Problems and Solutions

Chapter 1: Bases of Kinematics

1.1. Motion in One Dimension

Problems: 1, 2, 3, 4, 7, 12, 21, 30, 34, 48, 50
(Page 30 - 35)

1. While driving a car at 90 km/h, how far do you move while your eyes shut for 0.50 s during a hard sneeze?

90 km/h = 25 m/s

During the sneeze, the car travels a distance:

$S = v \times t$

$= 25 \times 0.50$

$= 12.5$ (m)

2. Compute your average velocity in the following two cases: (a) You walk 73.2 m at a speed of 1.22 m/s and then run 73.2 m at a speed of 2.85 m/s along a straight track. (b) You walk for 1.0 min at a speed 1.22 m/s and then run for 1.0 min at 3.05 m/s along a straight track. (c) Graph x versus t for both cases and indicate how the average velocity is found on the graph.

(a) the average velocity: $$v_{avg} = \frac{\text{displacement}}{\text{time interval}}$$

along a straight line: $$v_{avg} = \frac{73.2 + 73.2}{\frac{73.2}{1.22} + \frac{73.2}{2.85}} = 1.71(\text{m/s})$$

(b)

$$v_{avg} = \frac{1.22(m/s) \times 60s + 3.05(m/s) \times 60s}{2 \times 60s} \approx 2.14(m/s)$$

(c)

$$x = v_0 t$$

3. An automobile travels on a straight road for 40 km at 30 km/h. It then continues in the same direction for another 40 km at 60 km/h. (a) What is the average velocity of the car during this 80 km trip? (assume that it moves in the positive x direction) (b) What is the average speed? (c) Graph x versus t and indicate how the average velocity is found on the graph.

Following the definition: $v_{avg} = \frac{\text{displacement}}{\text{time interval}}$

(a) The car moves in the same direction, so the total displacement is: $\Delta x = 40 + 40 = 80(km)$

The total time:

$$\Delta t = \frac{40}{30} + \frac{40}{60} = 2(h)$$

So:

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{80}{2} = 40(km/h)$$

(b) Following the definition:

$$s_{avg} = \frac{\text{total distance}}{\text{time interval}}$$

So:

$$d = 40 + 40 = 80(km)$$

$$\Delta t = \frac{40}{30} + \frac{40}{60} = 2(h)$$

$$s_{avg} = \frac{d}{\Delta t} = \frac{80}{2} = 40(km/h)$$

(c) We use the following equation to graph x versus t:

$$x = v_0 t$$

4. A car travels up a hill at a constant speed of 35 km/h and returns down the hill at a constant speed of 60 km/h. Calculate the average speed for the round trip.

Average speed = total distance/time interval

$$D_{up} = D_{down}$$

$$s = \frac{D_{up} + D_{down}}{t_{up} + t_{down}} = \frac{D_{up} + D_{down}}{\dfrac{D_{up}}{s_{up}} + \dfrac{D_{down}}{s_{down}}} = 2\frac{s_{up} \times s_{down}}{s_{up} + s_{down}}$$

$$s = 44.2(km/h)$$

7. Two trains, each having a speed of 30 km/h, are headed at each other on the same straight track. A bird that can fly 60 km/h flies off the front of one train when they are 60 km apart and heads directly for the other train. On reaching the other train, the (crazy) bird flies directly back to the first train, and so forth. What is the total distance the bird travels before the trains collide?

The two trains are travelling with the same speed when they are 60km apart, so the collision will occur after each train travels 30km.

The trains collide after:

$$t = \frac{S\,train}{v\,train} = \frac{30}{30} = 1(h)$$

The total distance the bird travels is:

S = vbird × t = 60 × 1 = 60(km)

12. An abrupt slowdown in concentrated traffic can travel as a pulse, termed a shock wave, along the line of cars, either downstream (in the traffic direction) or upstream, or it can be stationary. Figure 2-25 shows a uniformly spaced line of cars moving at speed v = 25.0 m/s toward a uniformly spaced line of slow cars moving at speed v_s = 5.00 m/s. Assume that each faster car adds length L = 12.0 m (car length plus buffer zone) to the line of slow cars when it joins the line, and assume it slows abruptly at the last instant. (a) For what separation distance d between the faster cars does the shock wave remain stationary? If the separation is twice that amount, what are the (b) speed and (c) direction (upstream or downstream) of the shock wave?

