• PROGRAM OF "PHYSICS"

Lecturer: Dr. DO Xuan Hoi

Room A1. 503

E-mail : dxhoi@hcmiu.edu.vn

PHYSICS I
(General Mechanics)
02 credits (30 periods)

Chapter 1 Bases of Kinematics

- Motion in One Dimension
- Motion in Two Dimensions

Chapter 2 The Laws of Motion

Chapter 3 Work and Mechanical Energy

Chapter 4 Linear Momentum and Collisions

Chapter 5 Rotation of a Rigid Object About a Fixed Axis

Chapter 6 Static Equilibrium

Chapter 7 Universal Gravitation

PHYSICS 1

Chapter 6
Static Equilibrium

The Conditions for Equilibrium

The Center of Gravity

Examples of Rigid Objects in Static Equilibrium

1 The Conditions for Equilibrium

1.a The first condition for equilibrium

- The term ***equilibrium*** implies either that the object is **at rest** or that its **center of mass moves with constant velocity**

- The object is at rest : It is described as being in ***static equilibrium.***

- **A particle** is in equilibrium-that is, the particle does not accelerate-in an inertial frame of reference if the vector sum of all the forces acting on the particle is zero:

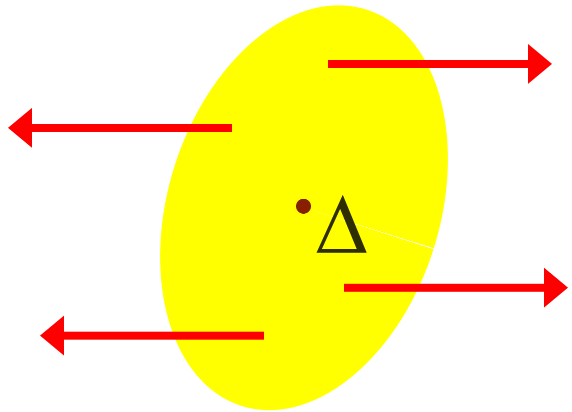
$$\sum \vec{F} = \vec{0}$$

- "For **an extended body**, the equivalent statement is that the center of mass of the body has zero acceleration if the vector sum of all external forces acting on the body is zero"

(The first condition for equilibrium)

1.b The second condition for equilibrium

- Suppose an object is capable of pivoting about an axis under influence of two forces of equal magnitude act in opposite directions along parallel lines of action (**a couple**)



- The net force is zero but the net torque is not zero; it has a magnitude of 2*Fd*.

→ The object is not in static equilibrium

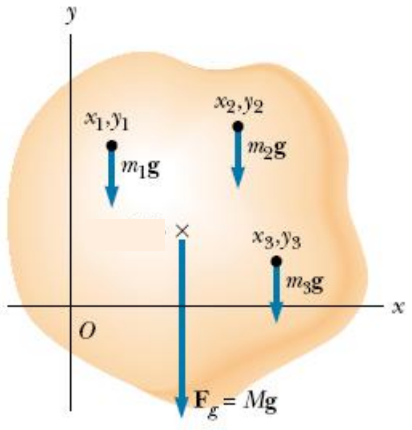
- From : $\tau = I\alpha = 0$

$\rightarrow$ **The second condition for equilibrium:**

"The resultant external torque about *any* axis must be zero"

$$\boxed{\sum \vec{\tau} = \vec{0}}$$

2 The Center of Gravity



- Consider the **gravitational torque** on a body of arbitrary shape

A typical particle has mass m_i and weight $\boldsymbol{w}_i = m_i\mathbf{g}$

The torque vector τ_i of the weight $\boldsymbol{w}_i$ with respect to 0 : $\vec{\tau}_i = \vec{r}_i \times \vec{w}_i = \vec{r}_i \times m_i\vec{g}$

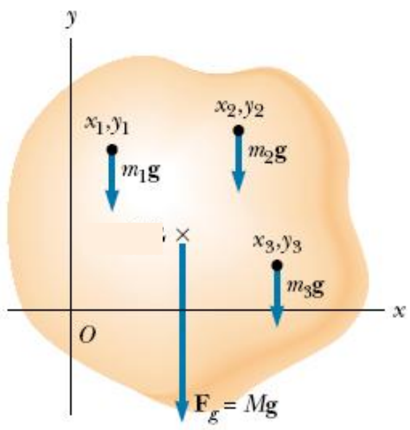
if the acceleration due to gravity $\boldsymbol{g}$ has the same magnitude and direction at every point in the body.

