# • PROGRAM OF “PHYSICS”

**Lecturer: Dr. DO Xuan Hoi**
**Room A1. 503**
**E-mail : dxhoi@hcmiu.edu.vn**

# PHYSICS I
# (General Mechanics)
## 02 credits (30 periods)

**Chapter 1 Bases of Kinematics**
- Motion in One Dimension
- Motion in Two Dimensions

**Chapter 2 The Laws of Motion**

**Chapter 3 Work and Mechanical Energy**

**Chapter 4 Linear Momentum and Collisions**

**Chapter 5 Rotation of a Rigid Object About a Fixed Axis**

**Chapter 6 Static Equilibrium**

**Chapter 7 Universal Gravitation**

# PHYSICS I

## Chapter 4
## Linear Momentum and Collisions

**Linear Momentum and Its Conservation**

**Collisions in One Dimension**

**Two-Dimensional Collisions**

**The Center of Mass**

**Motion of a System of Particles**

# 1 Linear Momentum and Its Conservation

- From Newton's laws: force must be present to change an object's velocity (speed and/or direction)

Wish to consider effects of collisions and corresponding change in velocity

Golf ball initially at rest, so some of the KE of club transferred to provide motion of golf ball and its change in velocity

Method to describe is to use concept of **linear momentum**

Linear momentum = product of mass velocity

scalar

vector

$$\boxed{\vec{p} = m\vec{v}}$$

kg.m/s

**<u>Linear momentum</u>** : Vector quantity, the direction of the momentum is the same as the velocity's

Applies to three-dimensional motion as well

$$p_x = mv_x \; ; p_y = mv_y \; ; p_z = mv_z$$

- Force and linear momentum

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt}$$

$$\boxed{\sum \vec{F} = \frac{d\vec{p}}{dt}}$$

$$d\vec{p} = \vec{F}dt$$

**Impulse :** $\boxed{I = \Delta\vec{p} = \int d\vec{p} = \int_{t_i}^{t_f} \vec{F}\Delta t}$

# Test

Suppose a ping-pong ball and a bowling ball are rolling toward you. Both have the same momentum, and you exert the same force to stop each. How do the time intervals to stop them compare?

1. It takes less time to stop the ping-pong ball.
2. Both take the same time.
3. It takes more time to stop the ping-pong ball.

# *EXAMPLE 1*

A 50-g golf ball at rest is hit by "Big Bertha" club with 500-g mass. After the collision, golf leaves with velocity of 50 m/s.

**(a)** Find impulse imparted to ball

**(b)** Assuming club in contact with ball for 0.5 ms, find average force acting on golf ball

**(a)** $\Delta p = mv_f - mv_i = (0.050\ kg)(50\ m/s) - 0 = \underline{2.50\ kg \cdot m/s}$

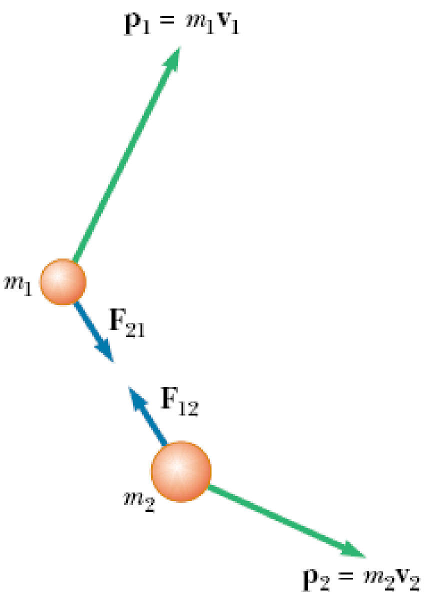
**(b)** $$F = \frac{\Delta p}{\Delta t}$$

$$= \frac{2.50\ kg \cdot m/s}{0.5 \times 10^{-3}\ s} = \underline{5.00 \times 10^3\ N}$$

- **Conservation of momentum for a two-particle system**

Consider two particles 1 and 2 interacting with each other :

$$\vec{F}_{21} = \frac{d(\vec{p}_1)}{dt}\ ;\ \vec{F}_{12} = \frac{d(\vec{p}_2)}{dt}$$




Newton's third law : $\vec{F}_{21} = -\vec{F}_{12}\ ;$

$$\frac{d(\vec{p}_1)}{dt} = -\frac{d(\vec{p}_2)}{dt}$$

$$\frac{d(\vec{p}_1 + \vec{p}_2)}{dt} = 0\ ;\quad \vec{p}_1 + \vec{p}_2 = const$$

In general, for an isolated system : $\boxed{\sum_i \vec{p}_i = const}$

Whenever two or more particles in an **isolated system** interact, the total momentum of the system remains **constant**

# *EXAMPLE 2*

**One-dimensional explosion**: A box with mass $m = 6.0$ kg slides with speed $v = 4.0$ m/s across a frictionless floor in the positive direction of an x axis. The box explodes into two pieces. One piece, with mass $m_1 = 2.0$ kg, moves in the positive direction of the x axis at $v_1 = 8.0$ m/s.
What is the velocity of the second piece?