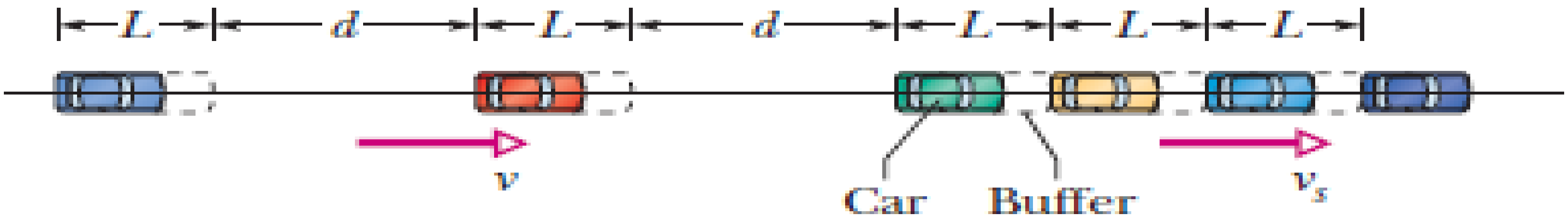
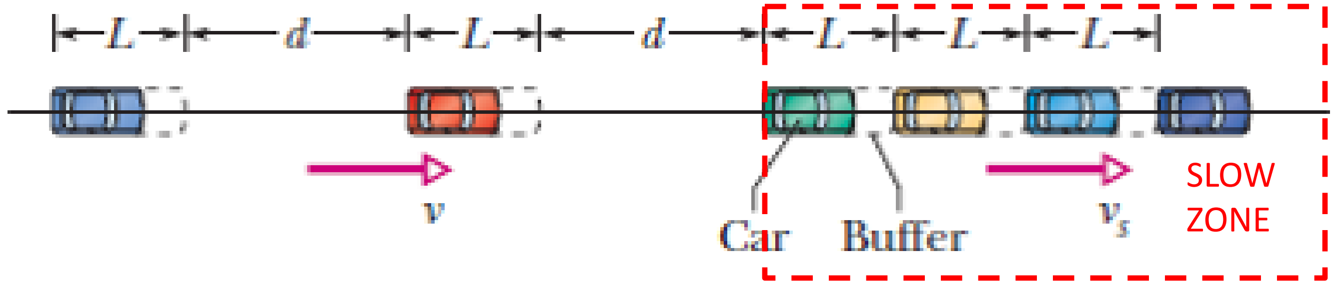


Figure 2-25 Problem 12.



(a) Imagine that when a car crosses the slow zone (picture), it will slow down from 25 m/s to 5 m/s. Time for the slow car to move a distance L in the slow zone:

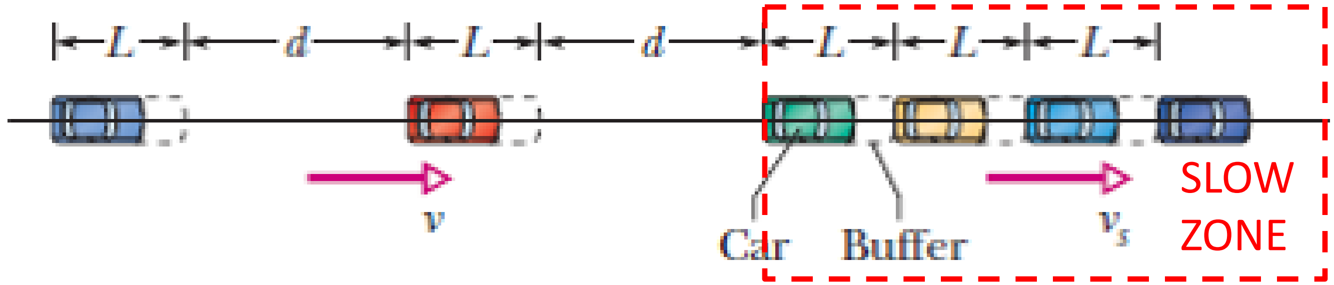
$$t = \frac{L}{v_s} = \frac{12}{5} = 2.4\ (s)$$

Time for the fast car to join the slow zone:

$$t = \frac{L+d}{v} = \frac{L+d}{25}\ (s)$$

The shock wave remains stationary when:

$$\frac{L+d}{25} = 2.4 \Rightarrow d = 25 \times 2.4 - 12 = 48 (m)$$



(b) d' = 2d = 96m

Let x is the distance between 2 cars in the slow zone (not including Buffer). Time for the fast car to join the slow zone:

t = $\frac{d'+L+x}{v}$ (1)

Time for the slow car to move a distance (L+x):

t = $\frac{L+x}{vs}$ (2)

(1)(2) → $\frac{d'+L+x}{v}$ = $\frac{L+x}{vs}$ → x = 12m→ t = $\frac{L+x}{vs} = 4.8(\mathrm{s})$

Speed of the shock wave:

V = $\frac{x}{t} = \frac{12}{4.8} = 2.5\ \left(\frac{m}{s}\right)$

(c) The direction of the shock wave is downstream because x > 0.

30. The brakes on your car can slow you at a rate of 5.2 m/s^2. (a) If you are going 146 km/h and suddenly see a state trooper, what is the minimum time in which you can get your car under the 90 km/h speed limit? (The answer reveals the futility of braking to keep your high speed from being detected with a radar or laser gun).
(b) Graph x versus t and v versus t for such a slowing.

