Solving problems

Chapter 2: Force and Motion

Problems: 3, 5, 7, 13, 24, 34, 45, 49, 51, 56, 57, 59
(Page 108 – 112)

5. Three astronauts propelled by jet backpacks, push and guide a 120 kg asteroid toward a processing dock, exerting the forces shown in the figure below, with F_1=32 N, F_2=55 N, F_3=41 N, $\theta_1=30^0$, and $\theta_3=60^0$. What is the asteroid's acceleration (a) in unit-vector notation and as (b) a magnitude and (c) a direction relative to the positive direction of the x axis?

a) $$F_x = F_2 + F_1\cos\theta_1 + F_3\cos\theta_3 \quad = 103.2\text{ N}$$

$$F_x = m\,a_x \quad \Rightarrow a_x = 0.86\text{ m/s}^2$$

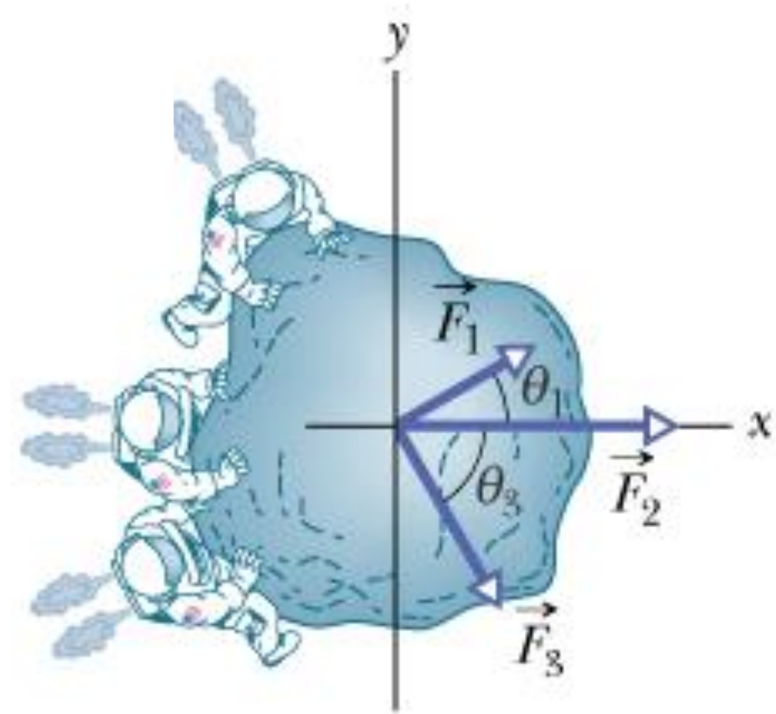
$$F_y = F_1\sin\theta_1 - F_3\sin\theta_3 \quad = -19.5\text{ N}$$

$$F_y = m\,a_y \quad \Rightarrow a_y = -0.163\text{ m/s}^2$$

$$\Rightarrow \vec{a} = a_x\hat{i} + a_y\hat{j}$$

b) $$|\vec{a}| = \sqrt{a_x^2 + a_y^2} \quad = 0.875\text{ m/s}^2$$

c) $$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) \quad = -11^0$$



7. There are two forces on the 2.00 kg box in the overhead view of Fig. 5-31, but only one is shown. For F1=20.0N, a=12.0 m/s2, and θ=30.0⁰, find the second force (a) in unit-vector notation and as (b) a magnitude and (c) an angle relative to the positive direction of the x axis.