The initial momentum of the system: $\vec{p}_i = m\vec{v}$

The final momenta of the two pieces: $\vec{p}_f = m_1\vec{v}_1 + m_2\vec{v}_2$

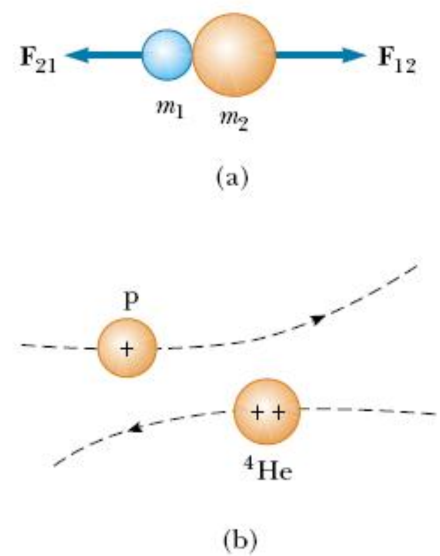
$$\vec{p}_i = \vec{p}_f \;; \quad mv = m_1 v_1 + m_2 v_2$$

$$v_2 = \frac{(6.0\ kg)(4.0\ m/s) - (2.0\ kg)(8.0\ m/s)}{4.0\ kg} = 2.0\ m/s$$

# 2 Collisions in One Dimension

## a. Collisions

**A collisions** is a event of two particles' coming together for a short time and thereby producing impulsive forces on each other




The total momentum of an isolated system just before a collision equals the total momentum of the system just after the collision

## PROBLEM 1

A car of mass 1800 kg stopped at a traffic light is struck from the rear by a 900-kg car, and the two become entangled. If the smaller car was moving at 20.0 m/s before the collision, what is the velocity of the entangled cars after the collision?

## SOLUTION

$$p_i = m_1 v_{1i} = (900\ \mathrm{kg})(20.0\ \mathrm{m/s}) = 1.80 \times 10^4\ \mathrm{kg \cdot m/s}$$

$$p_f = (m_1 + m_2) v_f = (2\ 700\ \mathrm{kg}) v_f$$

$$v_f = \frac{p_i}{m_1 + m_2} = \frac{1.80 \times 10^4\ \mathrm{kg \cdot m/s}}{2\ 700\ \mathrm{kg}} = \boxed{6.67\ \mathrm{m/s}}$$

## b. Types of Collisions

Momentum is conserved in any collision

What about kinetic energy?

## Inelastic collisions

- Kinetic energy is not conserved : $KE_i = KE_f + \text{lost energy}$
  - Some of the kinetic energy is converted into other types of energy such as heat, sound, work to permanently deform an object
- Perfectly inelastic collisions occur when the objects stick together
  - Not all of the KE is necessarily lost

# Perfectly Inelastic Collisions:

- When two objects stick together after the collision, they have undergone a perfectly inelastic collision
- Suppose, for example, $v_{2i} = 0$. Conservation of momentum becomes

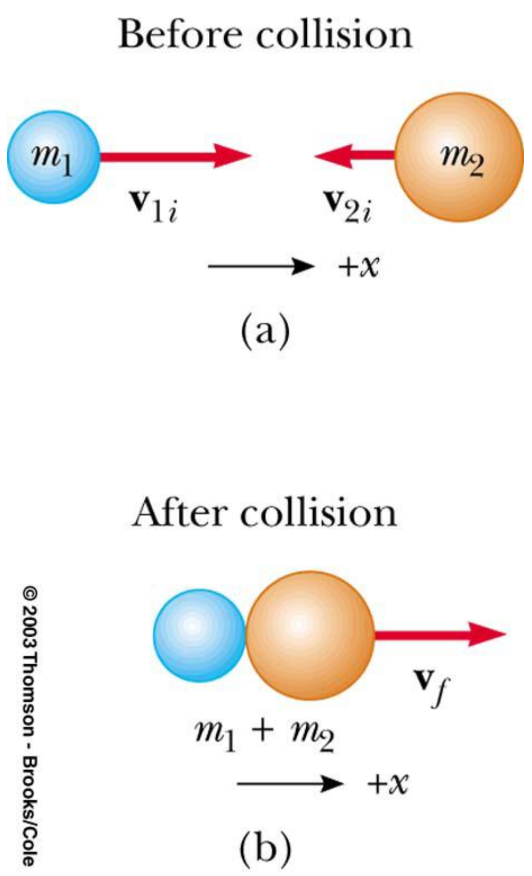
$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$$m_1 v_{1i} + 0 = (m_1 + m_2) v_f$$

E.g., if $m_1 = 1000\ kg,\ m_2 = 1500\ kg$:

$(1000 kg)(50\ m/s) + 0 = (2500 kg) v_f$ ,

$$v_f = \frac{5 \times 10^4\ kg \cdot m/s}{2.5 \times 10^3\ kg} = \underline{20\ m/s}.$$



# Perfectly Inelastic Collisions:

- What amount of KE lost during collision?

$$KE_{before} = \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2$$

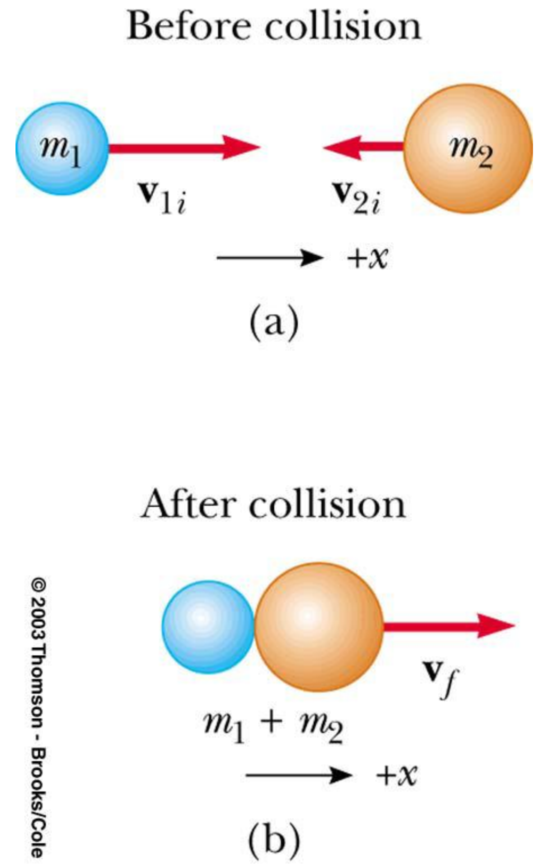
$$= \frac{1}{2}(1000\ kg)(50\ m/s)^2 = 1.25 \times 10^6\ J$$

