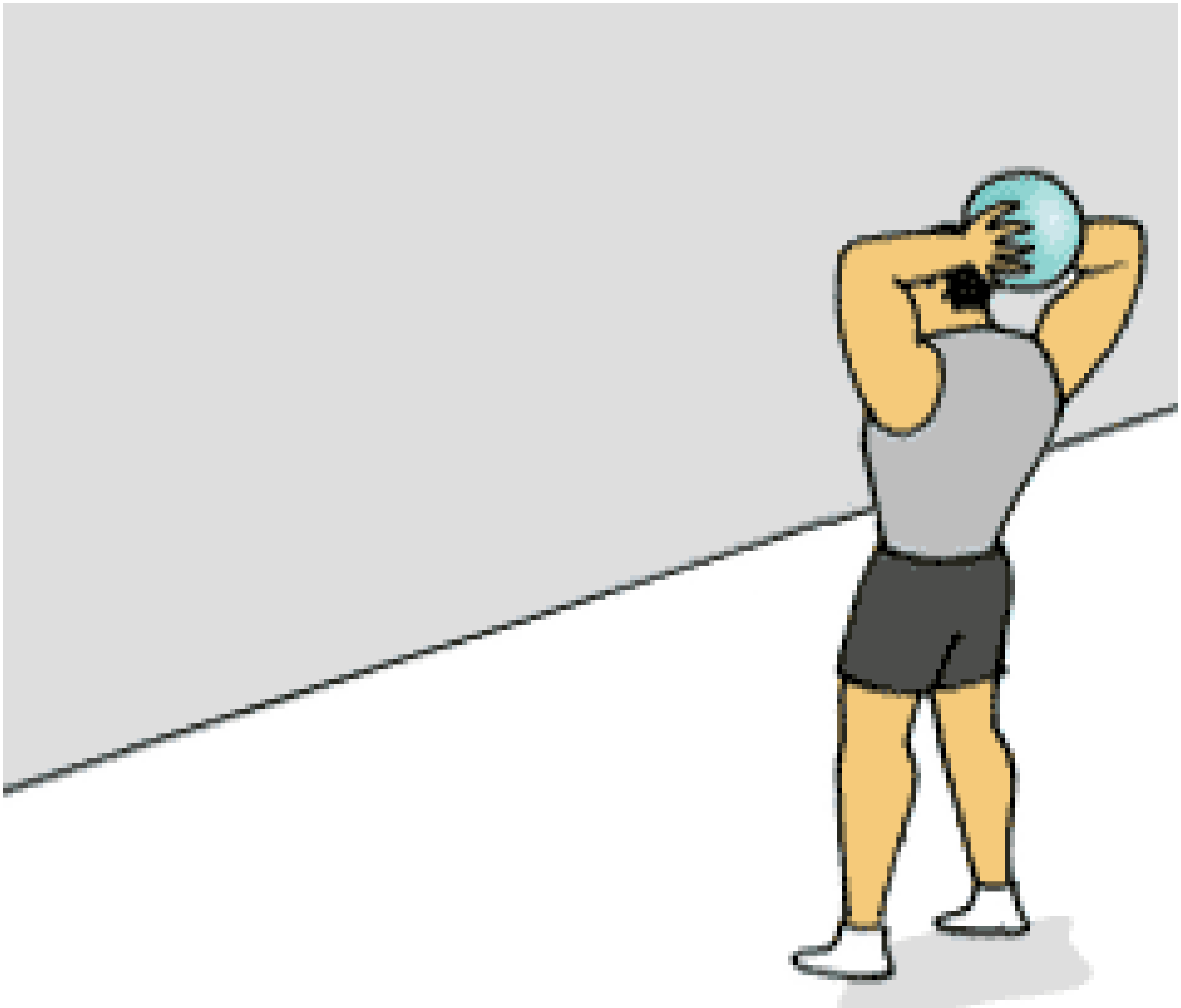
(c)

$$M\vec{a}_{com} = \sum_{i=1}^{n} m_i\vec{a}_i$$

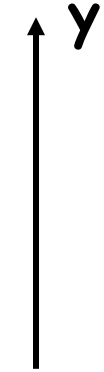
$$\vec{a}_{com} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$$

$$a_{com} = \frac{m_2 g}{m_1 + m_2} = 3.68\,(\text{m/s}^2)$$

$\vec{a}_{com}$ is downward, hence : $\vec{a}_{com} = (-3.68\ \text{m/s}^2)\,\hat{\text{j}}$



25. A 1.2 kg ball drops vertically onto a floor, hitting with a speed of 25 m/s. It rebounds with an initial speed of 10 m/s. (a) What impulse acts on the ball during the contact? (b) If the ball is in contact with the floor for 0.020 s, what is the magnitude of the average force on the floor from the ball?