a) $$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

a_x =- a. sin (30⁰) = -6 m/s²

a_y = -a. cos (30⁰) = -10.4 m/s²

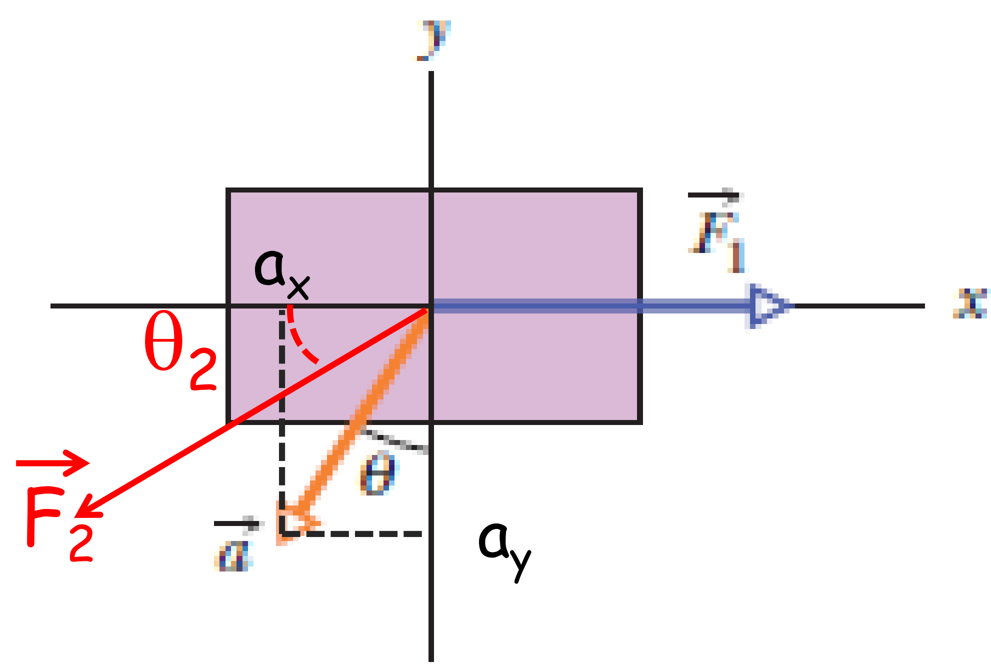
$$\Rightarrow \vec{a} = -6\hat{i} - 10.4\hat{j}$$

$$\vec{F}_2 = \vec{F}_{net} - \vec{F}_1 = m\vec{a} - \vec{F}_1$$

= 2 (-6)i +2x(-10.4)j- 20 i

= -32 i - 20.8 j

b) F_2 =38.17 N



c) $$\theta_2 = \tan^{-1}\left(\frac{F_{2y}}{F_{2x}}\right)$$ = 33°

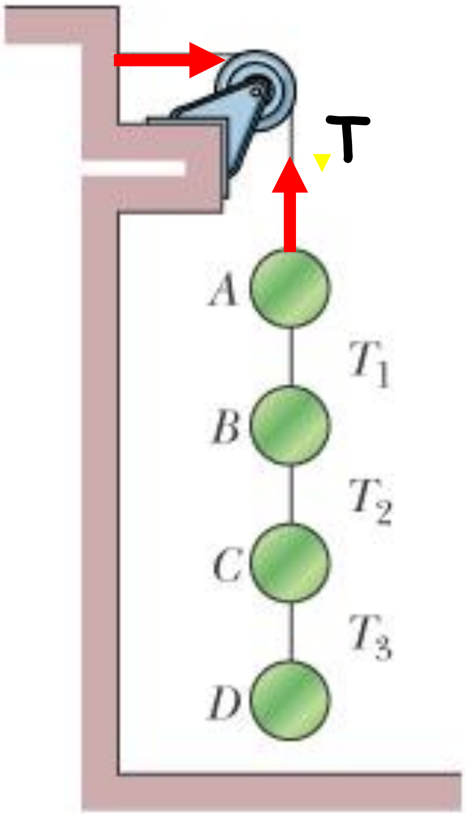
13. The figure below shows an arrangement in which four disks are suspended by cords. The longer, top cord loops over a frictionless pulley and pulls with a force of magnitude 98 N on the wall to which it is attached. The tension in the shorter cords are T_1=58.8 N, T_2=49.0 N, and T_3=9.8 N. What are the masses of (a) disk A, (b) disk B, (c) disk C, and (d) disk D?