$$KE_{after} = \frac{1}{2}(m_1 + m_2) v_f^2$$

$$= \frac{1}{2}(2500\ kg)(20\ m/s)^2 = 0.50 \times 10^6\ J$$

$$\Delta KE_{lost} = \underline{0.75 \times 10^6\ J}$$

lost in heat/"gluing"/sound/…



# Elastic collisions

both momentum and kinetic energy are conserved

Typically have two unknowns

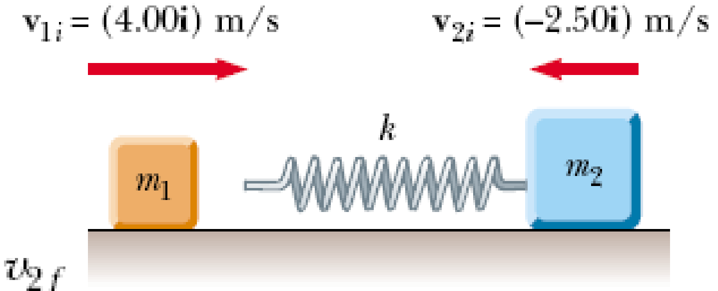
$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

Solve the equations simultaneously

# *EXAMPLE 2*

A block of mass $m_1$ = 1.60 kg initially moving to the right with a speed of 4.00 m/s on a frictionless horizontal track collides with a spring attached to a second block of mass $m_2$ = 2.10 kg initially moving to the left with a speed of 2.50 m/s. The spring constant is 600 N/m.
(a) At the instant block 1 is moving to the right with a speed of 3.00 m/s, determine the velocity of block 2.




$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$(1.60 \text{ kg})(4.00 \text{ m/s}) + (2.10 \text{ kg})(-2.50 \text{ m/s}) = (1.60 \text{ kg})(3.00 \text{ m/s}) + (2.10 \text{ kg})v_{2f}$$

$$v_{2f} = \boxed{-1.74 \text{ m/s}}$$

# *EXAMPLE 2*

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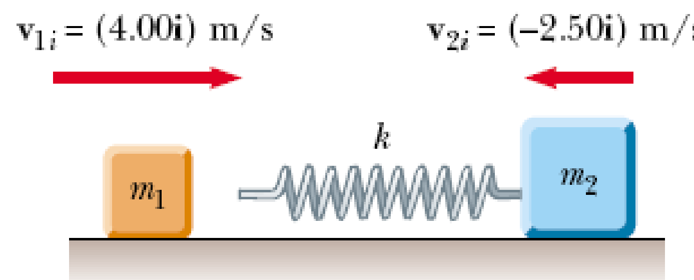
(b) Determine the distance the spring is compressed at that instant.

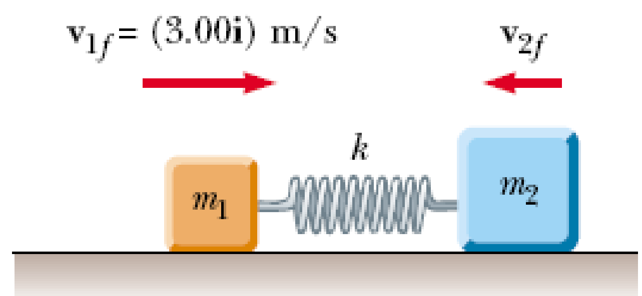



$$v_{2f} = \boxed{-1.74 \text{ m/s}}$$

Conservation of mechanical energy :

$$\frac{1}{2}m_1{v_{1i}}^2 + \frac{1}{2}m_2{v_{2i}}^2 = \frac{1}{2}m_1{v_{1f}}^2 + \frac{1}{2}m_2{v_{2f}}^2 + \frac{1}{2}kx^2$$

$$x = \boxed{0.173 \text{ m}}$$



# 3 Two-Dimensional Collisions

Momentum is conserved in any collision :

$$\boxed{m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}}$$

# Problem Solving Strategy

Set up coordinate axes and define your velocities with respect to these axes

It is convenient to choose the x axis to coincide with one of the initial velocities

Draw and label all the velocities and include all the given information

Write expressions for the total momentum before and after the collision in the x-direction

Repeat for the y-direction

Solve for the unknown quantities

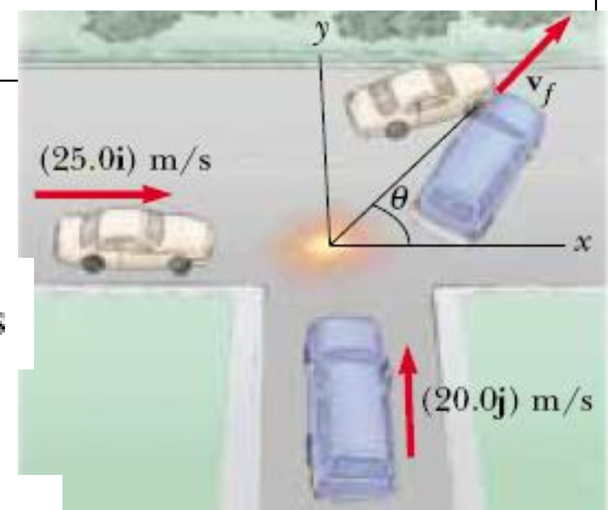
If the collision is inelastic, additional information is probably required

If the collision is perfectly inelastic, the final velocities of the two objects is the same

If the collision is elastic, use the KE equations to help solve for the unknowns

## PROBLEM 2

A 1 500-kg car traveling east with a speed of 25.0 m/s collides at an intersection with a 2 500-kg van traveling north at a speed of 20.0 m/s.
Find the direction and magnitude of the velocity of the wreckage after the collision, assuming that the vehicles stick together after the collision.