(a)

$$v_i = -25\ \text{m/s};\ v_f = 10\ \text{m/s}$$

Impulse J:

$$\vec{J} = \Delta\vec{p}$$

The ball is dropping vertically → one dimensional motion:

$$J = \Delta p_y = m(v_f - v_i) = 1.2 \times [10 - (-25)] = 42\ (\text{kg.m/s})$$

$$\vec{J} = (42\ \text{kg.m/s})\,\hat{j}$$

(b)

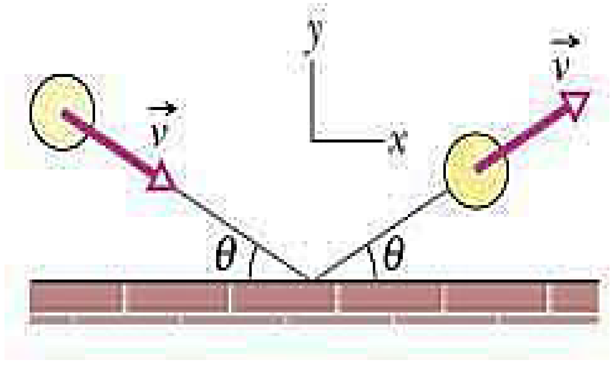
$$J = F_{avg}\Delta t \Rightarrow F_{avg} = \frac{J}{\Delta t} = \frac{42}{0.02} = 2100\ (\text{N})$$

38. In the overhead view of the figure below, a 300g ball with a speed v of 8.0 m/s strikes a wall at an angle θ of 30⁰ and then rebounds with the same speed and angle. It is in contact with the wall for 10 ms. In unit-vector notation, what are (a) the impulse on the ball from the wall and (b) the average force on the wall from the ball?

$$\vec{J} = \Delta\vec{p} = m(\vec{v}_f - \vec{v}_i)$$

$$\vec{v}_i = +6.9\hat{i} - 4.0\hat{j};\ \vec{v}_f = +6.9\hat{i} + 4.0\hat{j}$$

$$\vec{J} = (0.3\,\text{kg})(8.0\,\text{m/s})\,\hat{j} = (2.4\,\text{kg.m/s})\,\hat{\text{j}}$$



The average force on the ball from the wall:

$$\vec{F}_{avg} = \frac{\vec{J}}{\Delta t} = \left(\frac{2.4\,\text{kg.m/s}}{0.01\,\text{s}}\right)\hat{\text{j}} = (240\ \text{N})\,\hat{j}$$

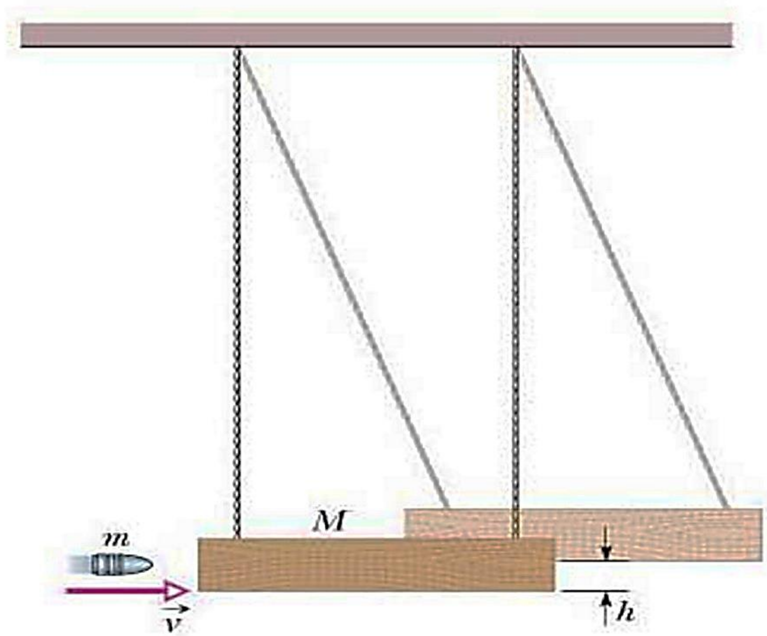
According to Newton's third law, the force on the wall from the ball:

$$-\vec{F}_{avg} = (-240\ \text{N})\,\hat{j}$$

Problems: 49, 56, 67, 60, 64, 74
(p. 234-237)

49. A bullet of mass 10g strikes a ballistic pendulum of mass 2kg. The center of mass of the pendulum rises a vertical distance of 12 cm. Assuming that the bullet remains embedded in the pendulum, calculate the bullet's initial speed.

