• PROGRAM OF "PHYSICS"

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PHYSICS I
(General Mechanics)
02 credits (30 periods)

Chapter 1 Bases of Kinematics

- Motion in One Dimension
- Motion in Two Dimensions

Chapter 2 The Laws of Motion

Chapter 3 Work and Mechanical Energy

Chapter 4 Linear Momentum and Collisions

Chapter 5 Rotation of a Rigid Object About a Fixed Axis

Chapter 6 Static Equilibrium

Chapter 7 Universal Gravitation

PHYSICS I

Chapter 7
Universal Gravitation

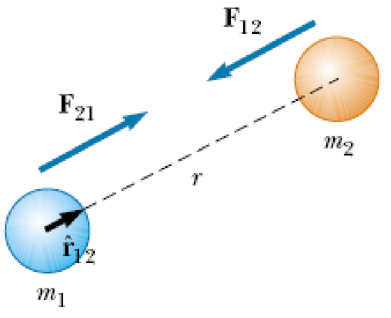
Newton's Law of Universal Gravitation
Kepler's Laws
Gravitational Potential Energy

1. Newton's Law of Universal Gravitation

• The Law: "Every particle in the Universe attracts every other particle with a force that is
directly proportional to the product of their masses and inversely proportional to the square of the distance between them."

$$F_g = G\frac{m_1 m_2}{r^2}$$

$$G = 6.673 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2$$



$$\overrightarrow{F_{12}} = -G\frac{m_1 m_2}{r^2}\overrightarrow{r_{12}} = -G\frac{m_1 m_2}{r^3}\vec{r}$$

- **Free-Fall Acceleration**

The force acting on a freely falling object of mass *m* near the Earth's surface. Newton's second law:

$$F = ma \equiv mg = F_g = G\frac{M_E m}{R_E^2}$$

(M_E: Earth's mass; R_E: Earth's radius)

$$\longrightarrow \quad g = G\frac{M_E}{R_E^2} \approx 9.81\, m/s^2$$

Object of mass *m* located a distance *h* above the Earth's surface or a distance *r* from the Earth's center:

$$\longrightarrow \quad g' = G\frac{M_E}{(R_E + h)^2}$$

EXAMPLE 1

Two objects attract each other with a gravitational force of magnitude 1.00×10^{-8} N when separated by 20.0 cm. If the total mass of the two objects is 5.00 kg, what is the mass of each?

$$F_g = G\frac{m_1 m_2}{r^2} = 6.673\times10^{-11}\frac{m_1 m_2}{(0.20)^2} = 1.00\times10^{-8}$$

$$m_1 m_2 = 6.00$$

$$m_1 + m_2 = 5.00\ kg$$

$$m_1 = 2.00\ kg;\ m_2 = 3.00\ kg$$

$$m_1 = 3.00\ kg;\ m_2 = 2.00\ kg$$

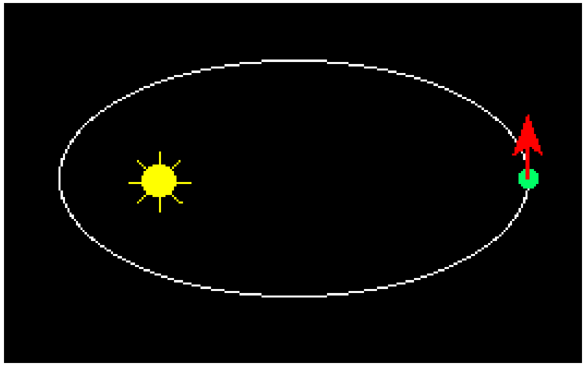
2. Kepler's laws

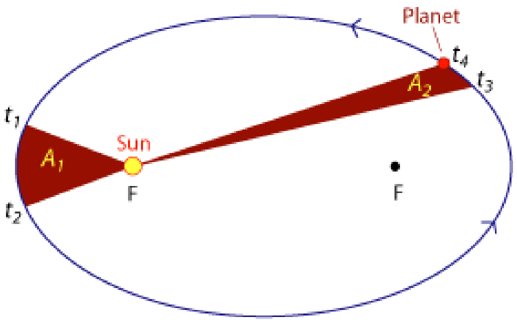
1. All planets move in elliptical orbits with the Sun at one focal point.