## SOLUTION

Stick together : perfectly inelastic collision

$$\sum p_{xi} = (1\,500 \text{ kg})(25.0 \text{ m/s}) = 3.75 \times 10^4 \text{ kg}\cdot\text{m/s}$$

$$\sum p_{xf} = (4\,000 \text{ kg})v_f \cos\theta$$

$$3.75 \times 10^4 \text{ kg}\cdot\text{m/s} = (4\,000 \text{ kg})v_f \cos\theta$$

$$\sum p_{yi} = \sum p_{yf}$$

$$(2\,500 \text{ kg})(20.0 \text{ m/s}) = (4\,000 \text{ kg})v_f \sin\theta$$

$$5.00 \times 10^4 \text{ kg}\cdot\text{m/s} = (4\,000 \text{ kg})v_f \sin\theta$$

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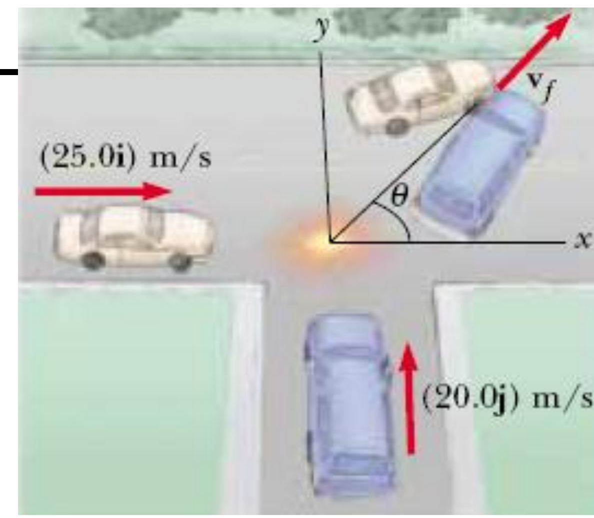



## SOLUTION

$$3.75 \times 10^4 \text{ kg}\cdot\text{m/s} = (4\,000 \text{ kg}) v_f \cos\theta$$

$$5.00 \times 10^4 \text{ kg}\cdot\text{m/s} = (4\,000 \text{ kg}) v_f \sin\theta$$

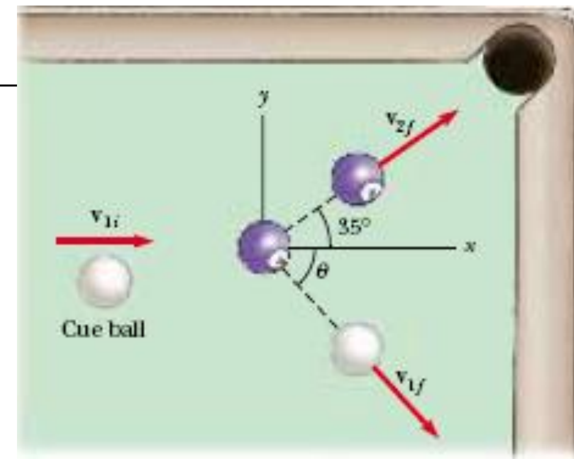
$$\frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{5.00 \times 10^4}{3.75 \times 10^4} = 1.33$$

$$\theta = \boxed{53.1^\circ}$$

$$v_f = \frac{5.00 \times 10^4 \text{ kg}\cdot\text{m/s}}{(4\,000 \text{ kg})\sin 53.1^\circ} = \boxed{15.6 \text{ m/s}}$$

## PROBLEM 3

In a game of billiards, a player wishes to sink a target ball 2 in the corner pocket as shown in the figure. If the angle to the corner pocket is 35°, at what angle is the cue ball 1 deflected?
Assume that friction and rotational motion are unimportant and that the collision is elastic




## SOLUTION

Conservation of energy :

$$\frac{1}{2}m_1 {v_{1i}}^2 = \frac{1}{2}m_1 {v_{1f}}^2 + \frac{1}{2}m_2 {v_{2f}}^2$$