→ The **total torque** due to the gravitational forces on all the particles is

$$\vec{\tau} = \sum_i \vec{\tau}_i = \sum_i (\vec{r}_i \times m_i\vec{g}) = \left(\sum_i m_i\vec{r}_i\right) \times \vec{g}$$

$$\vec{\tau} = \frac{\left(\sum_i m_i\vec{r}_i\right) \times M\vec{g}}{M} = \vec{r}_{CM} \times M\vec{g}$$

$$\vec{\tau} = \frac{\left(\sum_i m_i \vec{r}_i\right) \times M\vec{g}}{M} = \vec{r}_{CM} \times M\vec{g}$$



The **total weight** of the body :

$$\vec{w} = M\vec{g}$$

$$\longrightarrow \quad \vec{\tau} = \vec{r}_{CM} \times \vec{w}$$

The total gravitational torque is the same as though the total weight ***w*** were acting on the position $\boldsymbol{r}_{\text{CM}}$ of the center of mass

The center of mass ≡ **The center of gravity**

→ If ***g*** has the same value at all points on a body, its center of gravity is identical to its center of mass.

3 Examples of Rigid Objects in Static Equilibrium

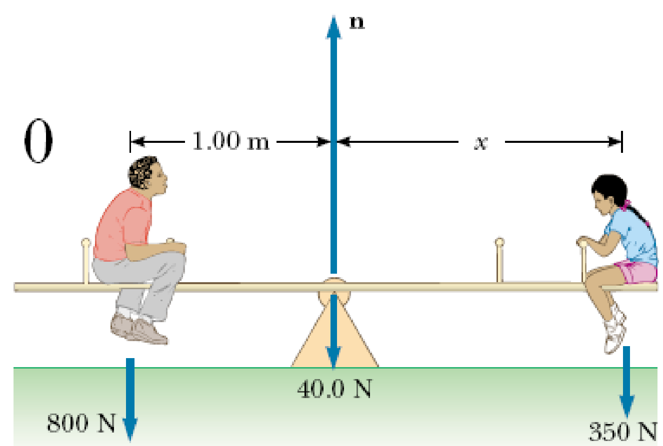
EXAMPLE 1

A uniform 40.0-N board supports a father and daughter weighing 800 N and 350 N, respectively. If the support is under the center of gravity of the board and if the father is 1.00 m from the center,
(a) determine the magnitude of the upward force ***n*** exerted on the board by the support.

(a) From $\sum F_y = 0$

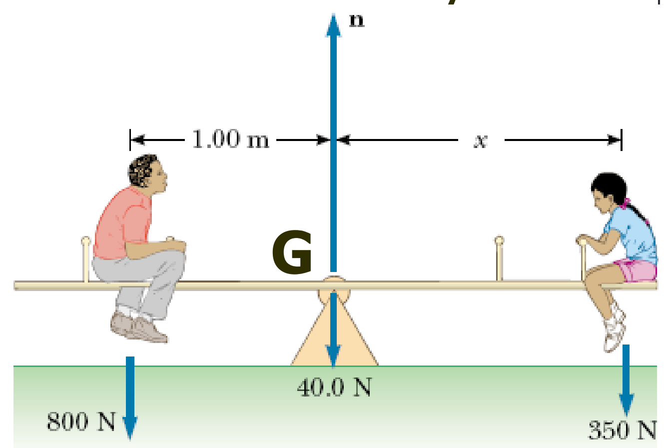
$$n - 800\text{ N} - 350\text{ N} - 40.0\text{ N} = 0$$

$$n = 1\,190\text{ N}$$



EXAMPLE 1

A uniform 40.0-N board supports a father and daughter weighing 800 N and 350 N, respectively. If the support is under the center of gravity of the board and if the father is 1.00 m from the center,
(b) Determine where the child should sit to balance the system.



(b) Take an axis perpendicular to the page through the center of gravity **G** as the axis for our torque