2. The radius vector drawn from the Sun to a planet sweeps out equal areas in equal time intervals.

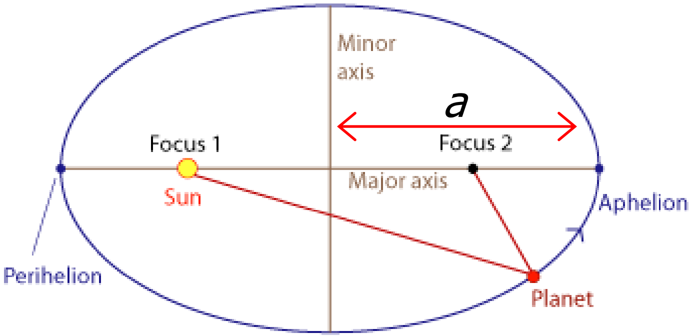
3. The square of the orbital period of any planet is proportional to the cube of the semimajor axis of the elliptical orbit.

T : period of revolution, a : semimajor axis → $T^2 = K_S a^3$



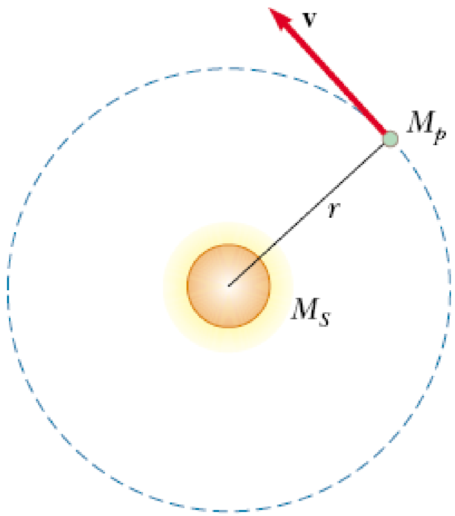


Kepler's Second Law:
Law of Equal Areas
(greatly exaggerated)



An elliptical orbit of a planet
(greatly exaggerated)

CIRCULAR ORBIT : A planet of mass M_p moving around the Sun of mass M_S in a circular orbit



$$\frac{GM_S M_p}{r^2} = \frac{M_p v^2}{r}$$

$$\frac{GM_S}{r^2} = \frac{(2\pi r/T)^2}{r}$$

$$T^2 = \left(\frac{4\pi^2}{GM_S}\right) r^3 = K_S r^3$$

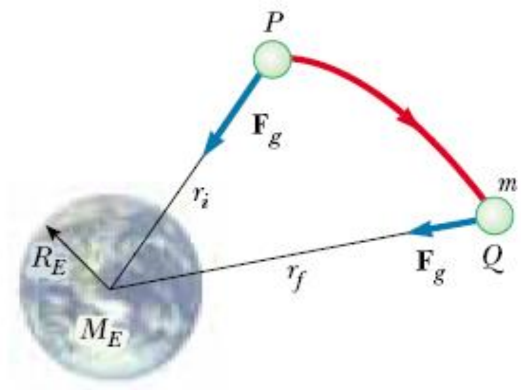
EXAMPLE 2

Calculate the mass of the Sun using the fact that the period of the Earth's orbit around the Sun is 3.156×10^{7} s and its distance from the Sun is 1.496×10^{11} m.

$$M_S = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2(1.496\times10^{11}\,\mathrm{m})^3}{(6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2})(3.156\times10^{7}\,\mathrm{s})^2}$$

$$= \boxed{1.99\times10^{30}\,\mathrm{kg}}$$

3. Gravitational Potential Energy



$$F = -\frac{dU}{dr}$$

$$dU = -Fdr = -\left[-G\frac{m_1 m_2}{r^2}\right]dr$$

$$U_f - U_i = \int_{U_i}^{U_f} dU = \int_{r_i}^{r_f} -G\frac{m_1 m_2}{r^2}dr = Gm_1 m_2 \int_{r_i}^{r_f} -\frac{dr}{r^2}$$

$$U_f - U_i = -Gm_1 m_2\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$$

We put :

$$U_f = -G\frac{m_1 m_2}{r_f}\ ; \quad U_i = -G\frac{m_1 m_2}{r_i}$$

In general:

$$\boxed{U = -G\frac{m_1 m_2}{r}}$$

To compare:

- Gravitational force near the Earth's surface: $F = mg$

Gravitational potential near the Earth's surface: $U = -mgx$

- Elastic force : $F = -kx$

Elastic potential: $U = \frac{1}{2}kx^2$

- Gravitational force : $F_g = G\frac{m_1 m_2}{r^2}$

Gravitational potential : $U = -G\frac{m_1 m_2}{r}$

- Coulomb force : $F_C = K\frac{q_1 q_2}{r^2}$

Gravitational potential : $U_C = -K\frac{q_1 q_2}{r}$

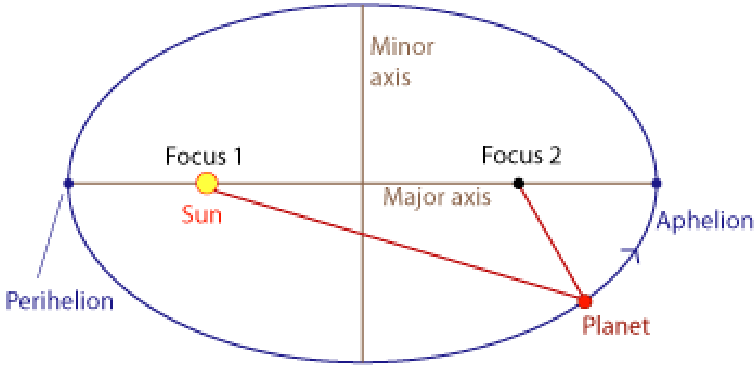
General formula: $F = -\frac{dU}{dr}$ $\vec{F} = -gradU \equiv -\nabla U$

EXAMPLE 3

As Mars orbits the sun in its elliptical orbit, its distance of closest approach to the center of the sun (at perihelion) is 2.067×10^{11} m, and its maximum distance from the center of the sun (at aphelion) is 2.492×10^{11} m. If the orbital speed of Mars at aphelion is 2.198×10^{4} m/s, what is its orbital speed at perihelion? (You can ignore the influence of the other planets.)

Apply conservation of energy:

$$\frac{1}{2}m_M v_a^2 - \frac{GM_S m_M}{r_a} = \frac{1}{2}m_M v_p^2 - \frac{GM_S m_M}{r_p}$$

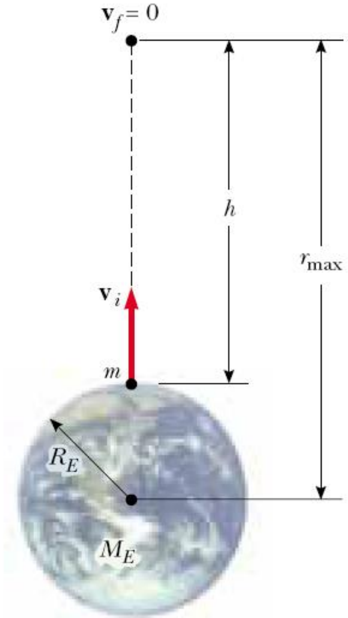


An elliptical orbit of a planet
(greatly exaggerated)

$$v_p = \sqrt{v_a^2 - 2GM_S\left(\frac{1}{r_a} - \frac{1}{r_p}\right)} = 2.650\times10^4 \text{ m/s.}$$

EXAMPLE 4 Escape Speed

Escape speed: The minimum speed the object must have at the Earth's surface in order to escape from the influence of the Earth's gravitational field.



$$\frac{1}{2}mv_i^2 - \frac{GM_E m}{R_E} = -\frac{GM_E m}{r_{max}}$$

$$v_i^2 = 2GM_E\left(\frac{1}{R_E} - \frac{1}{r_{max}}\right)$$

$$h = r_{max} - R_E$$

$r_{max} \longrightarrow \infty \quad v_i = v_{esc}$

$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}}$$

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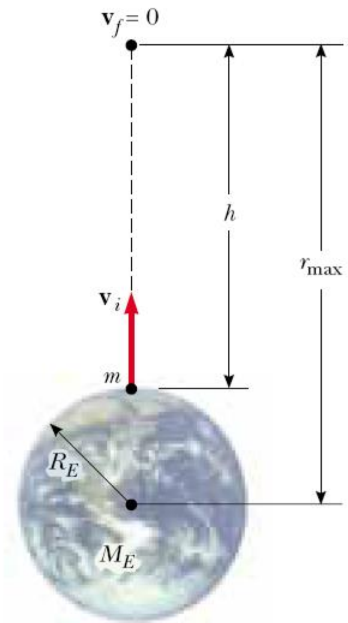

$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}}$$

$$= \sqrt{\frac{2(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(5.98 \times 10^{24}\ \mathrm{kg})}{6.37 \times 10^{6}\ \mathrm{m}}}$$

$$= 1.12 \times 10^{4}\ \mathrm{m/s}$$