$${v_{1i}}^2 = {v_{1f}}^2 + {v_{2f}}^2 \quad (1)$$

Conservation of momentum : $\mathbf{v}_{1i} = \mathbf{v}_{1f} + \mathbf{v}_{2f}$

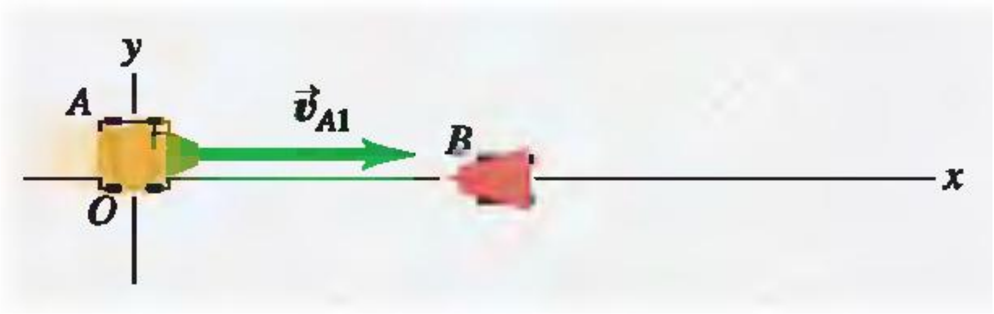
$${v_{1i}}^2 = (\mathbf{v}_{1f} + \mathbf{v}_{2f}) \cdot (\mathbf{v}_{1f} + \mathbf{v}_{2f}) = {v_{1f}}^2 + {v_{2f}}^2 + 2\mathbf{v}_{1f} \cdot \mathbf{v}_{2f}$$

$${v_{1i}}^2 = {v_{1f}}^2 + {v_{2f}}^2 + 2v_{1f}v_{2f}\cos(\theta + 35°) \quad (3)$$

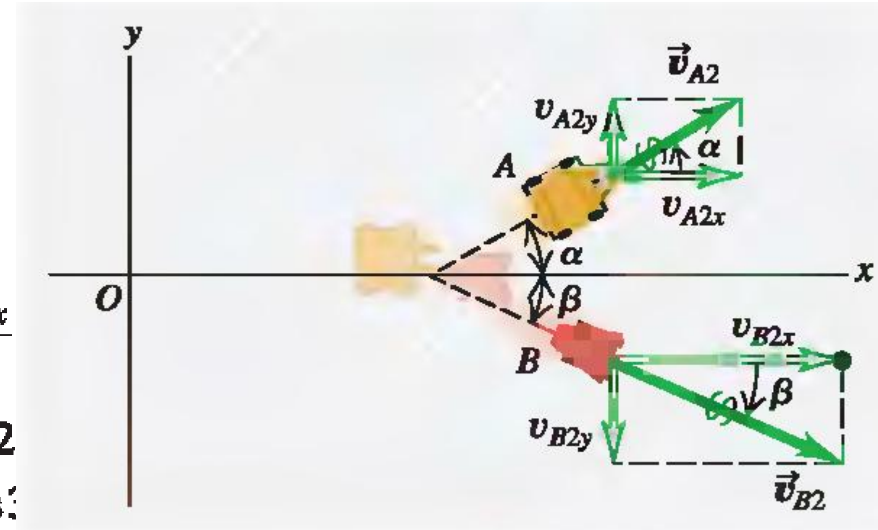
(1) and (3) : $0 = 2v_{1f}v_{2f}\cos(\theta + 35°)$ $\quad \theta = \boxed{55°}$

# PROBLEM 4

The figure shows two battling robots sliding on a frictionless surface. Robot A, with mass 20 kg, initially moves at 2.0 m/s parallel to the x-axis. It collides with robot B, which has mass 12 kg and is initially at rest. After the collision, robot A is moving at 1.0 m/s in a direction that makes an angle a = 30° with its initial direction. What is the final velocity of robot B?



## SOLUTION




$$m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$$

$$v_{B2x} = \frac{m_A v_{A1x} + m_B v_{B1x} - m_A v_{A2x}}{m_B}$$

$$= \frac{\begin{bmatrix} (20\,\text{kg})(2.0\,\text{m/s}) + (12 \\ -(20\,\text{kg})(1.0\,\text{m/s})(\cos \end{bmatrix}}{12\,\text{kg}}$$

$$= 1.89\,\text{m/s}$$

$$m_A v_{A1y} + m_B v_{B1y} = m_A v_{A2y} + m_B v_{B2y}$$

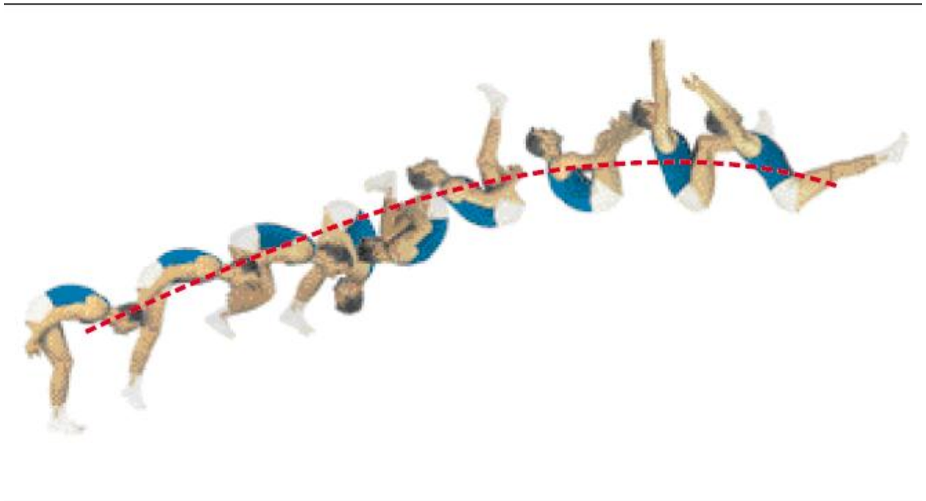
$$v_{B2y} = \frac{m_A v_{A1y} + m_B v_{B1y} - m_A v_{A2y}}{m_B}$$