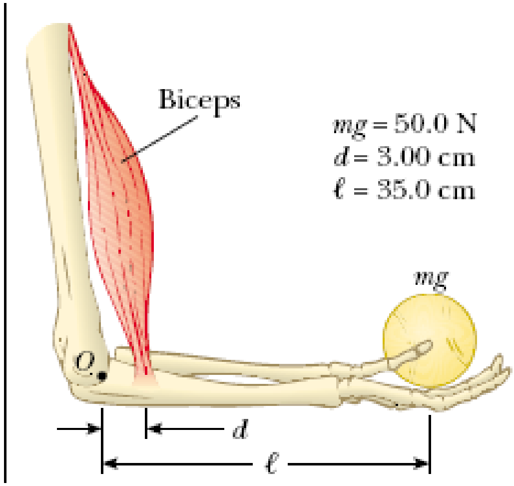
$\Sigma\tau = 0$ yields

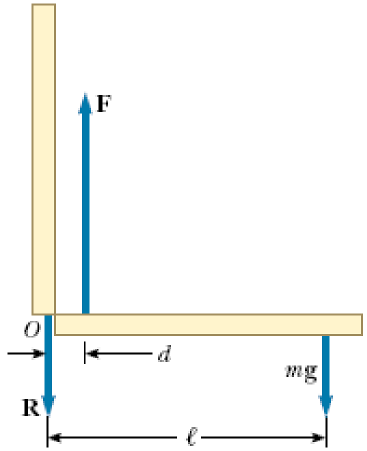
$$n(1.00 \text{ m}) - (40.0 \text{ N})(1.00 \text{ m}) - (350 \text{ N})(1.00 \text{ m} + x) = 0$$

$$x = 2.29 \text{ m.}$$

EXAMPLE 2

A person holds a 50.0-N sphere in his hand. The forearm is horizontal. The biceps muscle is attached 3.00 cm from the joint, and the sphere is 35.0 cm from the joint. Find the upward force exerted by the biceps on the forearm and the downward force exerted by the upper arm on the forearm and acting at the joint. Neglect the weight of the forearm.





$$F = R + W = R + 50.0\ N$$

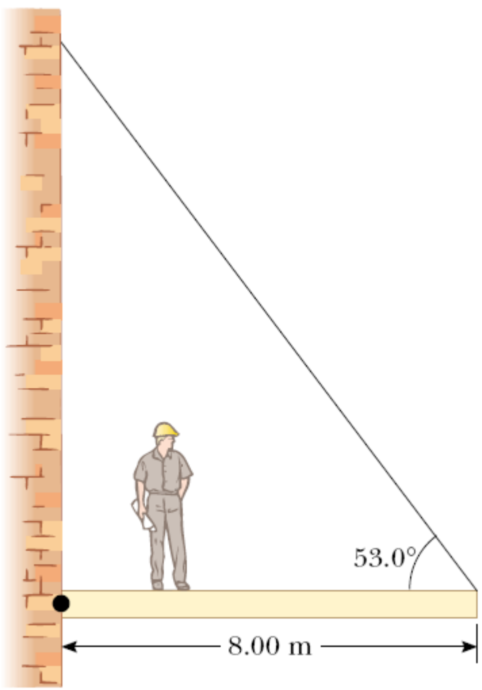
$$Fd = mgl$$

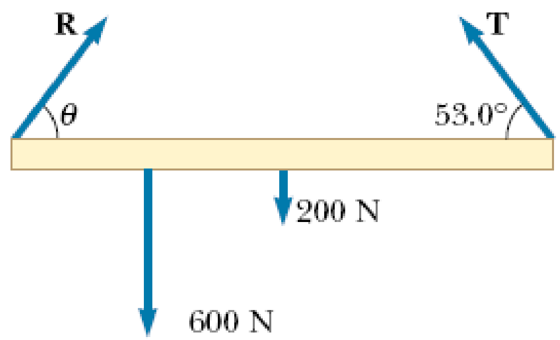
$$F \times 3.00 = 50.0 \times 35.0$$

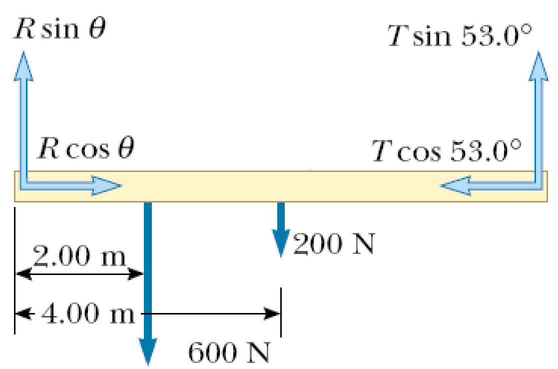
$$F = 583\ N$$

$$R = 533\ N$$

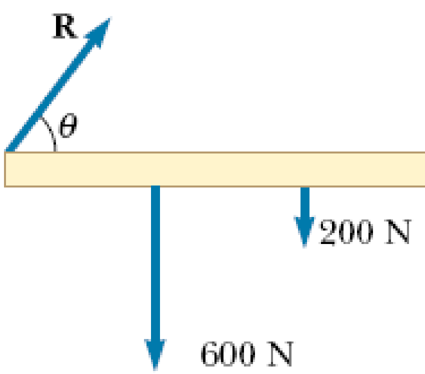
EXAMPLE 3 A uniform horizontal beam with a length of 8.00 m and a weight of 200 N is attached to a wall by a pin connection. Its far end is supported by a cable that makes an angle of 53.0° with the horizontal . If a 600-N person stands 2.00 m from the wall, find the tension in the cable, as well as the magnitude and direction of the force exerted by the wall on the beam.