Disk A: $T=T_1+m_A g \rightarrow m_A=4.0$ (kg)

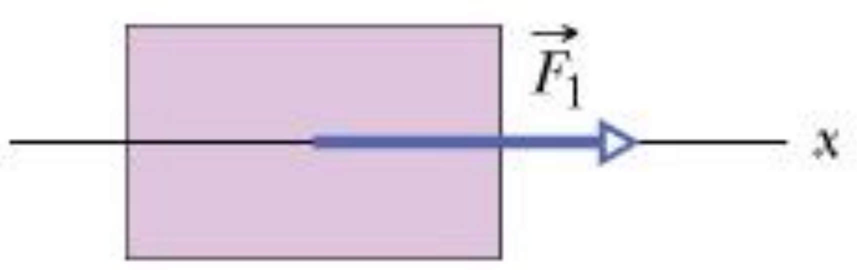
Disk B: $T_1=T_2+m_B g \rightarrow m_B=1.0$ (kg)

Disk C: $T_2=T_3+m_C g \rightarrow m_C=4.0$ (kg)

Disk D: $T_3=m_D g \rightarrow m_D=1.0$ (kg)



24. There are two horizontal forces on the 2.0 kg box in the overhead view of the figure below but only one (of magnitude F_1=30 N) is shown. The box moves along the x axis. For each of the following values for the acceleration a_x of the box, find the second force in unit-vector notation: (a) 10 m/s^2, (b) 20 m/s^2, (c) 0, (d) -10 m/s^2, and (e) -20 m/s^2.



$F_1+F_2=ma$

(a) $F_2=ma-F_1=2.0\times10-30=-10$ (N)

$$\vec{F}_2 = (-10\text{N})\hat{i}$$

(d) $F_2=ma-F_1=2.0\times(-10)-30=-50$ (N)

$$\vec{F}_2 = (-50\,\text{N})\hat{\text{i}}$$

45. An elevator cab that weighs 27.8 kN moves upward. What is the tension in the cable if the cab's speed is (a) increasing at a rate of 1.22 m/s^2 and (b) decreasing at a rate of 1.22 m/s^2?

(a) Applying Newton's second law, $a=+1.22$ m/s^2:

+

$$T - mg = ma$$

$$m = (27.8 \times 1000)/9.8 = 2837 \text{ (kg)}$$

Therefore, $T = 2837\,(9.8+1.22) = 31.3 \times 10^3$ (N)

(b) $a=-1.22$ m/s^2: $T = 2837\,(9.8-1.22) = 24.3 \times 10^3$ (N)

••49 In Fig. 5-45, a block of mass $m = 5.00$ kg is pulled along a horizontal frictionless floor by a cord that exerts a force of magnitude $F = 12.0$ N at an angle $\theta = 25.0°$. (a) What is the magnitude of the block's acceleration? (b) The force magnitude F is slowly increased. What is its value just before the block is lifted (completely) off the floor? (c) What is the magnitude of the block's acceleration just before it is lifted (completely) off the floor?

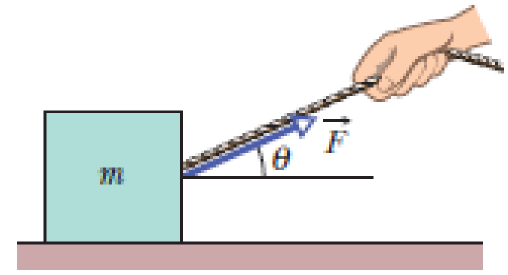
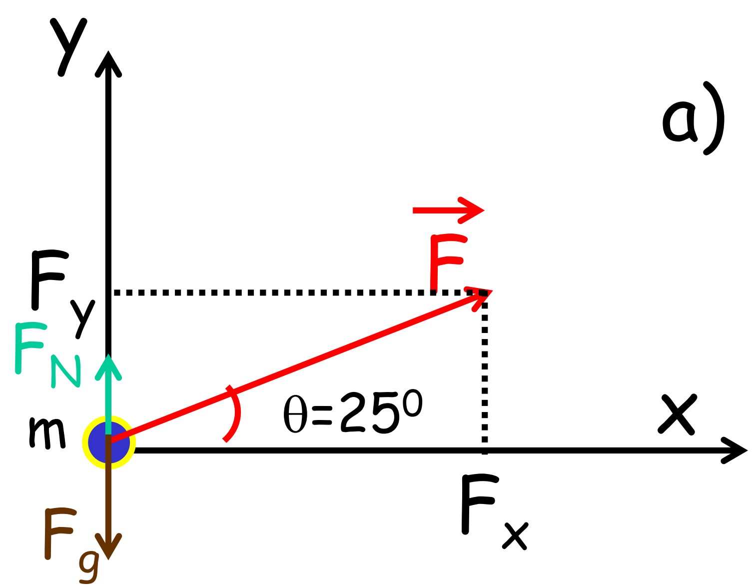