- $a=-5.2\ m/s^2$; $v_0=146$ km/h or $v_0=40.6$ m/s;
- $v_1=90$ km/h or $v_1=25$ m/s

(a) The minimum time t_{min} must match:

$$v = v_0 + at \le v_1 \quad \xrightarrow{(a<0)} \quad \frac{v_1 - v_0}{a} \le t$$

$$t_{min} = \frac{v_1 - v_0}{a} = \frac{(25 - 40.6)}{-5.2} = 3.0(s)$$

(b) x vs. t and v vs. t:

$$x = v_0 t + \frac{1}{2}at^2 = 40.6t - 2.6t^2; \quad v = v_0 + at = 40.6 - 5.2t$$

48. A hoodlum throws a stone vertically downward with an initial speed of 15.0 m/s from the roof of a building, 30.0 m above the ground. (a) How long does it take the stone to reach the ground? (b) What is the speed of the stone at impact?

This is a freely falling object problem:

$$y = y_0 + v_0 t + \frac{1}{2}at^2$$

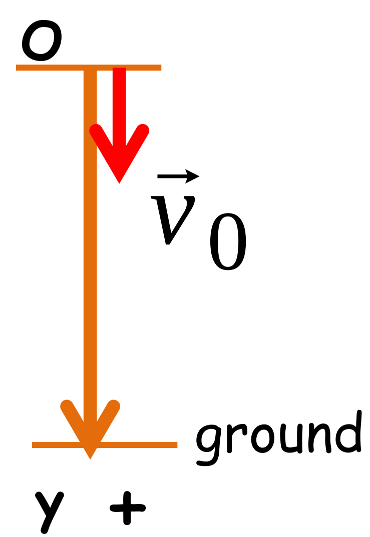
We choose the positive direction is downward, the origin O at the roof $y_0 = 0$, so:

(a) When the stone hits the ground, we have $y = 30$ m:

$$30 = 0 + 15t + \frac{1}{2}9.8t^2$$

$$\Rightarrow t = 1.38(s)$$

(discard t < 0)



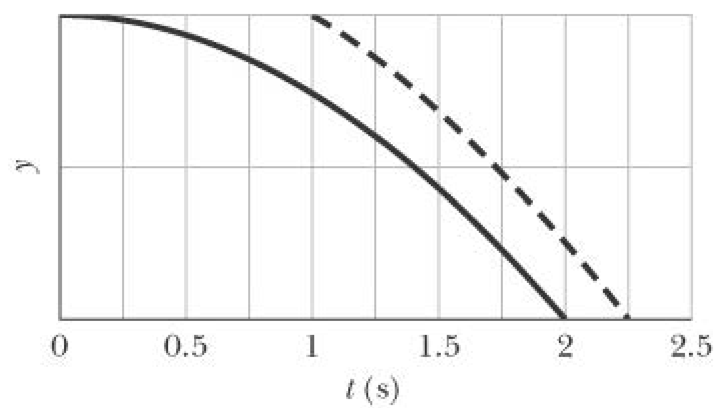
(b) $$v = v_0 + at = v_0 + gt = 15 + 9.8 \times 1.38 \approx 28.5(m/s)$$

50. At time t=0, apple 1 is dropped from a bridge onto a roadway beneath the bridge; somewhat later, apple 2 is thrown down from the same height. Figure below gives the vertical positions y of the apples versus t during the falling, until both apples have hit the roadway. With approximately what speed is apple2 thrown down?

- Apple 1 hits the roadway at t=2 s:

$$y = y_0 - \frac{1}{2}gt^2$$

$$0 = y_0 - \frac{1}{2}\times 9.8\times 2^2 \Rightarrow y_0 = 19.6\,(\mathrm{m})$$



- Apple 2 is thrown down at t=1 s and hits the roadway at t=2.25 s:

$$y = y_0 + v_0 t - \frac{1}{2}gt^2 \Rightarrow 0 = y_0 + v_0 t_2 - \frac{1}{2}g{t_2}^2$$

$$t_2 = 1.25\,(\mathrm{s}) \Rightarrow \boxed{v_0 \approx -9.6\,(\mathrm{m/s})}$$

Chapter 1: Bases of Kinematics

1.2. Motion in Two Dimension

Homework: 6, 11, 20, 27, 29, 54, 66, 70, 76
(page 78-83)

6. An electron's position is given by $\vec{r} = 3.00t\hat{i} - 4.00t^2\hat{j} + 2.00\hat{k}$ with t in seconds and $\vec{r}$ in meters. (a) In unit-vector notation, what is the $\vec{v}(t)$ electron's velocity ? At t=3.00 s, what is (b) $\vec{v}$ in unit vector notation and as (c) a magnitude and (d) an angle relative to the positive direction of the x axis?