The collision here is a perfectly inelastic collision, the linear momentum of the system bullet + pendulum is conserved because the external impulse J on the system is zero:



$$mv = (m+M)V \Rightarrow V = \frac{m}{m+M}v$$

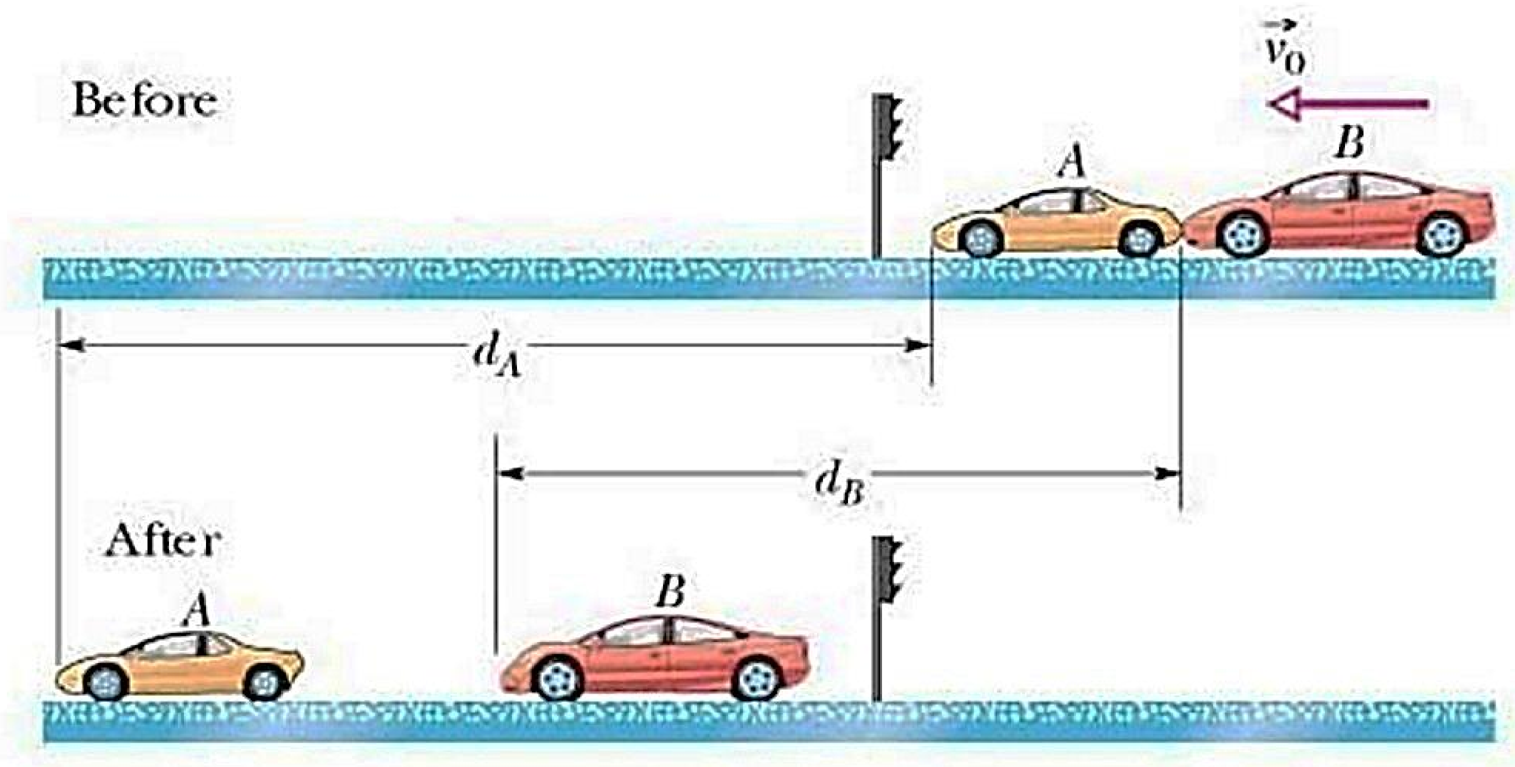
After the collision, the mechanical energy of the system bullet-block-Earth is conserved:

$$\frac{1}{2}(m+M)V^2 = (m+M)gh \Rightarrow V = \sqrt{2gh}$$

$$\Rightarrow v = \frac{m+M}{m}\sqrt{2gh}$$

Read also Sample Problem 9-8 (page 218)