Figure 5-45



a) $a_x = F_x/m$

$= F\cos(25^0)/m = 2.17\ m/s^2$

m only accelerate in x direction , so $a_y = 0$;

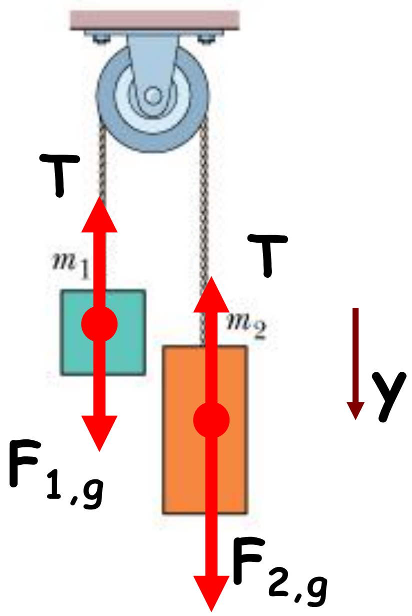
$a = a_x = 2.17\ m/s^2$

b) Block is lifted when $F_N = 0 \Leftrightarrow F = F_g/\sin(25^0) = 116$ N

c) $a = F\cos(25^0)/m = 116\cos(25^0)/5 = 21\ m/s^2$

51. The figure below shows two blocks connected by a cord (of negligible mass) that passes over a frictionless pulley (also of negligible mass). The arrangement is known as Atwood's machine. One block has mass $m_1=1.3$ kg; the other has mass $m_2=2.8$ kg. What are (a) the magnitude of the block's acceleration and (b) the tension in the cord?

$$m_1 g - T = m_1 a_1$$
$$m_2 g - T = m_2 a_2$$



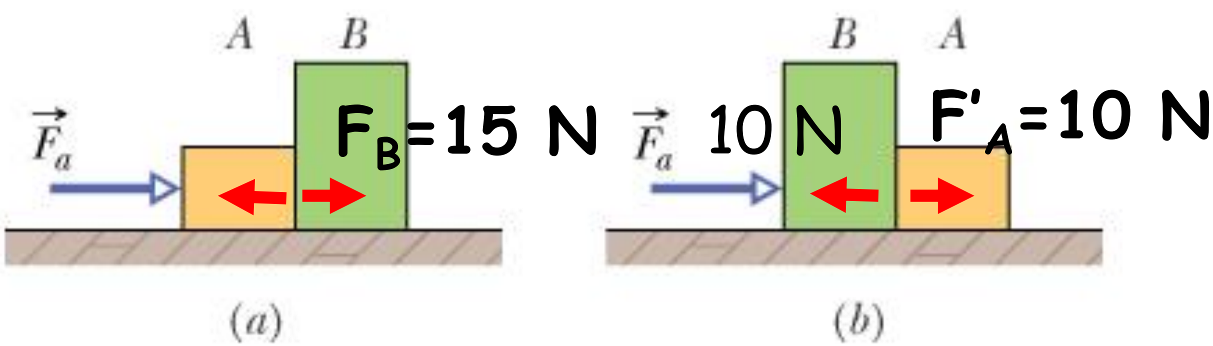
$a_1 = -a_2 = -a$:

$$m_1 g - T = -m_1 a$$
$$m_2 g - T = m_2 a$$

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2} = 3.6(m/s^2)$$

$$T = m_1(g+a)$$
$$T = 17.4\ (N)$$

56. In Figure a, a constant horizontal force $\vec{F}_a$ is applied to block A, which pushes against block B with a 15.0 N force directed horizontally to the right. In Figure b, the same force $\vec{F}_a$ is applied to block B; now block A pushes on block B with a 10.0 N force directed horizontally to the left. The blocks have a combined mass of 12.0 kg. What are the magnitudes of (a) their acceleration in Figure a and (b) force $\vec{F}_a$?