(a)

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = \frac{d}{dt}(3.00t\hat{i} - 4.00t^2\hat{j} + 2.00\hat{k})$$

$$\vec{v}(t) = 3.00\hat{i} - 8.00t\hat{j}$$

(b) at t = 3s: $\vec{v} = 3.00\,\hat{i} - 24.00\,\hat{j}$

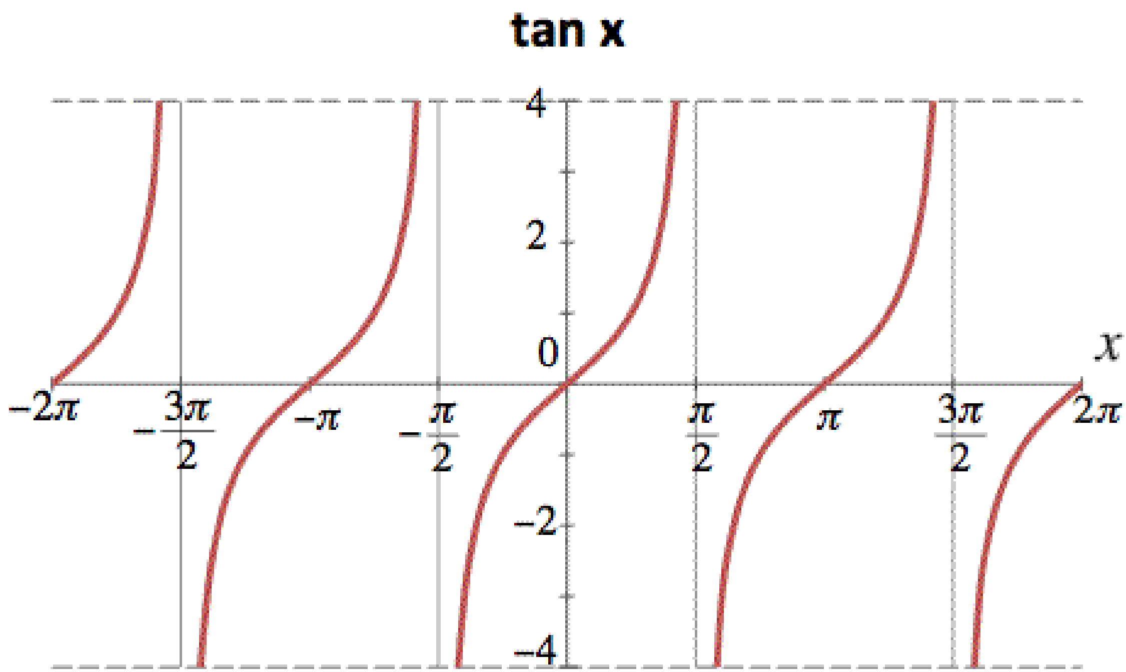
(c) the magnitude of the velocity:

$$v = \sqrt{3.00^2 + (-24.00)^2} = 24.2\ (m/s)$$

(d) $$\tan\theta = \frac{v_y}{v_x} \Rightarrow \theta = \tan^{-1}\left(\frac{-24}{3}\right)$$

$$\theta_1 = -82.9^0 \quad \text{or} \quad \theta_2 = \theta_1 + 180^0 = -82.9^0 + 180^0 = 97.1^0$$

We choose $\theta = -82.9^0$ (the fourth quadrant) since $v_x>0$ and $v_y<0$



11. The position $\vec{r}$ of a particle moving an xy plane is given by $\vec{r} = (2.0t^3 - 5.0t)\hat{i} + (6.0 - 7.0t^4)\hat{j}$, with r in meter and t in second. In unit-vector notation, calculate (a) $\vec{r}$, (b) $\vec{v}$, and (c) $\vec{a}$ for t=2.0 s. (d) What is the angle between the positive direction of the x axis and a line tangent to the particle's path at t = 2.0s?