56. In the "before" part of the figure below, car A (mass 1100 kg) is stopped at a traffic light when it is rear-ended by car B (mass 1400 kg). Both cars then slide with locked wheels until the frictional force from the slick road (with a low μ_k of 0.10) stops them, at distance d_A=8.2 m and d_B=6.1 m. What are the speeds of (a) car A and (b) car B at the start of the sliding, just after the collision? (c) Assuming that linear momentum is conserved during the collision, find the speed of car B just before the collision. (d) Explain why this assumption may be invalid.



$$v^2 - v_0^2 = -2ad \Rightarrow v_0 = \sqrt{2ad} = \sqrt{2\mu_k g d}$$

The magnitude of the acceleration of each car is determined by:

$$a = \frac{F_k}{m} = \frac{\mu_k m g}{m} = \mu_k g$$

Velocities of car A and B after the collision:

$$\Rightarrow v_A = \sqrt{2\mu_k g d_A}; \quad v_B = \sqrt{2\mu_k g d_B}$$

(c) If p conserved: $\Delta p = J = F_{avg}\Delta t = 0$

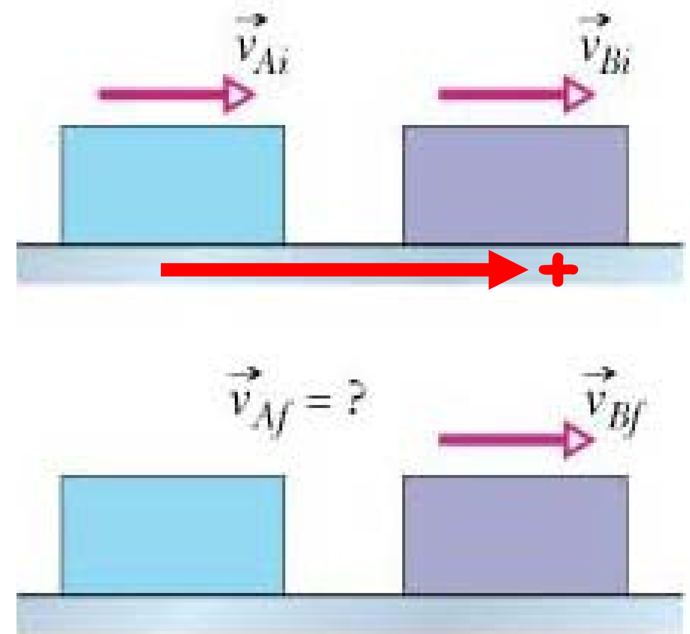
$$m_B v_{0,B} = m_A v_A + m_B v_B \Rightarrow v_{0,B} = \frac{m_A v_A + m_B v_B}{m_B}$$

(d) p is conserved if the frictional force exerted on the cars from the road is negligible during the collision.

However, $\Delta p = J = F_{avg}\Delta t \neq 0$

Therefore, the assumption that p is conserved (or Δp = 0) may be invalid

60. Block A (mass 1.6 kg) slides into block B (mass 2.4 kg), along a frictionless surface. The directions of three velocities before (i) and after (f) the collision are indicated; the corresponding speeds are v_{Ai} = 5.5 m/s, v_{Bi} = 2.5 m/s, and v_{Bf} = 4.9 m/s. What are the (a) speed and (b) direction (left or right) of velocity v_{Af}? (c) Is the collision elastic?