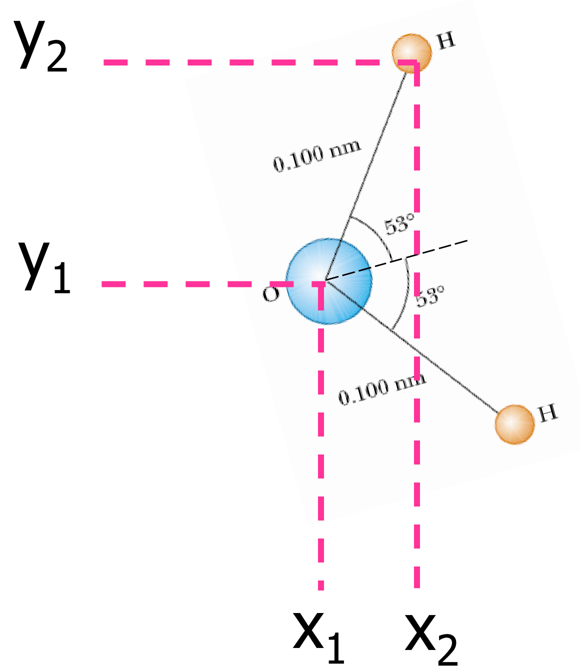
$$= \frac{\begin{bmatrix} (20\,\text{kg})(0) + (12\,\text{kg})(0) \\ -(20\,\text{kg})(1.0\,\text{m/s})(\sin 30^\circ) \end{bmatrix}}{12\,\text{kg}}$$

$$= -0.83\,\text{m/s}$$

$$v_{B2} = \sqrt{(1.89\,\text{m/s})^2 + (-0.83\,\text{m/s})^2} = 2.1\,\text{m/s} \qquad \beta = \arctan\frac{-0.83\,\text{m/s}}{1.89\,\text{m/s}} = -24^\circ$$

# 4. The center of mass






The coordinates of the center of mass of *n* particles

$$x_{\mathrm{CM}} \equiv \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots + m_n x_n}{m_1 + m_2 + m_3 + \cdots + m_n} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

$$y_{\mathrm{CM}} \equiv \frac{\sum_i m_i y_i}{M} \qquad \text{and} \qquad z_{\mathrm{CM}} \equiv \frac{\sum_i m_i z_i}{M}$$

The coordinates of the center of mass of *n* particles

$$x_{CM} \equiv \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots + m_n x_n}{m_1 + m_2 + m_3 + \cdots + m_n} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

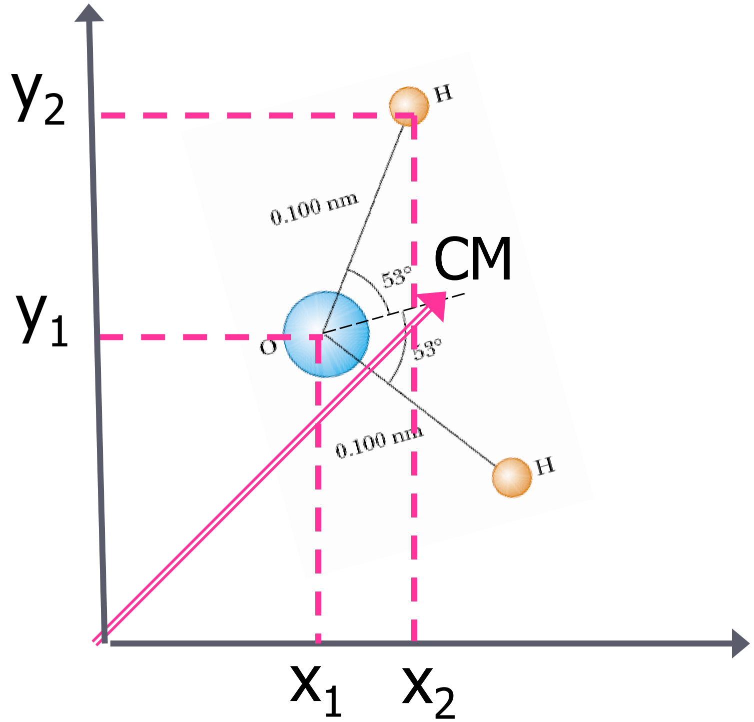
$$y_{CM} \equiv \frac{\sum_i m_i y_i}{M} \quad \text{and} \quad z_{CM} \equiv \frac{\sum_i m_i z_i}{M}$$

The position vector of the center of mass :

$$\mathbf{r}_{CM} = x_{CM}\mathbf{i} + y_{CM}\mathbf{j} + z_{CM}\mathbf{k}$$

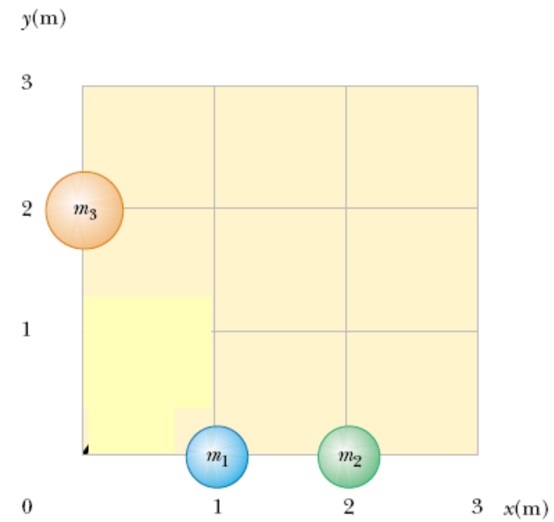
$$= \frac{\sum_i m_i x_i \mathbf{i} + \sum_i m_i y_i \mathbf{j} + \sum_i m_i z_i \mathbf{k}}{M}$$

$$\mathbf{r}_{CM} \equiv \frac{\sum_i m_i \mathbf{r}_i}{M}$$



# EXAMPLE 3

A system consists of three particles located as shown in the figure. Find the center of mass of the system.