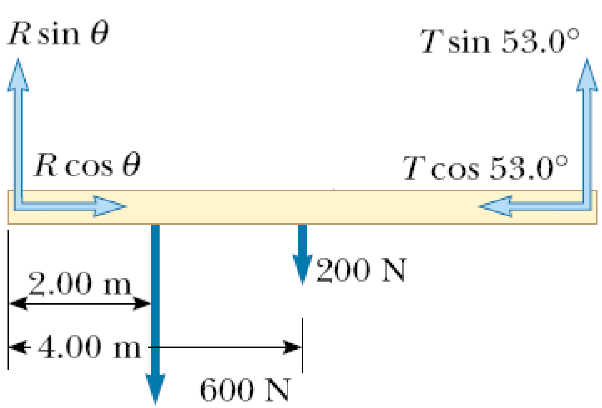






EXAMPLE 3 A uniform horizontal beam with a length of 8.00 m and a weight of 200 N is attached to a wall by a pin connection. Its far end is supported by a cable that makes an angle of 53.0° with the horizontal . If a 600-N person stands 2.00 m from the wall, find the tension in the cable, as well as the magnitude and direction of the force exerted by the wall on the beam.





$$(1) \qquad \sum F_x = R\cos\theta - T\cos 53.0^\circ = 0$$

$$(2) \qquad \sum F_y = R\sin\theta + T\sin 53.0^\circ - 600\text{ N} - 200\text{ N} = 0$$

$$\sum \tau = (T\sin 53.0^\circ)(8.00\text{ m}) - (600\text{ N})(2.00\text{ m}) - (200\text{ N})(4.00\text{ m}) = 0$$

$$T = \boxed{313\text{ N}}$$

$$R\cos\theta = 188\text{ N}$$

$$R\sin\theta = 550\text{ N}$$

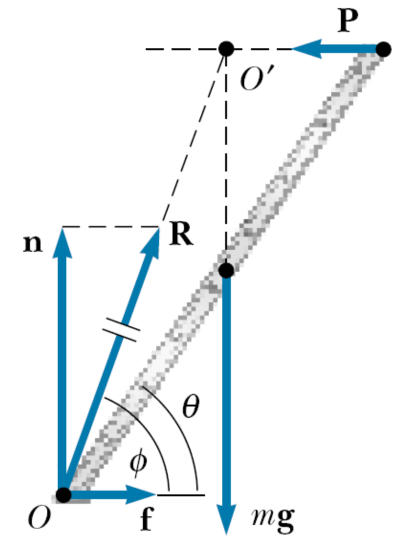
$$\tan\theta = \frac{550\text{ N}}{188\text{ N}} = 2.93$$

$$\theta = \boxed{71.1^\circ}$$

$$R = \frac{188\text{ N}}{\cos\theta} = \frac{188\text{ N}}{\cos 71.1^\circ} = \boxed{580\text{ N}}$$

EXAMPLE 4

A uniform ladder of length l and weight 50 N rests against a smooth, vertical wall. If the coefficient of static friction between the ladder and the ground is $\mu_s = 0.40$, find the minimum angle θ_{min} at which the ladder does not slip.



$$\sum F_x = f - P = 0$$

$$\sum F_y = n - mg = 0 \longrightarrow n = mg = 50\text{ N.}$$

When the ladder is on the verge of slipping, the force of friction must be a maximum :

$$f_{s,\max} = \mu_s n = 0.40(50\text{ N}) = 20\text{ N.}$$

(Because : $f_s \leq \mu_s n$ **)** ⟶ At this angle : $P = 20$ N.

The torques about an axis through O :

$$\sum \tau_O = P\ell \sin\theta - mg\frac{\ell}{2}\cos\theta = 0 \quad \tan\theta_{\min} = \frac{mg}{2P} = \frac{50\text{ N}}{40\text{ N}} \quad \theta_{\min} = 51^\circ$$