We choose the positive direction is rightward

(a) The linear momentum of the system (A+B) is conserved (no friction):

$$m_A v_{Ai} + m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf}$$

$$v_{Af} = v_{Ai} + \frac{m_B}{m_A}(v_{Bi} - v_{Bf})$$

$$v_{Af} = 5.5 + \frac{2.4}{1.6}(2.5 - 4.9) = 1.9 (\text{m/s})$$

(b) $v_{Af} > 0$, so the v_{Af} direction is to the right

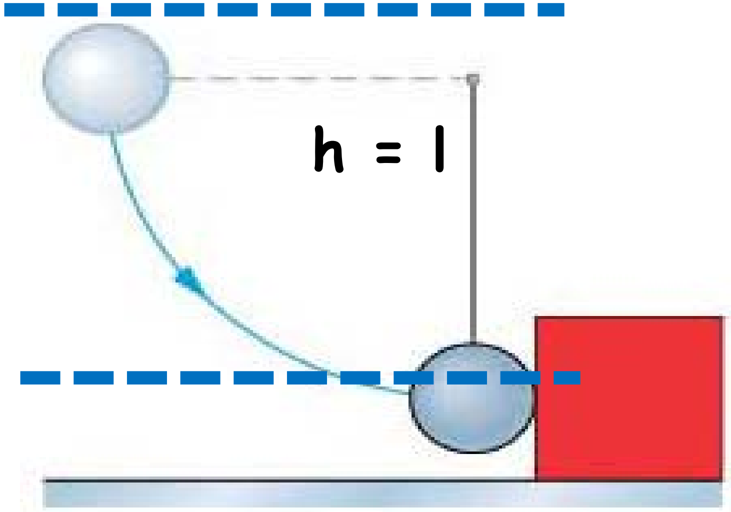
(c)
$$\Delta K = K_f - K_i = \frac{1}{2}\left(m_A v_{Af}^2 + m_B v_{Bf}^2\right) - \frac{1}{2}\left(m_A v_{Ai}^2 + m_B v_{Bi}^2\right) = 0 \text{ (J)}$$

$\Rightarrow$ the collision is elastic

64. A steel ball of mass 2.5 kg is fastened to a cord that is 70cm long and fixed at the far end. The ball is then released when the cord is horizontal. At the bottom of its path, the ball strikes a 2.8 kg steel block initially at rest on a frictionless surface. The collision is elastic. Find (a) the speed of the ball and (b) the speed of the block, both just after the collision.

Conservation of mechanical energy: $U_g = mgh$

$$mgh = mgl = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gh}$$



Conservation of linear momentum (no friction): $U_g = 0$

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \quad (1)$$

$$v_{1i} = \sqrt{2gl}$$

The collision is elastic:
$$\Delta K = \frac{1}{2}\left(m_1 v_{1f}^2 + m_2 v_{2f}^2\right) - \frac{1}{2}\left(m_1 v_{1i}^2\right) = 0$$

$$m_1 v_{1f}^2 + m_2 v_{2f}^2 = m_1 v_{1i}^2 \quad (2)$$

$$\begin{cases} m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} & (1) \\ m_1 v_{1f}^2 + m_2 v_{2f}^2 = m_1 v_{1i}^2 & (2) \end{cases}$$

$$\begin{cases} m_1 (v_{1i} - v_{1f}) = m_2 v_{2f} \\ m_1 (v_{1i}^2 - v_{1f}^2) = m_2 v_{2f}^2 \end{cases}$$

$$\begin{cases} m_1 (v_{1i} - v_{1f}) = m_2 v_{2f} \\ v_{1i} + v_{1f} = v_{2f} \end{cases}$$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} ; v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

74. Two 2.0kg bodies, A and B, collide. The velocities before the collision are $\vec{v}_A = 15\hat{i} + 30\hat{j}$ m/s and $\vec{v}_B = -10\hat{i} + 5\hat{j}$ m/s. After the collision, $\vec{v}'_A = -5\hat{i} + 20\hat{j}$ m/s. What are (a) the final velocity of B and (b) the change in the total kinetic energy (including sign)?

We assume that the total linear momentum of the two bodies is conserved:

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B$$

$m_A = m_B$:

$$\vec{v}'_B = \vec{v}_A + \vec{v}_B - \vec{v}'_A$$

$$\vec{v}'_B = 10\hat{i} + 15\hat{j}$$

(b)

$$K_f = \frac{1}{2}m_A v'^2_A + \frac{1}{2}m_B v'^2_B$$

$$K_i = \frac{1}{2}m_A v^2_A + \frac{1}{2}m_B v^2_B$$

$$\Delta K = K_f - K_i = -500\,(\mathrm{J})$$

➔ The collision here is an inelastic collision since KE is not a constant.