$$x_{CM} = \frac{\sum_i m_i x_i}{M} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$= \frac{(1.0\text{ kg})(1.0\text{ m}) + (1.0\text{ kg})(2.0\text{ m}) + (2.0\text{ kg})(0\text{ m})}{1.0\text{ kg} + 1.0\text{ kg} + 2.0\text{ kg}}$$

$$= \frac{3.0\text{ kg}\cdot\text{m}}{4.0\text{ kg}} = 0.75\text{ m}$$

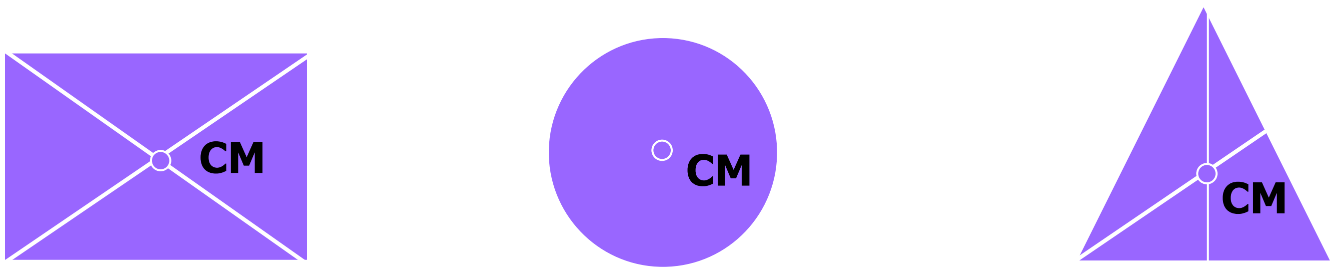
$$y_{CM} = \frac{\sum_i m_i y_i}{M} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

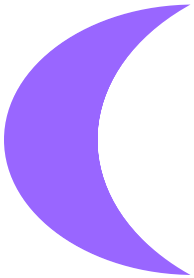
$$= \frac{(1.0\text{ kg})(0) + (1.0\text{ kg})(0) + (2.0\text{ kg})(2.0\text{ m})}{4.0\text{ kg}}$$

$$= \frac{4.0\text{ kg}\cdot\text{m}}{4.0\text{ kg}} = 1.0\text{ m}$$

$$\mathbf{r}_{CM} = x_{CM}\mathbf{i} + y_{CM}\mathbf{j} = \boxed{0.75\mathbf{i}\text{ m} + 1.0\,\mathbf{j}\text{ m}}$$

- **The center of mass of a symmetric system**

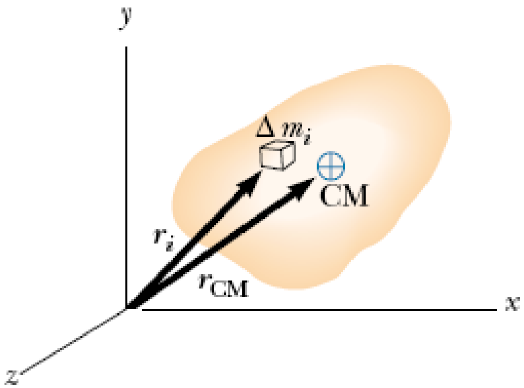


**CM ?**

**The center of mass of any symmetric object lies on an axis of symmetry and on any plane of symmetry.**

- **The center of mass of a rigid body**




$$x_{CM} \approx \frac{\sum_i x_i \, \Delta m_i}{M}$$

$$x_{CM} = \lim_{\Delta m_i \to 0} \frac{\sum_i x_i \, \Delta m_i}{M} = \frac{1}{M} \int x \, dm$$

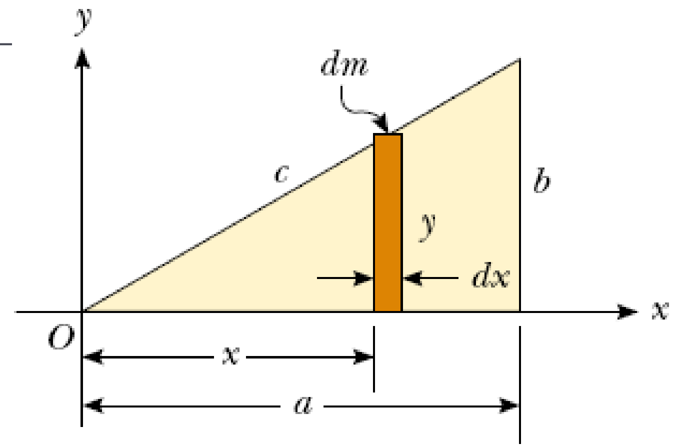
$$y_{CM} = \frac{1}{M} \int y \, dm \quad \text{and} \quad z_{CM} = \frac{1}{M} \int z \, dm$$

The vector position of the center of mass :

$$\mathbf{r}_{CM} = \frac{1}{M} \int \mathbf{r} \, dm$$

# *EXAMPLE 4*

An object of mass *M* is in the shape of a right triangle whose dimensions are shown in the figure. Locate the coordinates of the center of mass, assuming the object has a uniform mass per unit area.