(a) Figure a: $F_B = m_B a$

Figure b: $F'_A = m_A a$

$a = F_B/m_B = F_a/m_a$

F_B=15 N; F'_A=10 N since $F_B = 1.5\ F_a$, so $m_B = 1.5\ m_a$

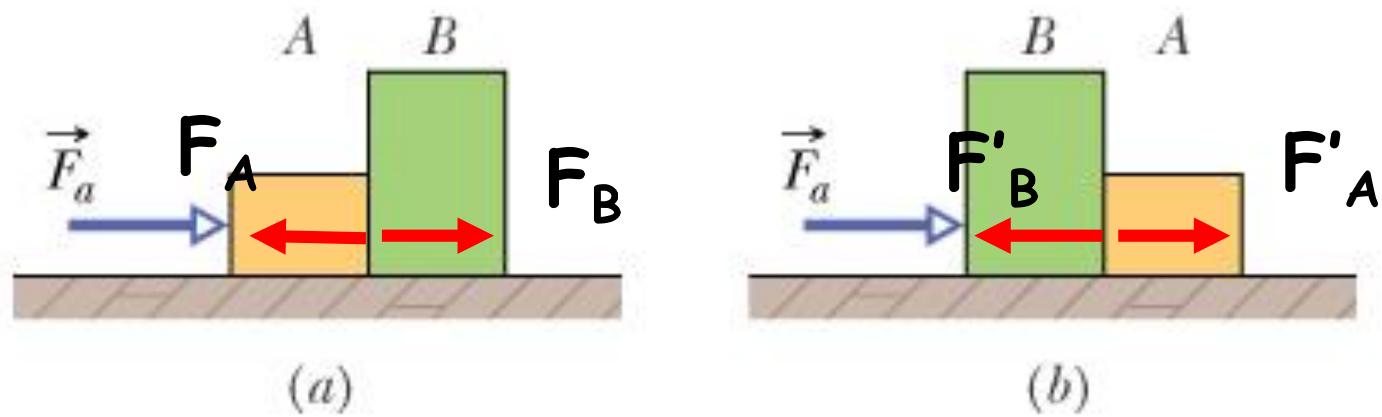
$m_a + m_B = 12$ kg ➔ $m_a = 4.8$ kg

$a = 10/4.8 = 2.08\ (m/s^2)$

(b) $F_a = (m_A + m_B)a = 25$ (N)

Inverse problem:

If we know F_a = 25 N, m_A = 4.8 kg and m_B = 7.2 kg, Determine contact forces between the blocks in Figure a and b.



$F_a = (m_A+m_B)a \rightarrow a = F_a/(m_A+m_B) = 25/12 \approx 2.08\ m/s^2$

Figure a: $F_B = m_Ba \rightarrow F_B= 7.2 \times 2.08 \approx 15$ N

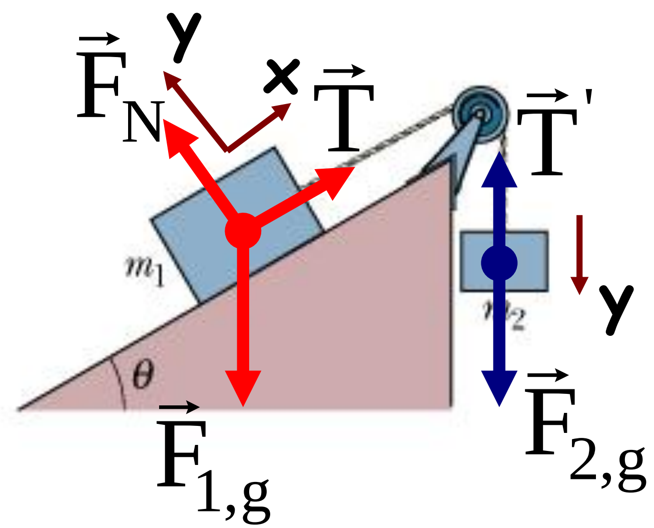
$F_a - F_A = m_Aa \rightarrow F_A= 25 - 4.8 \times 2.08 \approx 15$ N

Figure b: $F'_A = m_Aa \rightarrow F'_A= 4.8 \times 2.08 \approx 10$ N

$F_a - F'_B = m_Ba \rightarrow F'_B= 25 - 7.2 \times 2.08 \approx 10$ N

57. A block of mass m_1=3.7 kg on a frictionless plane inclined at angle $\theta=30.0^0$ is connected by a cord over a massless, frictionless pulley to a second block of mass m_2=2.30 kg hanging vertically. What are (a) the magnitude of the acceleration of each block, (b) the direction of the acceleration of the hanging block, and (c) the tension in the cord?