(a) at t = 2 s: $\vec{r} = (6m)\hat{i} - (106m)\hat{j}$

(b) $\vec{v} = \dfrac{d\vec{r}}{dt} = (6.0t^2 - 5.0)\hat{i} - (28.0t^3)\hat{j}$

So, at t = 2 s: $\vec{v} = (19m/s)\hat{i} - (224m/s)\hat{j}$

(c) $\vec{a} = \dfrac{d\vec{v}}{dt} = (12.0t)\hat{i} - (84.0t^2)\hat{j}$

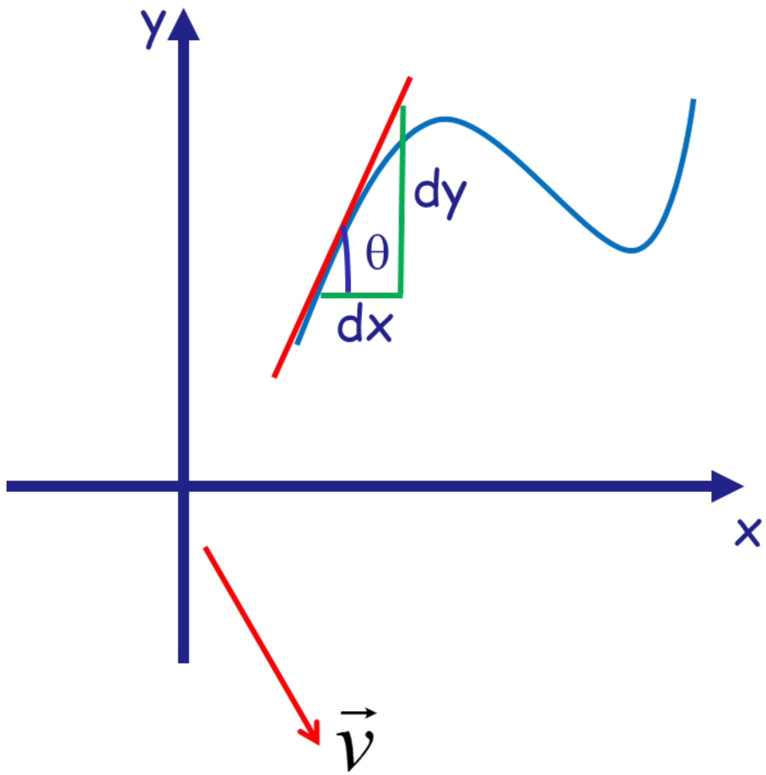
So, at t = 2 s: $\vec{a} = (24m/s^2)\hat{i} - (336m/s^2)\hat{j}$

(d) a line tangent to the particle's path:

$$\tan\theta = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{v_y}{v_x}$$

$$\tan\theta = -\frac{224}{19}$$

$$\Rightarrow \theta_1 = -85.2^0; \theta_2 = 94.8^0$$



V_y<0, V_x>0, so the velocity should be in the fourth quadrant

•••20 In Fig. 4-32, particle A moves along the line $y = 30$ m with a constant velocity $\vec{v}$ of magnitude 3.0 m/s and parallel to the x axis. At the instant particle A passes the y axis, particle B leaves the origin with a zero initial speed and a constant acceleration $\vec{a}$ of magnitude 0.40 m/s^2. What angle θ between $\vec{a}$ and the positive direction of the y axis would result in a collision?

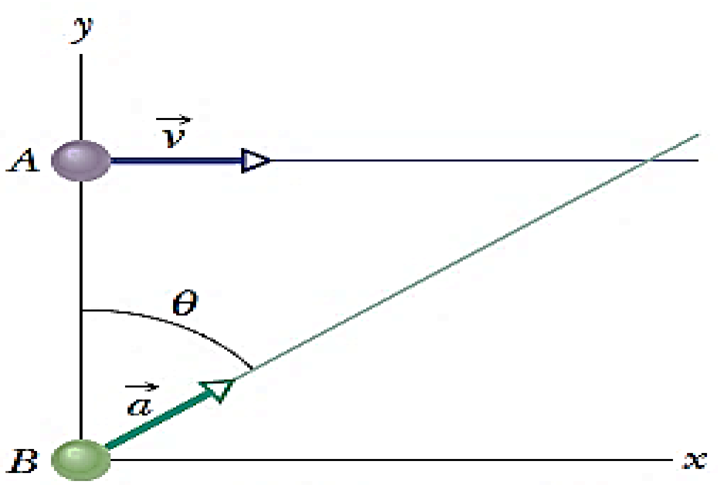


Figure 4-32 Problem 20.

After a time t

- A moves a distance along to x: $X_A = vt = 3t$ (m)
- B moves a distance:

On Ox: $X_{Bx} = a_x t^2/2 = 0.2 \sin(\theta)\, t^2$

On Oy: $X_{By} = a_y t^2/2 = 0.2 \cos(\theta)\, t^2$

At the time of collision: $\begin{cases} X_A = X_{Bx} \Leftrightarrow 3t - 0.2 \sin(\theta) t^2 = 0 \Rightarrow t = 15/\sin(\theta) \\ X_{By} = 30 \Leftrightarrow 0.2 \cos(\theta)\, t^2 = 30 \end{cases}$

$\Rightarrow 30\cos^2(\theta) + 45\cos(\theta) - 30 = 0 \Rightarrow \cos(\theta) = ½$, (eliminate $\cos(\theta) = -2$)