$$x_{\mathrm{CM}} = \frac{1}{M}\int x\,dm$$

$$dm = \frac{\text{total mass of object}}{\text{total area of object}} \times \text{area of strip}$$

$$= \frac{M}{1/2\,ab}(y\,dx) = \left(\frac{2M}{ab}\right)y\,dx$$

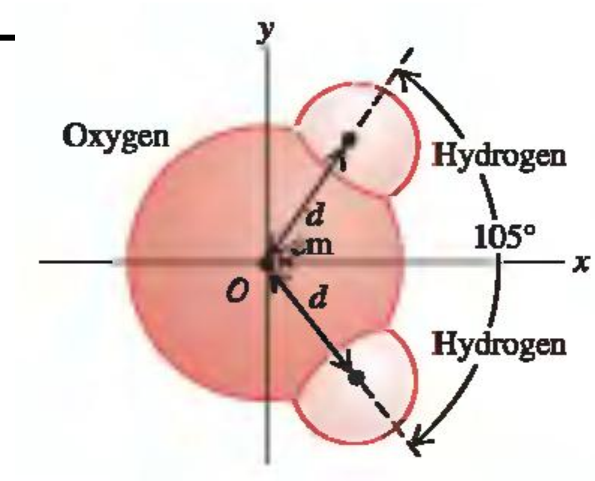
$$x_{\mathrm{CM}} = \frac{1}{M}\int x\,dm = \frac{1}{M}\int_0^a x\left(\frac{2M}{ab}\right)y\,dx = \frac{2}{ab}\int_0^a xy\,dx$$

$$x_{\mathrm{CM}} = \frac{2}{ab}\int_0^a x\left(\frac{b}{a}\,x\right)dx = \frac{2}{a^2}\int_0^a x^2\,dx = \boxed{\frac{2}{3}\,a}$$

$$y_{\mathrm{CM}} = \boxed{\frac{1}{3}\,b}$$

## PROBLEM 5

The figure shows a simple model of the structure of a water molecule. The separation between atoms is d = $9.57 \times 10^{-11}$ m. Each hydrogen atom has mass 1.0 u, and the oxygen atom has mass 16.0 u. Find the position of the center of mass.




## SOLUTION

$$x_{cm} = \frac{\begin{bmatrix}(1.0\,\mathrm{u})(d\cos 52.5°) + (1.0\,\mathrm{u}) \\ \times\,(d\cos 52.5°) + (16.0\,\mathrm{u})(0)\end{bmatrix}}{1.0\,\mathrm{u} + 1.0\,\mathrm{u} + 16.0\,\mathrm{u}}$$

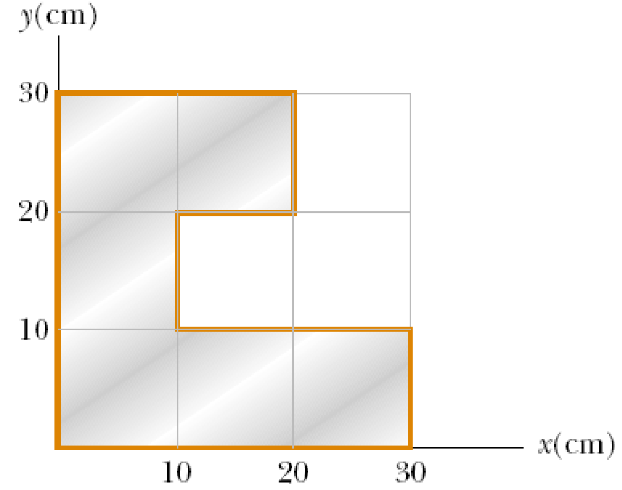
$$= 0.068d$$

$$x_{cm} = (0.068)(9.57 \times 10^{-11}\,\mathrm{m}) = 6.5 \times 10^{-12}\,\mathrm{m}$$

$$y_{cm} = \frac{\begin{bmatrix}(1.0\,\mathrm{u})(d\sin 52.5°) + (1.0\,\mathrm{u}) \\ \times\,(-d\sin 52.5°) + (16.0\,\mathrm{u})(0)\end{bmatrix}}{1.0\,\mathrm{u} + 1.0\,\mathrm{u} + 16.0\,\mathrm{u}} = 0$$

# PROBLEM 6

A uniform piece of sheet steel is shaped as shown in the figure. Compute the *x* and *y* coordinates of the center of mass of the piece




# SOLUTION

# 5 Motion of a System of Particles

The velocity of the center of mass of the system :

$$\mathbf{v}_{\mathrm{CM}} = \frac{d\mathbf{r}_{\mathrm{CM}}}{dt} = \frac{1}{M}\sum_i m_i \frac{d\mathbf{r}_i}{dt} = \frac{\sum_i m_i \mathbf{v}_i}{M}$$

$$M\mathbf{v}_{\mathrm{CM}} = \sum_i m_i \mathbf{v}_i = \sum_i \mathbf{p}_i = \mathbf{p}_{\mathrm{tot}}$$

The acceleration of the center of mass of the system :

$$\mathbf{a}_{\mathrm{CM}} = \frac{d\mathbf{v}_{\mathrm{CM}}}{dt} = \frac{1}{M}\sum_i m_i \frac{d\mathbf{v}_i}{dt} = \frac{1}{M}\sum_i m_i \mathbf{a}_i$$

$$M\mathbf{a}_{\mathrm{CM}} = \sum_i m_i \mathbf{a}_i = \sum_i \mathbf{F}_i \longrightarrow \boxed{\sum \mathbf{F}_{\mathrm{ext}} = M\mathbf{a}_{\mathrm{CM}} = \frac{d\mathbf{p}_{\mathrm{tot}}}{dt}}$$

The center of mass of a system of particles of combined mass *M* moves like an equivalent particle of mass *M* would move under the influence of the resultant external force on the system.