1. Force analysis



2. Applying Newton's second law:

Block 1: $$F_N - F_{1,g}\cos\theta = 0$$

$$T - F_{1,g}\sin\theta = m_1 a$$

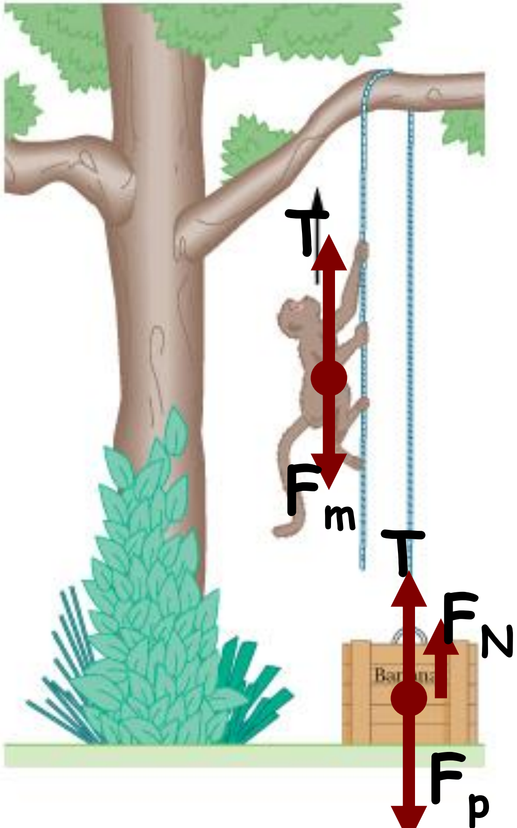
Block 2: $$F_{2,g} - T = m_2 a$$

$$\Rightarrow a = \frac{m_2 g - m_1 g \sin\theta}{m_1 + m_2} = 0.735\,(m/s^2)$$

a>0: the direction of the acceleration of block 2 is downward.

$$T = F_{2,g} - m_2 a = m_2(g - a) = 20.9\ (N)$$

59. A 10 kg monkey climbs up a massless rope that runs over a frictionless tree limb and back down to a 15 kg package on the ground (a) What is the magnitude of the least acceleration the monkey must have if it is to lift the package off the ground? If, after the package has been lifted, the monkey stops its climb and holds onto the rope, what are the (b) magnitude and (c) direction of the monkey's acceleration and (d) the tension in the rope?



(a) T: the force the rope pulls upward on the monkey: $T - mg = ma_m$

For the package:

$$T + F_N - Mg = Ma_p$$

To lift the package off the ground: $F_N = 0$, and the least acceleration a_m requires $a_p = 0$, so:

$$T = Mg$$

$$Mg - mg = ma_m$$

➔ $a_m = 4.9\ (m/s^2)$

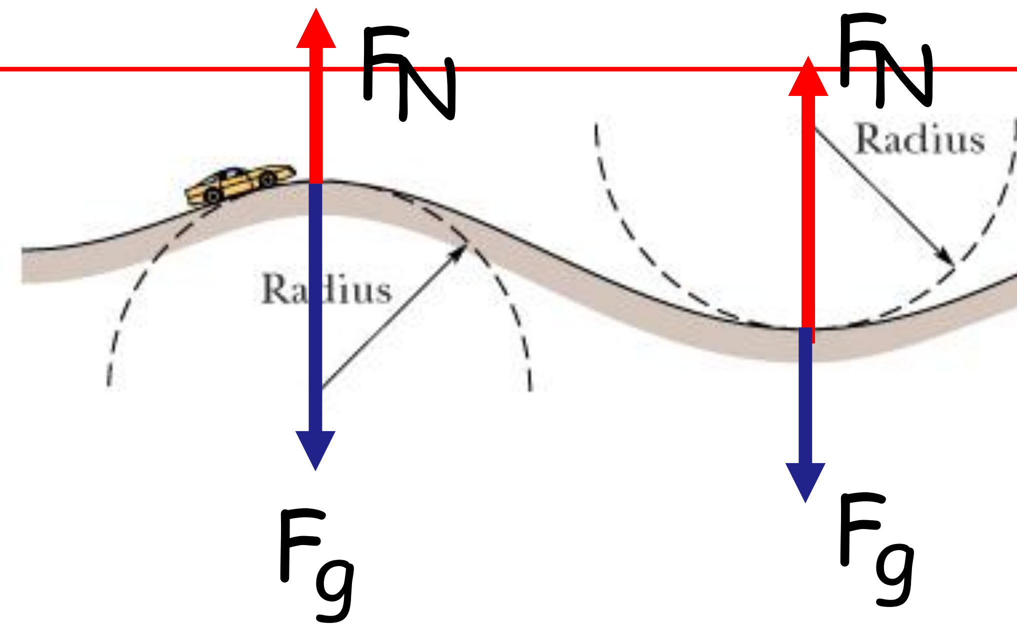
(b)