$\Rightarrow$ Angle $\theta = 60^0$

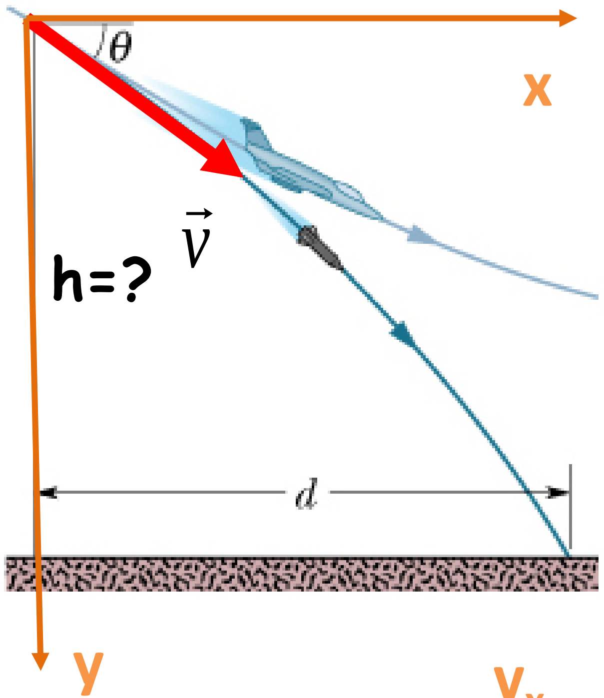
27. A certain airplane has a speed of 290 km/h and is diving at an angle of θ = 30⁰ below the horizontal when the pilot releases a radar decoy. The horizontal distance between the release point and the point where the decoy strikes the ground is d = 700 m. (a) How long is the decoy in the air? (b) How high was the release point?

This is a projectile motion

$$v_x = v\cos\theta$$

$$v_y = v\sin\theta$$

$$v = 290\,km/h = 80.6\,m/s$$



(a) Motion along the x axis:

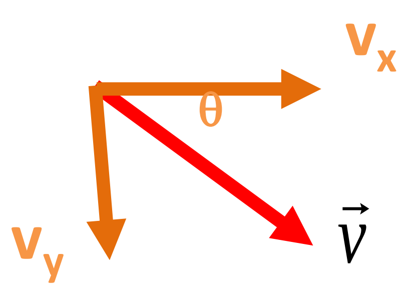
$$t = \frac{d}{v\cos\theta} = \frac{700}{80.6\times 0.866} \approx 10\ (\text{s})$$

(b) We have:

$$y = y_0 + v_{0y}t + \frac{1}{2}gt^2$$

$$h = y = (v\sin\theta)t + \frac{1}{2}gt^2$$

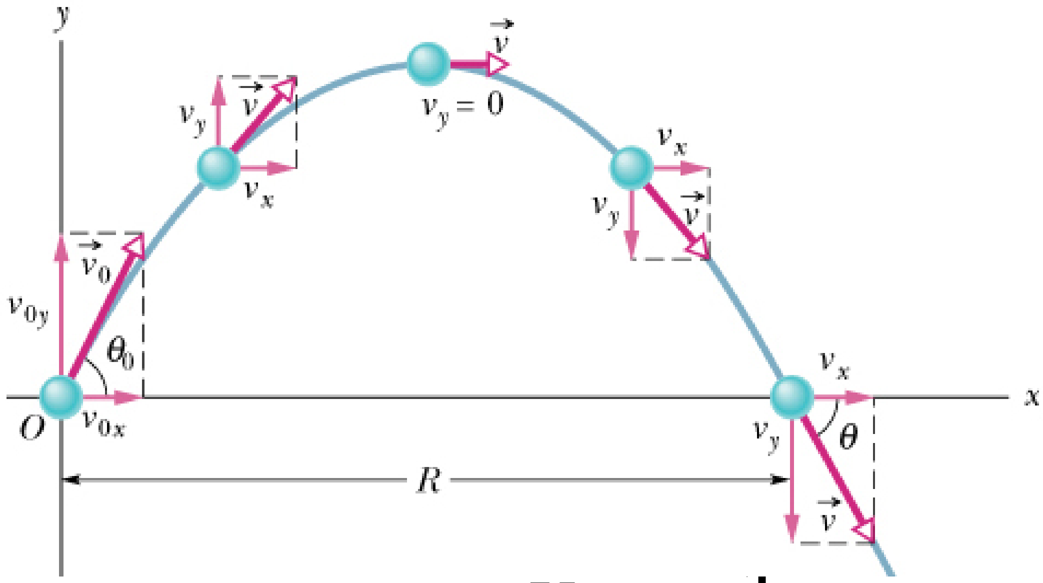
$$h = 893(m)$$



29. A projectile's launch speed is five times its speed at maximum height. Find launch angle θ_0.

- At maximum height: $v_y=0$; $v=v_x=v_{0x}$

We have $v_0 = 5\ v_{0x}$



$$\cos(\theta_0) = \frac{v_{0x}}{v_0} = \frac{1}{5}$$

$$\Rightarrow \theta_0 = 78.5^0$$

•••54 GO A ball is to be shot from level ground with a certain speed. Figure 4-45 shows the range R it will have versus the launch angle θ_0. The value of θ_0 determines the flight time; let t_{max} represent the maximum flight time. What is the least speed the ball will have during its flight if θ_0 is chosen such that the flight time is $0.500t_{max}$?

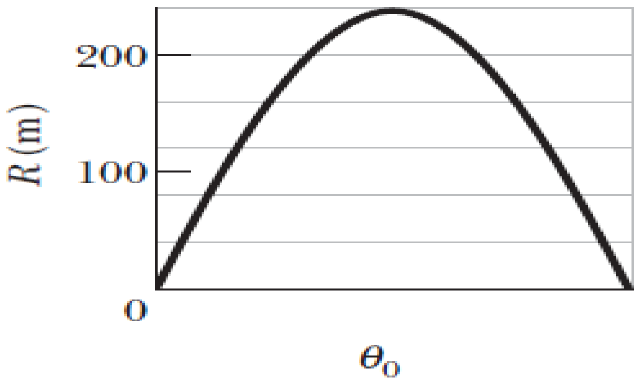


Figure 4-45 Problem 54.

$R = \frac{v^2{}_0}{g} sin2\theta_0$ From the figure R_{max}= 240 when $\theta_0 = 45^0$

$\Rightarrow v_o = \sqrt{R_{max}g}$ = 48.5 (m/s)

$y=y_0+v_{0y}t - gt^2/2$, when the ball reach the ground y=0

$\Rightarrow$ $v_0 \sin\theta_0 t - gt^2/2 = 0$ => flight time $t=2v_0\sin\theta_0/g$

$t = t_{max} = 2v_0/g$ when $\sin\theta_0 = 1$ or $\theta_0 = 90^0$

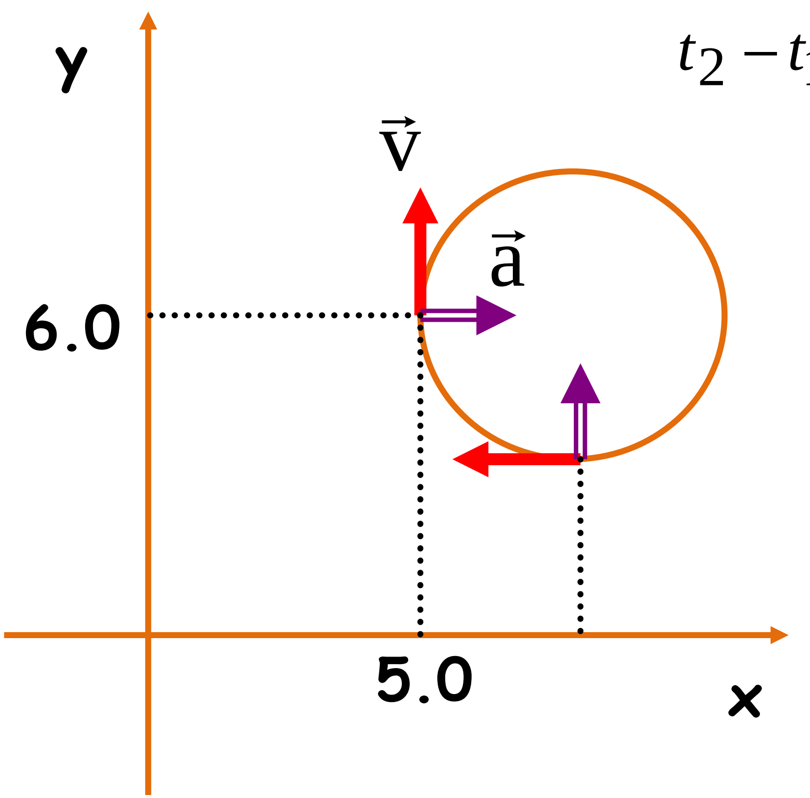
In case of $\theta_0 = 0.5\ t_{max}$, $\sin\theta_0 = 1/2$ => $\theta_0 = 30^0$

The least speed $v=v_x$ (when $v_y=0$)

$v=v_0 \cos(\theta_0) = 48.5 \cos(30^0) = 42$ (m/s)

66. A particle moves along a circular path over a horizontal xy coordinate system, at constant speed. At time t_1=5.0 s, it is at point (5.0 m, 6.0 m) with velocity (3.0 m/s) $\hat{j}$ and acceleration in the positive x direction. At time t_2=10.0 s, it has velocity (-3.0 m/s) $\hat{i}$ and acceleration in the positive y direction. What are the (a) x and (b) y coordinates of the center of the circular path if t_2-t_1 is less than one period?