$$a = \frac{(M - m)g}{M + m} = 1.96\ (m/s^2)$$

(c) $T = m(g + a_m) \approx 118\ (N)$

Problems: 49, 51, 70 (p. 134-137)

49. In the figure below, a car is driven at constant speed over a circular hill and then into a circular valley with the same radius. At the top of the hill, the normal force on the driver from the car seat is 0. The driver's mass is 80.0 kg. What is the magnitude of the normal on the driver from the seat when the car passes through the bottom of the valley?



At the top of the hill:

$$F_{centripetal} = F_g - F_N = m\frac{v^2}{R}$$

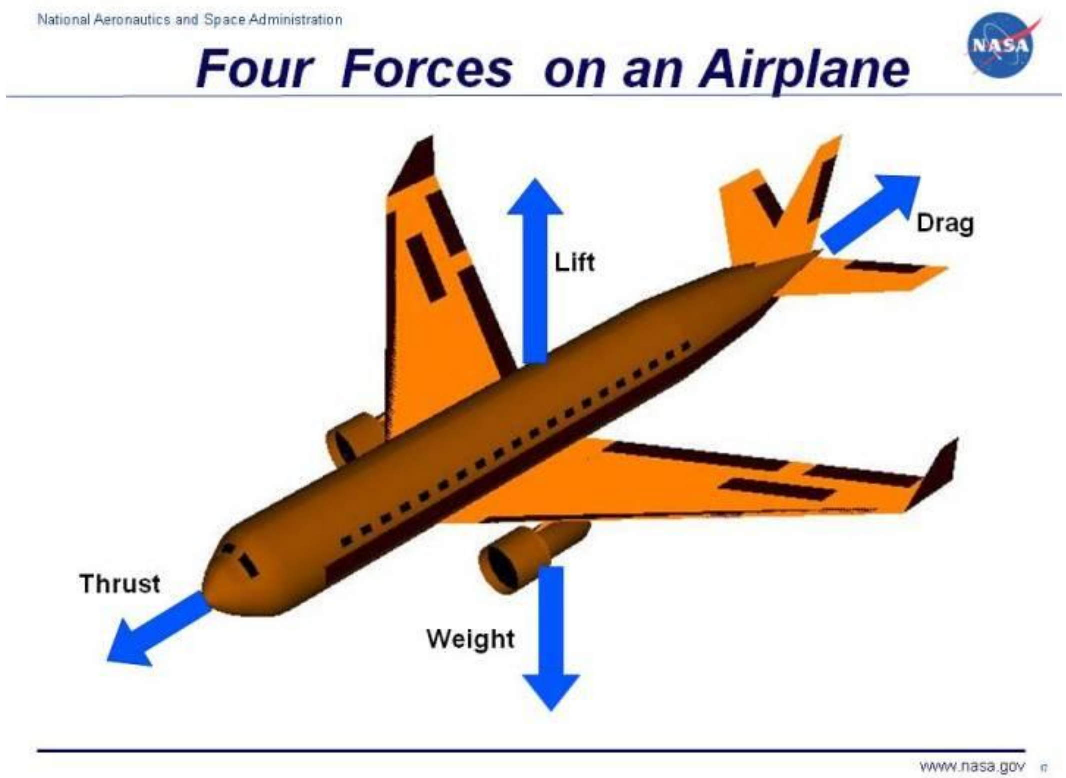
$$F_N = 0 \Rightarrow F_g = m\frac{v^2}{R}$$

At the bottom of the valley:

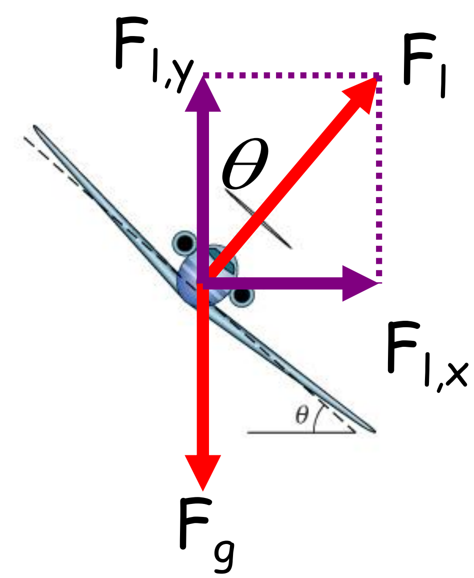
$$F_{centripetal} = F_N - F_g = m\frac{v^2}{R} \Rightarrow F_N = m\frac{v^2}{R} + F_g$$

$$F_N = 2F_g = 2mg = 2\times 80\times 9.8 = 1568\,(\text{N})$$

51. An airplane is flying in a horizontal circle at a speed of 600 km/h. If its wings are tilted at angle $\theta = 40^0$ to the horizontal, what is the radius of the circle in which the plane is flying? Assume that the required force is provided entirely by an "aerodynamic lift" that is perpendicular to the wing surface.



•According to the Bernoulli's principle, the aerodynamic lift appears due to the air-stream velocity over the top of the airplane greater than that at the bottom.



$$F_{l,y} = F_g$$

$$F_l \cos\theta = mg \ (1)$$

$F_{l,x}$ is the centripetal force: $F_{l,x} = m\frac{v^2}{R}$

$$F_l \sin\theta = m\frac{v^2}{R} (2)$$

$$v = 600 \ \text{km/h} = 166.7 \ \text{m/s}$$

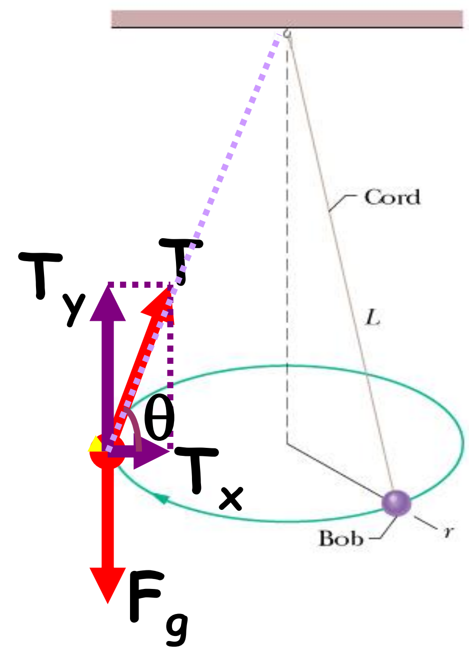
$$(1) \text{ and } (2) \Rightarrow R = \frac{v^2}{g\tan\theta} = \frac{166.7^2}{9.8\tan(40)} \approx 3379 \ (\text{m}) \text{ or } 3.38 \ (\text{km})$$

70. The figure below shows a *conical pendulum*, in which the bob (the small object at the lower end of the cord) moves in a horizontal circle at constant speed. The cord sweeps out a cone as the bob rotates. The bob has a mass of 0.050 kg, the string has length L=0.90 m and negligible mass, and the bob follows a circular path of circumference 0.94 m. What are (a) the tension in the string and (b) the period of the motion?

$$T_y - F_g = 0 \Rightarrow T\sin\theta = mg$$

T_x: the centripetal force

θ: the angle between the cord and the horizontal circle.



$$T_x = m\frac{v^2}{R} \Rightarrow T\cos\theta = m\frac{v^2}{R}$$

$$\cos\theta = \frac{R}{L};\ R = \frac{C}{2\pi} = \frac{0.94\,(m)}{2\times 3.14} = 0.15\,m$$

$$\theta = \arccos\left(\frac{R}{L}\right) = 80.4^0$$

(a) $T = \dfrac{mg}{\sin\theta} \approx 0.5\,(N)$ **(b)** $v = \sqrt{\dfrac{T\times R\cos\theta}{m}} = 0.5\,(m/s);\ P = \dfrac{C}{v} = 1.88\,(s)$