This is a uniform circular motion



$$t_2 - t_1 = \frac{3}{4}T + nT = 5.0\ (s)\ (n = 0 \text{ as } t_2 - t_1 < T)$$

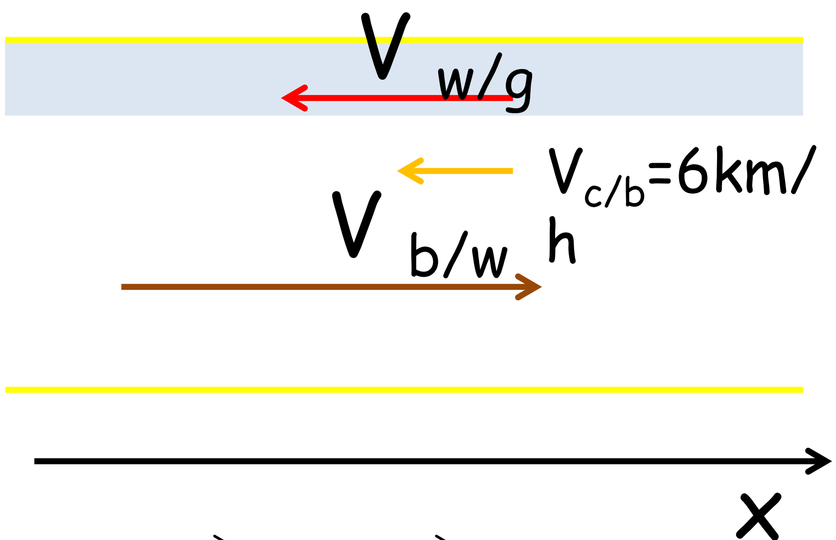
$$T = 6.67\ (s)$$

$$T = \frac{2\pi r}{v} \Rightarrow r = \frac{Tv}{2\pi} = 3.18\ (m)$$

$$x_{center} = 5.0 + 3.18 = 8.18\ (m)$$

$$y_{center} = 6.0\ (m)$$

•70 A boat is traveling upstream in the positive direction of an x axis at 14 km/h with respect to the water of a river. The water is flowing at 9.0 km/h with respect to the ground. What are the (a) magnitude and (b) direction of the boat's velocity with respect to the ground? A child on the boat walks from front to rear at 6.0 km/h with respect to the boat. What are the (c) magnitude and (d) direction of the child's velocity with respect to the ground?



a,

$$\vec{V}_{b/g} = \vec{V}_{b/w} + \vec{V}_{w/g}$$

$$= 14 - 9 = 5 \text{ (km/h)}$$

$V_{b/g}$ = 5 km/h

b) Positive x direction

c,

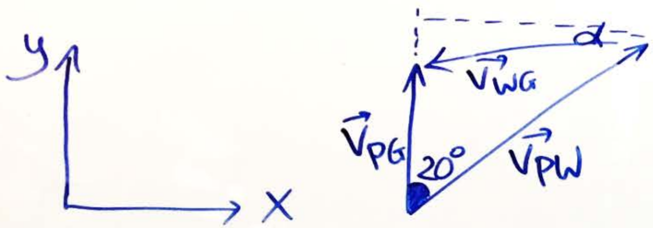
$$\vec{V}_{c/g} = \vec{V}_{c/b} + \vec{V}_{b/g}$$

$$= -6 + 5 = -1 \text{ (km/h)}$$

$V_{c/g}$ = 1 km/h

d) Negative x direction

••76 A light plane attains an airspeed of 500 km/h. The pilot sets out for a destination 800 km due north but discovers that the plane must be headed 20.0° east of due north to fly there directly. The plane arrives in 2.00 h. What were the (a) magnitude and (b) direction of the wind velocity?



We have: $\vec{v}_{PG} = \vec{v}_{PW} + \vec{v}_{WG}$

$$\vec{v}_{PG} = \frac{d}{t} = \frac{800}{2} = 400 \text{ km/h}$$

On Oy: $\vec{v}_{PG,y} = \vec{v}_{PW,y} + \vec{v}_{WG,y}$

$$400 = 500 . \cos 20° + \vec{v}_{WG,y}$$

$$\Rightarrow \vec{v}_{WG,y} = -70 \text{ km/h}$$

On Ox: $\vec{v}_{PG,x} = \vec{v}_{PW,x} + \vec{v}_{WG,x}$

$$0 = 500 . \sin 20° + \vec{v}_{WG,x}$$

$$\Rightarrow \vec{v}_{WG,x} = -171 \text{ km/h}$$

$$v_{WG} = \sqrt{v_{WG,x}^2 + v_{WG,y}^2} = \sqrt{70^2 + 171^2} = 185 \text{ km/h}$$

$$\alpha = \tan^{-1} \frac{v_{WG,y}}{v_{WG,x}}$$

$$\alpha = 22.3°$$

So the wind blows 22.3° west of due south